Use the Fundamental Counting Principle to determine the number of outcomes.

1. **FOOD** How many different combinations of sandwich, side, and beverage are possible?

Sandwiches	Sides	Beverages
hot dog hamburger veggie burger bratwurst grilled chicken	chips apple pasta salad	bottled water soda juice milk

SOLUTION:

There are 5 sandwich choices, 3 sides, and 4 beverages. The number of possible combinations is 5 × 3 × 4 or 60.

2. **QUIZZES** Each question on a five question multiple-choice quiz has answer choices labeled A, B, C, and D. How many different ways can a student answer the five questions?

SOLUTION:

There are 4 ways to answer each of the five questions. The number of different ways a students can answer is 4 × 4 × 4 × 4 × 4 or 1024.

3. **DANCES** Dane is renting a tuxedo for prom. Once he has chosen his jacket, he must choose from three types of pants and six colors of vests. How many different ways can he select his attire for prom?

SOLUTION:

There are 3 choices of pants and 6 choices of vest for each jacket selection. The number of different ways to select his attire for the prom is 3 × 6 or 18.

4. **MANUFACTURING** A baseball glove manufacturer makes a glove with the different options shown in the table. How many different gloves are possible?

Option	Number of Choices
sizes	4
types by position	3
materials	2
levels of quality	2

SOLUTION:

There are 4 sizes, 3 position types, 2 types of material, and 2 levels of quality. The number of different possible gloves is 4 × 3 × 2 × 2 or 48.

Evaluate each permutation or combination.

5. ${}_6P_3$

SOLUTION:

$${}_nP_r = \frac{n!}{(n-r)!} \quad \text{Permutations formula}$$

$${}_6P_3 = \frac{6!}{(6-3)!} \quad n=6, r=3$$

$$= \frac{6!}{3!} \quad \text{Subtract.}$$

$$= \frac{6\cdot5\cdot4\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}{\cancel{3}\cdot\cancel{2}\cdot\cancel{1}} \quad \text{Divide.}$$

$$= 6\cdot5\cdot4 \quad \text{Simplify.}$$

$$= 120 \quad \text{Multiply.}$$

6. ${}_7P_5$

SOLUTION:

$${}_nP_r = \frac{n!}{(n-r)!} \quad \text{Permutations formula}$$

$${}_7P_5 = \frac{7!}{(7-5)!} \quad n=7, r=5$$

$$= \frac{7!}{2!} \quad \text{Subtract.}$$

$$= \frac{7\cdot6\cdot5\cdot4\cdot3\cdot\cancel{2}\cdot\cancel{1}}{\cancel{2}\cdot\cancel{1}} \quad \text{Divide.}$$

$$= 7\cdot6\cdot5\cdot4\cdot3 \quad \text{Simplify.}$$

$$= 2520 \quad \text{Multiply.}$$

7. $_4C_2$

SOLUTION:

$$\begin{aligned}
{}_nC_r &= \frac{n!}{(n-r)!r!} && \text{Combination formula}\\
{}_4C_2 &= \frac{4!}{(4-2)!2!} && n=4, r=2\\
&= \frac{4!}{2!2!} && \text{Subtract.}\\
&= \frac{4\cdot 3\cdot \cancel{2}\cdot \cancel{1}}{2\cdot 1\cdot \cancel{2}\cdot \cancel{1}} && \text{Divide.}\\
&= \frac{4\cdot 3}{2} && \text{Simplify.}\\
&= 6 && \text{Simplify.}
\end{aligned}$$

8. $_{12}C_7$

SOLUTION:

$$\begin{aligned}
{}_nC_r &= \frac{n!}{(n-r)!r!} && \text{Combination formula}\\
{}_{12}C_7 &= \frac{12!}{(12-7)!7!} && n=12, r=7\\
&= \frac{12!}{5!7!} && \text{Subtract.}\\
&= 6 && \text{Use a calculator.}
\end{aligned}$$

