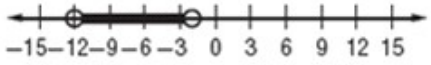
1-6 Solving Compound and Absolute Value Inequalities

Solve each inequality. Graph the solution set on a number line.

1. $-4 < g + 8 < 6$

ANSWER:

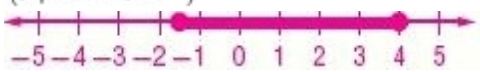
$\{g \mid -12 < g < -2\}$



2. $-9 \le 4y - 3 \le 13$

ANSWER:

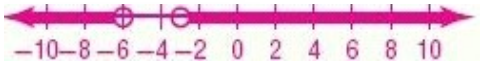
$\{y \mid -1.5 \le y \le 4\}$



3. $z + 6 > 3$ or $2z < -12$

ANSWER:

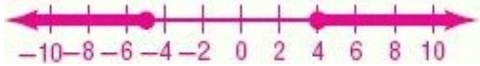
$\{z \mid z > -3 \text{ or } z < -6\}$



4. $m - 7 \ge -3$ or $-2m + 1 \ge 11$

ANSWER:

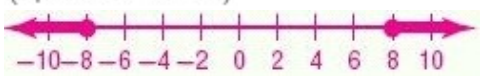
$\{m \mid m \ge 4 \text{ or } m \le -5\}$



5. $|c| \ge 8$

ANSWER:

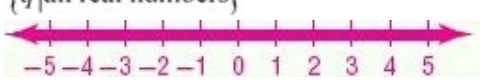
$\{c \mid c \ge 8 \text{ or } c \le -8\}$



6. $|q| \ge -1$

ANSWER:

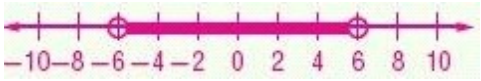
$\{q \mid \text{all real numbers}\}$



7. $|z| < 6$

ANSWER:

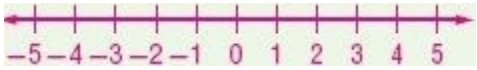
$\{z \mid -6 < z < 6\}$



8. $|x| \le -4$

ANSWER:

$\varnothing$



9. $|3v + 5| > 14$

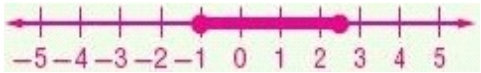
ANSWER:

$\left\{v \mid v > 3 \text{ or } v < -\frac{19}{3}\right\}$

10. $|4t - 3| \le 7$

ANSWER:

$\{t \mid -1 \le t \le 2.5\}$



11. **MONEY** Khalid is considering several types of paint for his bedroom. He estimates that he will need between 2 and 3 gallons. The table at the right shows the price per gallon for each type of paint Khalid is considering. Write a compound inequality and determine how much he could be spending.

Paint Type	Price per Gallon
Flat	$21.98
Satin	$23.98
Semi-Gloss	$24.98
Gloss	$25.98

ANSWER:

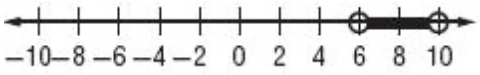
$43.96 \le c \le 77.94$; between $43.96 and $77.94

Solve each inequality. Graph the solution set on a number line.

12. $8 < 2v - 4 < 16$

ANSWER:

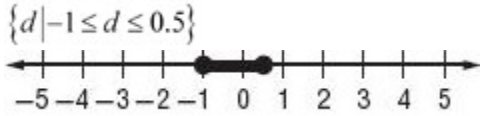
$\{v \mid 6 < v < 10\}$



13. $-7 \le 4d - 3 \le -1$

ANSWER:

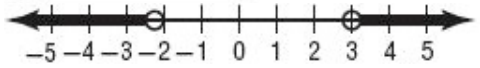
$\{d \mid -1 \le d \le 0.5\}$



14. $4r + 3 < -6$ or $3r - 7 > 2$

ANSWER:

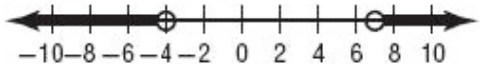
$\left\{r \mid r < -\frac{9}{4} \text{ or } r > 3\right\}$



15. $6y - 3 < -27$ or $-4y + 2 < -26$

ANSWER:

