0-6 Multiplying Probabilities

Determine whether the events are *independent* or *dependent*. Then find the probability.

1. A red die and a blue die are rolled. What is the probability of getting the result shown?

SOLUTION:

Since the outcome of tossing the red die does not affect the outcome of rolling the blue die, these events are independent.

$$\begin{aligned} &P(3 \text{ and } 5) \\ &= P(3) \cdot P(5) && \text{Probability of independent events} \\ &= \frac{1}{6} \cdot \frac{1}{6} \text{ or } \frac{1}{36} && P(3) = \frac{1}{6} \text{ and } P(5) = \frac{1}{6} \end{aligned}$$

The probability is $\frac{1}{36}$.

2. Yana has 4 black socks, 6 blue socks, and 8 white socks in his drawer. If he selects three socks at random with no replacement, what is the probability that he will first select a blue sock, then a black sock, and then another blue sock?

SOLUTION:

Since the socks are being selected with out replacement, the events are dependent.

$$P(\text{blue}) = \frac{6}{18} \text{ or } \frac{1}{3}$$

$$P(\text{black} | \text{blue}) = \frac{4}{17}$$

$$P(\text{blue} | (\text{blue and black})) = \frac{5}{16}$$

$$\begin{aligned} &P(\text{blue and black and blue}) \\ &= P(\text{black}) \cdot P(\text{blue} | \text{black}) \cdot P(\text{blue} | (\text{blue and black})) \\ &= \frac{1}{3} \cdot \frac{4}{17} \cdot \frac{5}{16} \text{ or } \frac{5}{204} \end{aligned}$$

The probability is $\frac{5}{204}$ or about 0.025.

A die is rolled twice. Find each probability.

3. *P*(2 and 3)

SOLUTION:

$$\begin{aligned} &P(2 \text{ and } 3) \\ &= P(2) \cdot P(3) && \text{Probability of independent events.} \\ &= \frac{1}{6} \cdot \frac{1}{6} \text{ or } \frac{1}{36} && P(2) = \frac{1}{6} \text{ and } P(3) = \frac{1}{6} \end{aligned}$$

The probability is $\frac{1}{36}$.

4. *P*(two 4s)

SOLUTION:

$$\begin{aligned} &P(4 \text{ and } 4) \\ &= P(4) \cdot P(4) && \text{Probability of independent events.} \\ &= \frac{1}{6} \cdot \frac{1}{6} \text{ or } \frac{1}{36} && P(4) = \frac{1}{6} \text{ and } P(4) = \frac{1}{6} \end{aligned}$$

The probability is $\frac{1}{36}$.

5. *P*(no 6s)

SOLUTION:

$$\begin{aligned} &P(\text{no 6 and no 6}) \\ &= P(\text{no 6}) \cdot P(\text{no 6}) && \text{Probability of independent events} \\ &= \frac{5}{6} \cdot \frac{5}{6} \text{ or } \frac{25}{36} && P(\text{no 6}) = \frac{5}{6} \end{aligned}$$

The probability is $\frac{25}{36}$.

6. *P*(two of the same number)

SOLUTION:

$$\begin{aligned} &P(\text{two of the same numbers}) \\ &= P(2\ 1\text{s}) + P(2\ 2\text{s}) + P(2\ 3\text{s}) + P(2\ 4\text{s}) + P(2\ 5\text{s}) + P(2\ 6\text{s}) \\ &= \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} && \text{P(2 1s)} \frac{1}{6} \cdot \frac{1}{6} \\ &= 6\left(\frac{1}{36}\right) = \frac{1}{6} && \text{Simplify.} \end{aligned}$$

The probability is $\frac{1}{6}$.

0-6 Multiplying Probabilities

A bag contains 8 blue marbles, 6 red marbles, and 5 green marbles. Three marbles are drawn one at a time. Find each probability.

7. The second marble is green, given that the first marble is blue and not replaced.

SOLUTION:

$P(\text{green}|\text{blue})$

$= \frac{P(\text{green and blue})}{P(\text{blue})}$ Conditional probability

$= \frac{\frac{40}{342}}{\frac{8}{19}}$ $P(\text{blue}) = \frac{8}{19}$ and $P(\text{green and blue}) = \frac{5}{19} \cdot \frac{8}{18}$

$= \frac{40}{342} \cdot \frac{19}{8} = \frac{5}{18}$ Simplify.

