1-2 Properties of Real Numbers

Name the sets of numbers to which each number belongs.

1. 62

   ANSWER:
   N, W, Z, Q, R

2. $\dfrac{5}{4}$

   ANSWER:
   Q, R

3. $\sqrt{11}$

   ANSWER:
   I, R

4. -12

   ANSWER:
   Z, Q, R

Name the property illustrated by each equation.

5. $(6 \cdot 8) \cdot 5 = 6 \cdot (8 \cdot 5)$

   ANSWER:
   Assoc. $(\times)$

6. $7(9-5) = 7 \cdot 9 - 7 \cdot 5$

   ANSWER:
   Dist.

7. $84 + 16 = 16 + 84$

   ANSWER:
   Comm. (+)

8. $(12+5)6 = 12 \cdot 6 + 5 \cdot 6$

   ANSWER:
   Dist.

Find the additive inverse and multiplicative inverse for each number.

9. -7

   ANSWER:
   $7; -\dfrac{1}{7}$

10. $\dfrac{4}{9}$

    ANSWER:
    $-\dfrac{4}{9}; \dfrac{9}{4}$

11. 3.8

    ANSWER:
    $-3.8; \dfrac{1}{3.8}$

12. $\sqrt{5}$

    ANSWER:
    $-\sqrt{5}; \dfrac{1}{\sqrt{5}}$

1-2 Properties of Real Numbers

13. **REASONING** Melba is mowing lawns for $22 each to earn money for a video game console that costs $550.
    a. Write an expression to represent the total amount of money Melba earned during this week.
    b. Evaluate the expression from part a by using the Distributive Property.
    c. When do you think Melba will earn enough for the video game console? Is this reasonable? Explain.

Lawns Mowed in One Week

Day	Lawns Mowed
Monday	2
Tuesday	4
Wednesday	3
Thursday	1
Friday	5
Saturday	6
Sunday	7

ANSWER:

a. 22(2 + 4 + 3 + 1 + 5 + 6 + 7) or 22(2) + 22(4) + 22(3) + 22(1) + 22(5) + 22(6) + 22(7)
b. $616
c. If she continues to mow the same number of lawns, at the end of next week she will have the money. This may not be reasonable because not all the lawns she mowed this week may need to be mowed again next week.

Simplify each expression.

14. $5(3x + 6y) + 4(2x - 9y)$

    ANSWER:
    $23x - 6y$

15. $6(6a + 5b) - 3(4a + 7b)$

    ANSWER:
    $24a + 9b$

16. $-4(6c - 3d) - 5(-2c - 4d)$

    ANSWER:
    $-14c + 32d$

17. $-5(8x - 2y) - 4(-6x - 3y)$

    ANSWER:
    $-16x + 22y$

Name the sets of numbers to which each number belongs.

18. $-\dfrac{4}{3}$

    ANSWER:
    Q, R

19. -8.13

    ANSWER:
    Q, R

20. $\sqrt{25}$

    ANSWER:
    N, W, Z, Q, R

21. $0.\overline{61}$

    ANSWER:
    Q, R

22. $\dfrac{9}{3}$

    ANSWER:
    N, W, Z, Q, R

23. $-\sqrt{144}$

    ANSWER:
    Z, Q, R

24. $\dfrac{21}{7}$

    ANSWER:
    N, W, Z, Q, R

25. $\sqrt{17}$

    ANSWER:
    I, R

Name the property illustrated by each equation.

26. $-7y + 7y = 0$

ANSWER:
Inverse (+)

27. $8\sqrt{11} + 5\sqrt{11} = (8+5)\sqrt{11}$

ANSWER:
Dist.

28. $(16+7) + 23 = 16 + (7+23)$

ANSWER:
Assoc. (+)

29. $\left(\dfrac{22}{7}\right)\left(\dfrac{7}{22}\right) = 1$

ANSWER:
Inverse $(\times)$

Find the additive inverse and multiplicative inverse for each number.