$\{y \mid y < -4 \text{ or } y > 7\}$



16. $|6h| < 12$

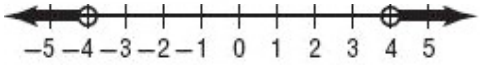
ANSWER:

$\{h \mid -2 < h < 2\}$

17. $|-4k| > 16$

ANSWER:

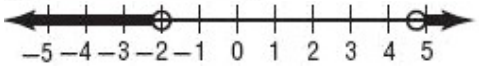
$\{k \mid k < -4 \text{ or } k > 4\}$



18. $|3x - 4| > 10$

ANSWER:

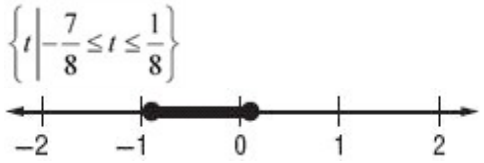
$\left\{x \mid x > \frac{14}{3} \text{ or } x < -2\right\}$



19. $|8t + 3| \le 4$

ANSWER:

$\left\{t \mid -\frac{7}{8} \le t \le \frac{1}{8}\right\}$



20. $|-9n - 3| < 6$

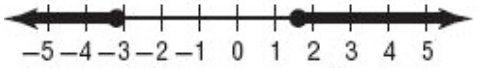
ANSWER:

$\left\{n \mid -1 < n < \frac{1}{3}\right\}$

21. $|-5j - 4| \ge 12$

ANSWER:

$\left\{j \mid j \ge \frac{8}{5} \text{ or } j \le -\frac{16}{5}\right\}$



22. **ANATOMY** Forensic scientists use the equation $h = 2.6f + 47.2$ to estimate the height h of a woman given the length in centimeters f of her femur bone.

a. Suppose the equation has a margin of error of ±3 centimeters. Write an inequality to represent the height of a woman given the length of her femur bone.

b. If the length of a female skeleton's femur is 50 centimeters, write and solve an absolute value inequality that describes the woman's height in centimeters.

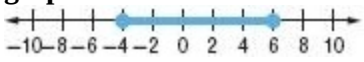
ANSWER:

a. $|2.6f + 47.2| < 3$

b. $|h - 177.2| < 3$; 174.2 cm $< h <$ 180.2 cm

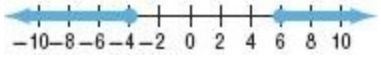
1-6 Solving Compound and Absolute Value Inequalities

Write an absolute value inequality for each graph.

23. 

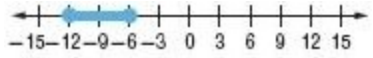
ANSWER:

$|x-1| \leq 5$

24. 

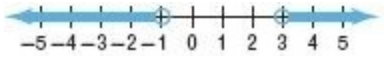
ANSWER:

$|x-1| \geq 5$

25. 

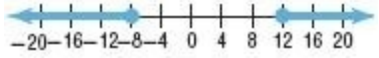
ANSWER:

$|x+9| \leq 3$

26. 

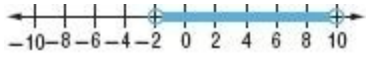
ANSWER:

$|x-1| > 2$

27. 

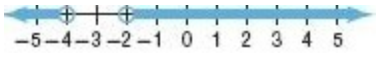
ANSWER:

$|x-2| \geq 10$

28. 

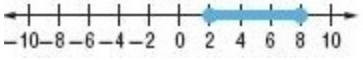
ANSWER:

$|x-4| < 6$

29. 

ANSWER:

$|x+3| > 1$

30. 

ANSWER:

$|x-5| \leq 3$

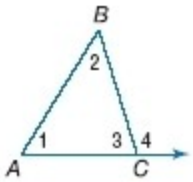
31. **DOGS** The Labrador retriever is one of the most recognized and popular dogs kept as a pet. Using the information given, write a compound inequality to describe the range of healthy weights for a fully grown female Labrador retriever.

Healthy Heights and Weights for Labrador Retrievers		
Gender	Height (in.)	Weight (lb)
Male	22.5–24.5	65–80
Female	21.5–23.5	55–70

ANSWER:

$55 \leq w \leq 70$

32. **GEOMETRY** The Exterior Angle Inequality Theorem states that an exterior angle measure is greater than the measure of either of its corresponding remote interior angles. Write two inequalities to express the relationships among the measures of the angles of ΔABC.