The probability is $\frac{5}{18}$.

8. The second marble is red, given that the first marble is green and is replaced.

SOLUTION:

$P(\text{red}|\text{green})$

$= \frac{P(\text{red and green})}{P(\text{green})}$ Conditional probability

$= \frac{\frac{30}{61}}{\frac{5}{19}}$ $P(\text{green}) = \frac{5}{19}$ and $P(\text{red and green}) = \frac{6}{19} \cdot \frac{5}{19}$

$= \frac{30}{361} \cdot \frac{19}{5} = \frac{6}{19}$ Simplify.

The probability is $\frac{6}{19}$.

9. The third marble is red, given that the first two are red and blue and not replaced.

SOLUTION:

$P(\text{red}|(\text{red and blue}))$

$= \frac{P(\text{red and (red and blue)})}{P((\text{red and blue}))}$ Conditional probability

$= \frac{\frac{240}{5814}}{\frac{48}{342}}$ $P(\text{red and blue}) = \frac{6}{19} \cdot \frac{8}{18}$ and

$P(\text{red and (red and blue)}) = \frac{6}{19} \cdot \frac{5}{18} \cdot \frac{8}{17}$

$= \frac{240}{5814} \cdot \frac{342}{48} = \frac{5}{17}$ Simplify.

The probability is $\frac{5}{17}$.

10. The third marble is green, given that the first two are red and are replaced.

SOLUTION:

$P(\text{green}|(\text{red and red}))$

$= \frac{P(\text{green and (red and red)})}{P((\text{red and red}))}$ Conditional probability

$= \frac{\frac{180}{19^3}}{\frac{36}{19^2}}$ $P(\text{red and red}) = \frac{6}{19} \cdot \frac{6}{19}$ and

$P(\text{green and (red and red)}) = \frac{5}{19} \cdot \frac{6}{19} \cdot \frac{6}{19}$

$= \frac{180}{19^3} \cdot \frac{19^2}{36} = \frac{5}{19}$ Simplify.

The probability is $\frac{5}{19}$.

DVDS There are 8 action, 3 comedy, and 5 drama DVDs on a shelf. Suppose three DVDs are selected at random from the shelf. Find each probability.

11. P(3 action), with replacement

SOLUTION:

$P(3 \text{ action}) = \frac{8}{16} \cdot \frac{8}{16} \cdot \frac{8}{16} = \frac{1}{8}$ $P(\text{action}) = \frac{8}{16}$

The probability is $\frac{1}{8}$.

12. P(2 action, then a comedy), without replacement

SOLUTION:

$P(\text{action, action, and comedy}) = \frac{3}{16} \cdot \frac{8}{15} \cdot \frac{7}{14} = \frac{1}{20}$

The probability is $\frac{1}{20}$.

13. **CARDS** You draw a card from a standard deck of cards and show it to a friend. The friend tells you that the card is red. What is the probability that you correctly guess that the card is the ace of diamonds?

SOLUTION:

Given that the card is red, the probability it is an ace is $\frac{1}{26}$.

The probability is $\frac{1}{26}$.

14. **HONOR ROLL** Suppose the probability that a student takes AP Calculus and is on the honor roll is 0.0035, and the probability that a student is on the honor roll is 0.23. Find the probability that a student takes AP Calculus given that he or she is on the honor roll.

SOLUTION:

$P(\text{AP Calc}|\text{honor roll})$

$= \frac{P(\text{AP Calc and honor roll})}{P(\text{honor roll})}$ Conditional probability

$= \frac{0.0035}{0.23}$ $P(\text{honor roll})=0.23$ and $P(\text{AP Calc and honor roll}) = 0.0035$

≈ 0.015 Simplify.

The probability is about 0.015.

15. **DRIVING TESTS** The table shows how students in Mr. Diaz's class fared on their first driving test. Some took a class to prepare, while others did not. Find each probability.

Status	Class	No Class
passed	64	48
failed	18	32

a. Paige passed, given that she took the class.
b. Madison failed, given that she did not take the class.
c. Jamal did not take the class, given that he passed.

SOLUTION:

Calculate the column and row totals.

Status	Class	No Class	Totals
passed	64	48	112
failed	18	32	50
Totals	82	80	162

a.