9. $_6C_1$

SOLUTION:

$$\begin{aligned}
{}_nC_r &= \frac{n!}{(n-r)!r!} && \text{Combination formula}\\
{}_6C_1 &= \frac{6!}{(6-1)!1!} && n=6, r=1\\
&= \frac{6!}{5!1!} && \text{Subtract.}\\
&= 6 && \text{Use a calculator.}
\end{aligned}$$

10. $_9P_5$

SOLUTION:

$$\begin{aligned}
{}_nP_r &= \frac{n!}{(n-r)!} && \text{Permutations formula}\\
{}_9P_5 &= \frac{9!}{(9-5)!} && n=9, r=5\\
&= \frac{9!}{4!} && \text{Subtract.}\\
&= 15,120 && \text{Use a calculator.}
\end{aligned}$$

Determine whether each situation involves *permutations* or *combinations*. Then solve the problem.

11. **SCHOOL** Charlita wants to take 6 different classes next year. Assuming that each class is offered each period, how many different schedules could she have?

SOLUTION:

Because the classes are different, the order in which the classes are taken is important. This situation involves permutations. Use the permutation formula for 6 things taken 6 at a time.

$$\begin{aligned}
{}_nP_r &= \frac{n!}{(n-r)!} && \text{Permutations formula}\\
{}_6P_6 &= \frac{6!}{(6-6)!} && n=6, r=6\\
&= \frac{6!}{0!} && \text{Subtract.}\\
&= 6! \text{ or } 720 && 0! = 1 \text{ and simplify.}
\end{aligned}$$

There are 720 different possible schedules that Charlita could have next year.

12. **BALLOONS** How many 4-colored groups can be selected from 13 different colored balloons?

SOLUTION:

Because the order in which the balloons are chosen is not important, this situation involves combinations. Use the combination formula for 13 things taken 4 at a time.

$$\begin{aligned}
{}_nC_r &= \frac{n!}{(n-r)!r!} && \text{Combination formula}\\
{}_{13}C_4 &= \frac{13!}{(13-4)!4!} && n=13, r=4\\
&= \frac{13!}{9!4!} && \text{Subtract.}\\
&= 715 && \text{Use a calculator.}
\end{aligned}$$

There are 715 different groups of 4 colored balloons possible.

13. **CONTEST** How many ways are there to choose the winner and first, second, and third runners-up in a contest with 10 finalists?

SOLUTION:

Because the contest winner and runner ups are different, the order in which they are chosen is important. This situation involves permutations. Use the Permutation formula for 10 things taken 4 at a time.

$$\begin{aligned} {}_nP_r &= \frac{n!}{(n-r)!} && \text{Permutations formula} \\ {}_{10}P_4 &= \frac{10!}{(10-4)!} && n=10, r=4 \\ &= \frac{10!}{6!} && \text{Subtract.} \\ &= 5040 && \text{Use a calculator.} \end{aligned}$$

There are 5040 different possible ways to choose the winner and runner ups.

14. **BANDS** A band is choosing 3 new backup singers from a group of 18 who try out. How many ways can they choose the new singers?

SOLUTION:

Because the order in which the singers are chosen is not important, this situation involves combinations. Use the Combination formula for 18 things taken 3 at a time.

$$\begin{aligned} {}_nC_r &= \frac{n!}{(n-r)!r!} && \text{Combination formula} \\ {}_{18}C_3 &= \frac{18!}{(18-3)!3!} && n=18, r=3 \\ &= \frac{18!}{15!3!} && \text{Subtract.} \\ &= 816 && \text{Use a calculator.} \end{aligned}$$

There are 816 different ways for the band to choose 3 new backup singers.

15. **PIZZA** How many different two-topping pizzas can be made if there are 6 options for toppings?