ANSWER:

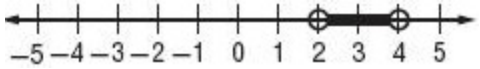
$m\angle 4 > m\angle 1,\ m\angle 4 > m\angle 2$

Solve each inequality. Graph the solution set on a number line.

33. $28 > 6k + 4 > 16$

ANSWER:

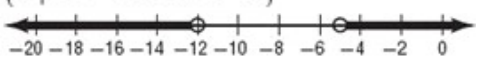
$\{k \mid 2 < k < 4\}$



34. $m - 7 > -12$ or $-3m + 2 > 38$

ANSWER:

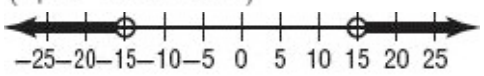
$\{m \mid m > -5 \text{ or } m < -12\}$



35. $|-6h| > 90$

ANSWER:

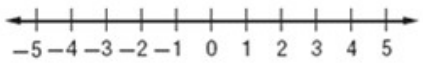
$\{h \mid h < -15 \text{ or } h > 15\}$



36. $-|-5k| > 15$

ANSWER:

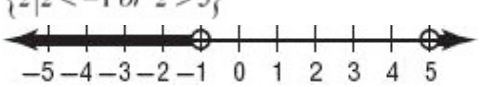
$\varnothing$



37. $3|2z-4|-6 > 12$

ANSWER:

$\{z \mid z < -1 \text{ or } z > 5\}$



38. $6|4p+2|-8 < 34$

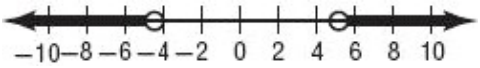
ANSWER:

$\left\{p \mid -\frac{9}{4} < p < \frac{5}{4}\right\}$

39. $\frac{|5f-2|}{6} > 4$

ANSWER:

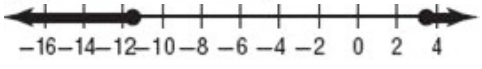
$\left\{f \mid f > \frac{26}{5} \text{ or } f < -\frac{22}{5}\right\}$



40. $\frac{|2w+8|}{5} \geq 3$

ANSWER:

$\left\{w \mid w \leq -\frac{23}{2} \text{ or } w \geq \frac{7}{2}\right\}$



Write an algebraic expression to represent each verbal expression.

41. numbers that are at least 4 units from –5

ANSWER:

$|x+5| \geq 4$

42. numbers that are no more than $\frac{3}{8}$ unit from 1

ANSWER:

$|x-1| \leq \frac{3}{8}$

43. numbers that are at least 6 units but no more than 10 units from 2

ANSWER:

$6 \leq |x-2| \leq 10$

44. **AUTO RACING** NASCAR rules stipulate that a car must conform to a set of 32 templates, each shaped to fit a different contour of the car. When a template is placed on a car, the gap between it and the car cannot exceed the specified tolerance. Each template is marked on its edge with a colored line that indicates the tolerance for the template.

Line Color	Tolerance (in.)
Red	0.07
Blue	0.25
Green	0.5

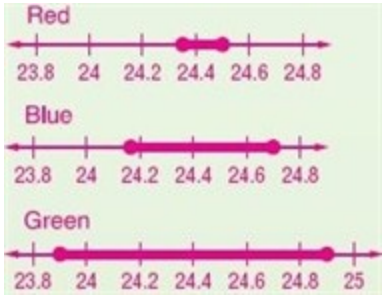
a. Suppose a certain template is 24.42 inches long. Use the information in the table at the right to write an absolute value inequality for templates with each line color.
b. Find the acceptable lengths for that part of a car if the template has each line color.
c. Graph the solution set for each line color on a number line.
d. The tolerance of which line color includes the tolerances of the other line colors? Explain your reasoning.

ANSWER:

a. red: $|x - 24.42| \le 0.07$;
blue: $|x - 24.42| \le 0.25$;
green: $|x - 24.42| \le 0.5$

b. red: $24.35 \le x \le 24.49$;
blue: $24.17 \le x \le 24.67$;
green: $23.92 \le x \le 24.92$

c.



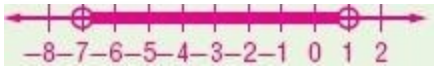
d. Red; the red line color has the smallest tolerance, $0.07 < 0.25 < 0.5$, so the other line colors would be well within their tolerances.