$P(\text{pass}|\text{class})$

$= \frac{P(\text{pass and class})}{P(\text{class})}$ Conditional probability

$= \frac{\frac{64}{162}}{\frac{82}{162}}$ $P(\text{class})=\frac{82}{162}$ and $P(\text{pass and class}) = \frac{64}{162}$

$= \frac{64}{82} = \frac{21}{41}$ Simplify.

b.

$P(\text{fail}|\text{class})$

$= \frac{P(\text{fail and no class})}{P(\text{no class})}$ Conditional probability

$= \frac{\frac{32}{162}}{\frac{80}{162}}$ $P(\text{no class})=\frac{80}{162}$ and $P(\text{fail and no class}) = \frac{32}{162}$

$= \frac{32}{80} = \frac{2}{5}$ Simplify.

c.

$P(\text{no class}|\text{pass})$

$= \frac{P(\text{no class and pass})}{P(\text{pass})}$ Conditional probability

$= \frac{\frac{48}{162}}{\frac{112}{162}}$ $P(\text{plass})=\frac{112}{162}$ and $P(\text{no class and pass}) = \frac{48}{162}$

$= \frac{48}{112} = \frac{3}{4}$ Simplify.

16. **SCHOOL CLUBS** King High School tallied the nu that were members of at least one after school club.

Gender	Clubs	No Clubs
male	156	242
female	312	108

a. A student is a member of a club given that he is m
b. A student is not a member of a club given that she
c. A student is a male given that he is not a member

SOLUTION:

Calculate the column and row totals.

Gender	Clubs	No Clubs	Totals
male	156	242	398
female	312	108	420
Totals	468	350	818

$P(\text{club member}|\text{male})$

$= \frac{P(\text{club member and male})}{P(\text{male})}$ Conditional probability

$= \frac{\frac{156}{818}}{\frac{398}{818}}$ $P(\text{male}) = \frac{398}{818}$ and

$P(\text{club member and male}) = \frac{156}{818}$

a. $= \frac{156}{398} = \frac{78}{199}$ Simplify.

$P(\text{club member}|\text{male}) = \frac{P(\text{club member and male})}{P(\text{male})}$ Conditional probability

$= \frac{\frac{156}{818}}{\frac{398}{818}}$ $P(\text{male}) = \frac{398}{818}$ and $P(\text{club member and male}) = \frac{156}{818}$

$= \frac{156}{398} = \frac{78}{199}$ Simplify.

$P(\text{non member}|\text{female})$

$= \frac{P(\text{non member and female})}{P(\text{female})}$ Conditional probability

$= \frac{\frac{108}{815}}{\frac{420}{815}}$ $P(\text{female}) = \frac{420}{818}$ and

$P(\text{non member and female}) = \frac{108}{818}$

b. $= \frac{108}{420} = \frac{9}{35}$ Simplify.

$P(\text{non member}|\text{female}) = \frac{P(\text{non member and female})}{P(\text{female})}$ Conditional probability

$= \frac{\frac{108}{815}}{\frac{420}{815}}$ $P(\text{female}) = \frac{420}{818}$ and $P(\text{non member and female}) = \frac{108}{818}$

$= \frac{108}{420} = \frac{9}{35}$ Simplify.

c.

$P(\text{male}|\text{non member})$

$= \frac{P(\text{male and non member})}{P(\text{non member})}$ Conditional probability

$= \frac{\frac{242}{818}}{\frac{350}{818}}$ $P(\text{non member}) = \frac{350}{818}$ and

$P(\text{male and non member}) = \frac{242}{818}$

$= \frac{242}{350} = \frac{121}{175}$ Simplify.

17. **FOOTBALL ATTENDANCE** The number of stu who have attended a football game at North Coast H School is shown. Find each probability.

Class	Freshman	Sophomore	Junior	Senior
attended	48	90	224	254
not attended	182	141	36	8

a. Given that a student is a freshman, the student has attended a game.
b. Given that a student has attended a game, the stud an upperclassman (a junior or senior).

SOLUTION:

Calculate the column and row totals.

class	Freshman	Sophomore	Junior	Senior
attended	48	90	224	254
not attended	182	141	36	8
Totals	230	231	260	262

a. Given that the student is a freshman, the *P*(not atte $= \frac{182}{230} = \frac{91}{115}$ or about 79.1%.

b. Given that the student attended the game, the *P* (upperclassman) $= \frac{478}{616} = \frac{239}{308}$ or about 77.6%.