SOLUTION:

Because the order in which the two toppings are chosen is not important, this situation involves combinations. Use the Combinations formula for 6 things taken 2 at a time.

$$\begin{aligned} {}_nC_r &= \frac{n!}{(n-r)!r!} && \text{Combination formula} \\ {}_6C_2 &= \frac{6!}{(6-2)!2!} && n=6, r=2 \\ &= \frac{6!}{4!2!} && \text{Subtract.} \\ &= 15 && \text{Use a calculator.} \end{aligned}$$

There are 15 different two-topping pizzas that can be made.

16. **SOFTBALL** How many ways can the manager of a softball team choose players for the top 4 spots in the lineup if she has 7 possible players in mind?

SOLUTION:

Because the player assigned to each position is different, the order in which they are chosen is important. This situation involves permutations. Use the Permutation formula for 7 things taken 4 at a time.

$$\begin{aligned} {}_nP_r &= \frac{n!}{(n-r)!} && \text{Permutations formula} \\ {}_7P_4 &= \frac{7!}{(7-4)!} && n=7, r=4 \\ &= \frac{7!}{3!} && \text{Subtract.} \\ &= 840 && \text{Use a calculator.} \end{aligned}$$

There are 840 different possible ways to choose players for the top four spots in the lineup.

17. **NEWSPAPERS** A newspaper has 9 reporters available to cover 4 different stories. How many ways can the reporters be assigned?

SOLUTION:

Because the order in which reporters are assigned to stories is different, the order is important. This situation involves permutations. Use the Permutations formula for 9 things taken 4 at a time.

$$\begin{aligned} {}_nP_r &= \frac{n!}{(n-r)!} && \text{Permutations formula} \\ {}_9P_4 &= \frac{9!}{(9-4)!} && n=9, r=4 \\ &= \frac{9!}{5!} && \text{Subtract.} \\ &= 3024 && \text{Use a calculator.} \end{aligned}$$

There are 3024 ways to assign the reporters to the stories.

18. **READING** Jack has a reading list of 12 books. How many ways can he select 9 books from the list to check out of the library?

SOLUTION:

The order in which the 9 books are selected is not important. This situation involves combinations. Use the Combination formula for 12 things taken 9 at a time.

$$\begin{aligned} {}_nC_r &= \frac{n!}{(n-r)!r!} && \text{Combination formula} \\ {}_{12}C_9 &= \frac{12!}{(12-9)!9!} && n=12, r=9 \\ &= \frac{12!}{3!9!} && \text{Subtract.} \\ &= 220 && \text{Use a calculator.} \end{aligned}$$

There are 220 ways Jack can select the 9 books to check out of the library.

19. **CHALLENGE** Abby is registering at a Web site and must select a six-character password. The password can contain either letters or digits.
a. How many passwords are possible if characters can be repeated? if no characters can be repeated?
b. How many passwords are possible if all characters are letters that can be repeated? if the password must contain exactly one digit? Which type of password is more secure? Explain.

SOLUTION:

a. There are 26 letters and 10 digits from which to choose, or 36 choices for each character in the password. If the characters can be repeated then there are 36 choices for each of the 6 characters or 36^6. This would equal 2,176,782,336 possible passwords.

If no characters can be repeated, then there are 36 choices for the first, 35 for the second, 34 for the third, 33 for the fourth, 32 for the fifth, and 31 for the sixth. The total number of passwords would be 36 × 35 × 34 × 33 × 32 × 31 or 1,402,410,240.

b. If the characters can be repeated, then there are 26 possible letters for each of the 6 characters. This would give a total of 26^6 or 308,915,776 possible passwords.

If the password must contain exactly one digit and it is placed first, then the password would have 10 choices for the first character and 26 for each of the next five.

If the digit is the second character, then there are 10 choices for the second and 26 choices for all the others. Likewise the digit could be placed in the third, fourth, fifth, or sixth spot with similar results. The total number of passwords would be six times the product of each way or 6(10 × 26 × 26 × 26 × 26 × 26) or 712,882,560.

The password with one digit is more secure, because the chance of someone guessing this password at random is $\frac{1}{712,882,560}$, which is less than guessing a 6-character password that contains only letters, $\frac{1}{308,915,776}$.