Solve each inequality. Graph the solution set on a number line.

45. $n + 6 > 2n + 5 > n - 2$

ANSWER:

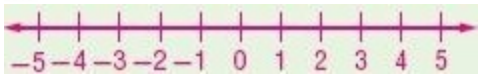
$\{n \mid -7 < n < 1\}$



46. $y + 7 < 2y + 2 < 0$

ANSWER:

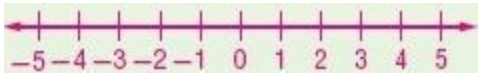
$\varnothing$



47. $2x + 6 < 3(x - 1) \le 2(x + 3)$

ANSWER:

$\varnothing$



48. $a - 16 \le 2(a - 4) < a + 2$

ANSWER:

$\{a \mid -8 \le a < 10\}$

49. $4g + 8 \ge g + 6$ or $7g - 14 \ge 2g - 4$

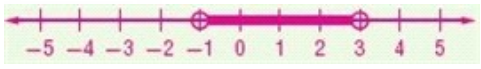
ANSWER:

$\left\{g \mid g \ge -\frac{2}{3}\right\}$

50. $5t + 7 > 2t + 4$ and $3t + 3 < 24 - 4t$

ANSWER:

$\{t \mid -1 < t < 3\}$



51. **HEALTH** Hypoglycemia (low blood sugar) and hyperglycemia (high blood sugar) are potentially dangerous and occur when a person's blood sugar fluctuates by more than 38 mg from the normal blood sugar level of 88 mg. Write and solve an absolute value inequality to describe blood sugar levels that are considered potentially dangerous.

ANSWER:

$|s - 88| > 38; \{s | s > 126 \text{ or } s < 50\}$

52. **AIR TRAVEL** The airline on which Drew is flying has weight restrictions for checked baggage. Drew is checking one bag.
a. Describe the range of weights that would classify Drew's bag as free, $25, $50, and unacceptable.
b. If Drew's bag weighs 68 pounds, how much will he pay to take it on the plane?

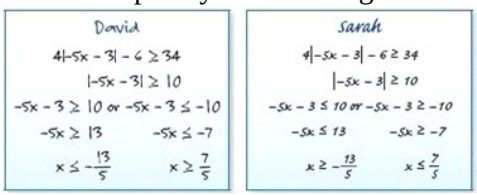
Cost for Checked Baggage	
Weight	**Cost**
Up to 50 lb limit	free
20 lb over limit	$25
More than 20, but less than 50 lb over limit	$50
More than 50 lb over limit	not accepted

ANSWER:

a. $x \leq 50; 50 < x < 70; 70 < x < 100; x > 100$

b. $25

53. **ERROR ANALYSIS** David and Sarah are solving $4|-5x - 3| - 6 \geq 34$. Is either of them correct? Explain your reasoning.



ANSWER:

Sample answer: David; when Sarah converted the absolute value into two inequalities, she mistakenly switched
the inequality symbols.

54. **CHALLENGE** Solve $|x - 2| - |x + 2| > x$.

ANSWER:

$x < 0$

REASONING Determine whether each statement is *true* or *false* . If false, provide a counterexample.

55. The graph of a compound inequality involving an *and* statement is bounded on the left and right by two values of x.

ANSWER:

False; sample answer: the graph of
$x > 2$ and $x > 5$ is a ray bounded only on one end.

56. The graph of a compound inequality involving an *or* statement contains a region of values that are not solutions.

ANSWER:

False; sample answer: the graph of
$x > 2$ or $x < 3$ includes the entire number line.

57. The graph of a compound inequality involving an *and* statement includes values that make all parts of the given statement true.

ANSWER:

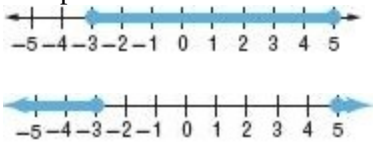
True

58. **WRITING IN MATH** An alternate definition of absolute value is to define $|a - b|$ as the distance between a and b on the number line. Explain how this definition can be used to solve inequalities of the form $|x - c| < r$.

ANSWER:

Sample answer: $|x - c|$ represents
the distance between some unknown value of the variable x and a point c on the number line. The solution set of the inequality is the set of all numbers such that the distance from the numbers to c is less than r units. Use a number line to find the numbers that are r units from c in either direction.