30. -8

ANSWER:
$8; -\dfrac{1}{8}$

31. 12.1

ANSWER:
$-12.1; \dfrac{1}{12.1}$

32. -0.25

ANSWER:
$0.25; -4$

33. $\dfrac{6}{13}$

ANSWER:
$-\dfrac{6}{13}, \dfrac{13}{6}$

34. $-\dfrac{3}{8}$

ANSWER:
$\dfrac{3}{8}; -\dfrac{8}{3}$

35. $\sqrt{15}$

ANSWER:
$-\sqrt{15}; \dfrac{1}{\sqrt{15}}$

36. **CONSTRUCTION** Jorge needs two different kinds of concrete: quick drying and slow drying. The quick-drying concrete mix calls for $2\dfrac{1}{2}$ pounds of dry cement, and the slow-drying concrete mix calls for $1\dfrac{1}{4}$ pounds of dry cement. He needs 5 times more quick-drying concrete and 3 times more slow-drying concrete than the mixes make.
a. How many pounds of dry cement mix will he need?
b. Use the properties of real numbers to show how Jorge could compute this amount mentally. Justify each step.

ANSWER:

a. $16\dfrac{1}{4}$ lb

b.

$$5\left(2\dfrac{1}{2}\right) + 3\left(1\dfrac{1}{4}\right)$$

$= 5\left(2+\dfrac{1}{2}\right) + 3\left(1+\dfrac{1}{4}\right)$	Definition of a mixed number
$= 5(2) + 5\left(\dfrac{1}{2}\right) + 3(1) + 3\left(\dfrac{1}{4}\right)$	Distributive Property
$= 10 + \dfrac{5}{2} + 3 + \dfrac{3}{4}$	Multiply.
$= 10 + 3 + \dfrac{5}{2} + \dfrac{3}{4}$	Commutative Property (+)
$= 13 + \dfrac{5}{2} + \dfrac{3}{4}$	Add.
$= 13 + \left(\dfrac{5}{2} + \dfrac{3}{4}\right)$	Associative Property (+)
$= 13 + 3\dfrac{1}{4}$ or $16\dfrac{1}{4}$	Add.

Simplify each expression.

37. $8b - 3c + 4b + 9c$

ANSWER:
$12b + 6c$

38. $-2a + 9d - 5a - 6d$

ANSWER:
$-7a + 3d$

39. $4(4x - 9y) + 8(3x + 2y)$

ANSWER:
$40x - 20y$

40. $6(9a - 3b) - 8(2a + 4b)$

ANSWER:
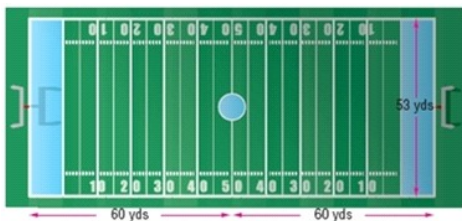$38a - 50b$

41. $-2(-5g + 6k) - 9(-2g + 4k)$

ANSWER:
$28g - 48k$

42. $-5(10x + 8z) - 6(4x - 7z)$

ANSWER:
$-74x + 2z$

43. **FOOTBALL** Illustrate the Distributive Property by writing two expressions for the area of a college football field. Then find the area of the football field.

ANSWER:

$53(60 + 60)$; $53(60) + 53(60)$; 6360 yd^2

44. **PETS** The chart shows the percent of dogs registered with the American Kennel Club that are of the eight most popular breeds.
a. Illustrate the Distributive Property by writing two expressions to represent the number of registered dogs of the top four breeds.
b. Evaluate the expressions you wrote to find the number of registered dogs of the top four breeds.