59. **REASONING** The graphs of the solutions of two different absolute value inequalities are shown. Compare and contrast the absolute value inequalities.



ANSWER:

Sample answer: The graph on the left indicates a solution set from –3 to 5. The graph on the right indicates a solution set of all numbers less than or equal to –3 or greater than or equal to 5.

60. **OPEN ENDED** Write an absolute value inequality with a solution of $a \le x \le b$.

ANSWER:

Sample answer:

$$\left|x - \frac{a+b}{2}\right| \le b - \frac{a+b}{2}$$

61. **WHICH ONE DOESN'T BELONG?** Identify the compound inequality that is not the same as the other three. Explain your reasoning.

$-3 < x < 5$ | $x > 2$ and $x < 3$ | $x > 5$ and $x < 1$ | $x > -4$ and $x > -2$

ANSWER:

Each of these has a non-empty solution set except for $x > 5$ and $x < 1$. There are no values of x that are simultaneously greater than 5 and less than 1.

62. **WRITING IN MATH** Summarize the difference between *and* compound inequalities and *or* compound inequalities.

ANSWER:

Sample answer: A compound inequality that contains *and* is true if and only if both individual inequalities are true, while an inequality containing *or* only needs one of the individual inequalities to be true.

63. Which of the following best describes the graph of the equations below?

$24y = 8x + 11$

$36y = 12x + 11$

A The lines have the same x-intercept.
B The lines have the same y-intercept.
C The lines are parallel.
D The lines are perpendicular.

ANSWER:

C

64. **SAT/ACT** Find an expression equivalent to $\left(\frac{3x^3}{y}\right)^3$

F $\frac{9x^6}{3y}$

G $\frac{9x^9}{y^3}$

H $\frac{9x^6}{3y^3}$

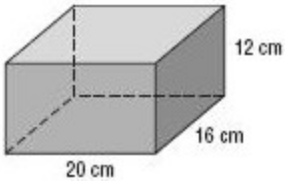
J $\frac{27x^6}{3y}$

K $\frac{27x^9}{y^3}$

ANSWER:

K

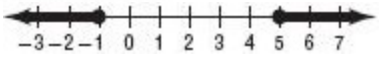
65. **GRIDDED RESPONSE** How many cubes that measure 4 centimeters on each side can be placed completely inside the box below?

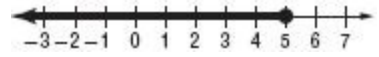


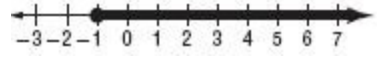
ANSWER:

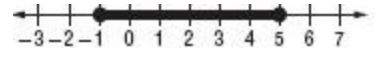
60

66. Which graph represents the solution set for $|3x-6|+8\geq 17$?

A 

B 

C 

D 

ANSWER:

A

67. **HEALTH** The National Heart Association recommends that less than 30% of a person's total daily caloric intake come from fat. One gram of fat yields nine Calories. Consider a healthy 21-year-old whose average caloric intake is between 2500 and 3300 Calories.

a. Write an inequality that represents the suggested fat intake for the person.

b. What is the greatest suggested fat intake for the person?

ANSWER:

a. $750 \leq x \leq 990$

b. 110g

68. **TRAVEL** Maggie is planning a 5-day trip to a family reunion. She wants to spend no more than $1000. Her plane ticket is $375, and the hotel is $85 per night.

a. Let f represent the cost of food for one day. Write an inequality to represent this situation.

b. Solve the inequality and interpret the solution.

ANSWER:

a. $800+5f\leq 1000$

b. She can spend no more than $40 per day on food.

Solve each equation. Check your solutions.

69. $4|x-5|=20$

ANSWER:

$\{0,10\}$

70. $|3y+10|=25$

ANSWER:

$\left\{-\frac{35}{3},5\right\}$

71. $|7z+8|=-9$

ANSWER:

$\varnothing$

Name the property illustrated by each statement.

72. If $5x=7$, then $5x+3=7+3$.

ANSWER:

Addition (=)

73. If $-3x+9=11$ and $6x+2=11$, then $-3x+9=6x+2$.

ANSWER:

Transitive (=)

74. If $\left[x+(-2)\right]+(-4)=5$, then $x+\left[-2+(-4)\right]=5$.

ANSWER:

Assoc. (+)