Top Dogs	
Breed	**Percent of Registered Dogs**
Labrador Retrievers	14.2
Yorkshire Terriers	5.6
German Shepherds	5.0
Golden Retrievers	4.9
Beagles	4.5
Dachshunds	4.1
Boxers	4.1
Poodles	3.4
Total Registered Dogs	**870,192**

Source: American Kennel Club

ANSWER:

a. $870,192(0.142 + 0.056 + 0.05 + 0.049) = 870,192(0.142) + 870,192(0.056) + 870,192(0.05) + 870,192(0.049)$
b. 258,447

45. **FINANCIAL LITERACY** Billie is given $20 in lunch money by her parents once every two weeks. On some days, she packs her lunch, and on other days, she buys her lunch. A hot lunch from the cafeteria costs $4.50, and a cold sandwich from the lunch line costs $2.
a. Billie decides that she wants to buy a hot lunch on Thursday and Friday of the first week and on Wednesday of the second week. Use the Distributive Property to determine how much that will cost.
b. How many cold sandwiches can Billie buy with the amount left over?
c. Assuming that both weeks are Monday through Friday, how many times will Billie have to pack her lunch?

ANSWER:
a. $13.50
b. 3
c. 4 times

Simplify each expression.

46. $\dfrac{2}{5}\left(6c-8d\right)+\dfrac{3}{4}\left(4c-9d\right)$

ANSWER:

$\dfrac{27}{5}c-\dfrac{199}{20}d$

47. $\dfrac{1}{3}\left(5x+8y\right)+\dfrac{1}{4}\left(6x-2y\right)$

ANSWER:

$\dfrac{19}{6}x+\dfrac{13}{6}y$

48. $-6\left(3a+5b\right)-3\left(6a-8c\right)$

ANSWER:

$-36a-30b+24c$

49. $-9\left(3x+8y\right)-3\left(5x+10z\right)$

ANSWER:

$-42x-72y-30z$

50. **MODELING** Mary is making curtains out of the same fabric for 5 windows. The two larger windows are the same size, and the three smaller windows are the same size. One larger window requires

$3\dfrac{3}{4}$ yards of fabric, and one smaller window needs

$2\dfrac{1}{3}$ yards of fabric.

a. How many yards of material will Mary need?
b. Use the properties of real numbers to show how Mary could compute this amount mentally.

ANSWER:

a. $14\dfrac{1}{2}$ yd

b.

$2\left(3\dfrac{3}{4}\right)+3\left(2\dfrac{1}{3}\right)=2\left(3+\dfrac{3}{4}\right)+3\left(2+\dfrac{1}{3}\right)$ (Definition of a mixed number)

$\qquad=2(3)+2\left(\dfrac{3}{4}\right)+3(2)+3\left(\dfrac{1}{3}\right)$ (Distributive Property)

$\qquad=6+\dfrac{3}{2}+6+1$ (Multiply)

$\qquad=6+6+1+\dfrac{3}{2}$ (Commutative Property of Addition)

$\qquad=13+\dfrac{3}{2}$ (Addition)

$\qquad=14\dfrac{1}{2}$

51. **MULTIPLE REPRESENTATIONS** Consider the following real numbers.

$$-\sqrt{6},\,3,\,\dfrac{-15}{3},\,4.\overline{1},\,\pi,\,0,\,\dfrac{3}{8},\,\sqrt{36}$$

a. TABULAR Organize the numbers into a table according to the sets of numbers to which each belongs.
b. ALGEBRAIC Convert each number to decimal form. Then list the numbers from least to greatest.
c. GRAPHICAL Graph the numbers on a number line.
d. VERBAL Make a conjecture about using decimal form to list real numbers in order.

ANSWER:

a. Sample answer:

irrational	rational	integer	whole	natural
$-\sqrt{6},\pi$	$3,\dfrac{-15}{3},4.1,0,$ $\dfrac{3}{8},\sqrt{36}$	$3,\dfrac{-15}{3},0,$ $\sqrt{36}$	$3,0,$ $\sqrt{36}$	$3,$ $\sqrt{36}$

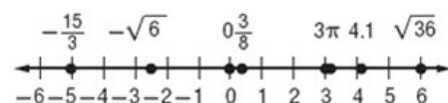**b.** $-\sqrt{6}\approx-2.449,$

$3=3.0,\dfrac{-15}{3}=-5;$

$4.\overline{1}=4.\overline{1},\pi\approx3.14,$

$0=0,\dfrac{3}{8}=.375;$

$\sqrt{36}=6;\dfrac{-15}{3},-\sqrt{6},$

$0,\dfrac{3}{8},3,\pi,4.\overline{1},\sqrt{36}$

c.

d. Sample answer: By converting the real numbers into decimal form, they can be easily lined up and compared.

52. **CLOTHING** A department store sells shirts for $12.50 each. Dalila buys 2, Latisha buys 3, and Pilar buys 1.
    a. Illustrate the Distributive Property by writing two expressions to represent the cost of these shirts.
    b. Use the Distributive Property to find how much money the store received from selling these shirts.

    ANSWER:
    a. $12.50(2+3+1); 12.50 \cdot 2 + 12.50 \cdot 3 + 12.50 \cdot 1$
    b. $75

53. **WHICH ONE DOESN'T BELONG?** Identify the number that does not belong with the other three. Explain your reasoning.

$\sqrt{21}$	$\sqrt{35}$	$\sqrt{67}$	$\sqrt{81}$

    ANSWER:

    $\sqrt{81}$; It is a rational number, while the other three are irrational numbers.

54. **CHALLENGE** If $12(5r + 6t) = w$, then in terms of w, what is $48(30r + 36t)$?

    ANSWER:
    $24w$

55. **ERROR ANALYSIS** Luna and Sophia are simplifying $4(14a - 10b) - 6(b + 4a)$. Is either of them correct? Explain your reasoning.

Luna	Sophia
$4(14a - 10b) - 6(b + 4a)$	$4(14a - 10b) - 6(b + 4a)$
$56a - 40b - 6b + 24a$	$56a - 40b - 6a - 24b$
$80a - 46b$	$50a - 64b$

    ANSWER:

    No; Luna did not distribute the negative sign to the second term and Sophia switched the a and b terms because usually a comes first. The correct answer is $32a - 46b$.

56. **REASONING** Determine whether the following statement is sometimes, always, or never true. Explain your reasoning.
    An irrational number is a real number underneath a radical sign.

    ANSWER:

    Sometimes; π and e are two examples of irrational numbers that do not involve the radical symbol.

57. **OPEN ENDED** Determine whether the Closure Property of Multiplication applies to irrational numbers. If not, provide a counterexample.

    ANSWER:

    Sample answer: $\sqrt{5} \cdot \sqrt{5} = \sqrt{25}$ or 5, which is not irrational.

OPEN ENDED The set of all real numbers is *dense,* meaning between any two distinct members of the set there lies infinitely many other members of the set. Find an example of (a) a rational number, and (b) an irrational number between the given numbers.

58. 2.45 and 2.5

    ANSWER:
    Sample answer: (a) 2.46 and (b) 2.48448444844448…

59. π and $\dfrac{10}{3}$

    ANSWER:
    Sample answer: (a) 3.2 and (b) $\sqrt{10}$

60. $1.\overline{9}$ and 2.01

    ANSWER:
    Sample answer: (a) 2.001 and (b) 2.001000100001 . . .

61. **WRITING IN MATH** Explain and provide examples to show why the Commutative Property does not hold true for subtraction or division.

    ANSWER:
    Sample answer: The Commutative Property does not hold for subtraction or division because order matters with these two operations. In addition or multiplication, the order does not matter.
    For example, $2 + 4 = 4 + 2$ and $2 \cdot 4 = 4 \cdot 2$. However, with subtraction, $2 - 4 \neq 4 - 2$, and with division,
    $\dfrac{2}{4} \neq \dfrac{4}{2}$.

62. **EXTENDED RESPONSE** Lenora bought several pounds of cashews and several pounds of almonds for a party. The cashews cost $8 per pound, and the almonds cost $6 per pound. Lenora bought a total of 7 pounds and paid a total of $48. Write and solve equations to determine the pounds of cashews and the pounds of almonds that Lenora purchased.
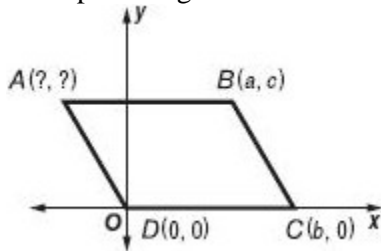
ANSWER:

$8c + 6a = 48$ and $c + a = 7$; $c = 3$ lb; $a = 4$ lb

63. **SAT/ACT** Find the 10th term in the series 2, 4, 7, 11, 16, …
    A 41
    B 46
    C 56
    D 67
    E 72

ANSWER:

C

64. **GEOMETRY** What are the coordinates of point A in the parallelogram?

    F $(b - a, c)$ **H** (b, c)
    G $(a - b, c)$ **J** (c, c)

ANSWER:

G

65. What is the domain of the function that contains the points $(-3, 0)$, $(0, 4)$, $(-2, 5)$, and $(6, 4)$?
    A $\{-3, 6\}$ **C** $\{0, 4, 5, 6\}$
    B $\{-3, -2, 0, 6\}$ **D** $\{-3, -2, 0, 4, 5, 6\}$
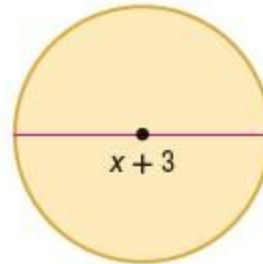
ANSWER:

B

66. Evaluate $8(4 - 2)^3$.

ANSWER:

64

67. Evaluate $a + 3(b + c) - d$, if $a = 5$, $b = 4$, $c = 3$, and $d = 2$.

ANSWER:

24

68. **GEOMETRY** The formula for the area A of a circle with diameter d is $A = \pi\left(\frac{d}{2}\right)^2$. Write an expression to represent the area of the circle.

ANSWER:

$\pi\left(\frac{x+3}{2}\right)^2$

69. **CONSTRUCTION** A 10-meter ladder leans against a building so that the top is 9.64 meters above the ground. How far from the base of the wall is the bottom of the ladder?

ANSWER:

about 2.66 m

Factor each polynomial.

70. $14x^2 + 10x - 8$

ANSWER:

$2\left(7x^2 + 5x - 4\right)$

71. $9x^2 - 3x + 18$

ANSWER:

$3\left(3x^2 - x + 6\right)$

72. $8x^2 + 16x + 12$

ANSWER:

$4\left(2x^2 + 4x + 3\right)$

73. $10x^2 - 20x$

ANSWER:
$10x(x-2)$

74. $7x^2 - 14x - 21$

ANSWER:
$7(x-3)(x+1)$

75. $12x^2 - 18x - 24$

ANSWER:
$6(2x^2 - 3x - 4)$

Find each product.

76. $(x+2)(x-3)$

ANSWER:
$x^2 - x - 6$

77. $(y+2)(y-1)$

ANSWER:
$y^2 + y - 2$

78. $(a-5)(a+4)$

ANSWER:
$a^2 - a - 20$

79. $(b-7)(b-3)$

ANSWER:
$b^2 - 10b + 21$

80. $(n+6)(n+8)$

ANSWER:
$n^2 + 14n + 48$

81. $(p-9)(p+1)$

ANSWER:
$p^2 - 8p - 9$

Evaluate each expression if $a = 3$, $b = \dfrac{2}{3}$, and $c = -1.7$.

82. $6b - 5$

ANSWER:
-1

83. $\dfrac{1}{6}b + 1$

ANSWER:
$\dfrac{10}{9}$

84. $2.3c - 7$

ANSWER:
-10.91

85. $-8(a-4)$

ANSWER:
8

86. $a + b + c$

ANSWER:
≈ 1.967

87. $\dfrac{a \cdot b}{c}$

ANSWER:
≈ -1.176

88. $a^2 - c$

ANSWER:
10.7

89. $\dfrac{a \cdot c}{a}$

ANSWER:
≈ -1.7