10-2 Arithmetic Sequences and Series

Find the indicated term of each arithmetic sequence.

1. $a_1 = 14, d = 9, n = 11$

ANSWER:
104

2. a_{18} for 12, 25, 38, …

ANSWER:
233

Write an equation for the *n*th term of each arithmetic sequence.

3. 13, 19, 25, …

ANSWER:
$a_n = 6n + 7$

4. $a_5 = –12, d = –4$

ANSWER:
$a_n = –4n + 8$

Find the arithmetic means in each sequence.

5. 6, ?, ?, ?, 42

ANSWER:
15, 24, 33

6. –4, ?, ?, ?, 8

ANSWER:
–1, 2, 5

Find the sum of each arithmetic series.

7. the first 50 natural numbers

ANSWER:
1275

8. 4 + 8 + 12 + … + 200

ANSWER:
5100

9. $a_1 = 12, a_n = 188, d = 4$

ANSWER:
4500

10. $a_n = 145, d = 5, n = 21$

ANSWER:
1995

Find the first three terms of each arithmetic series.

11. $a_1 = 8, a_n = 100, S_n = 1296$

ANSWER:
8, 12, 16

12. $n = 18$, $a_n = 112$, $S_n = 1098$

ANSWER:
10, 16, 22

13. **MULTIPLE CHOICE** Find $\sum_{k=1}^{12}(3k+9)$.

A 45

B 78

C 342

D 410

ANSWER:
C

Find the indicated term of each arithmetic sequence.

14. $a_1 = -18$, $d = 12$, $n = 16$

ANSWER:
162

15. $a_1 = -12$, $n = 66$, $d = 4$

ANSWER:
248

16. $a_1 = 9$, $n = 24$, $d = -6$

ANSWER:
–129

17. a_{15} for –5, –12, –19, …

ANSWER:
–103

18. a_{10} for –1, 1, 3, …

ANSWER:
17

19. a_{24} for 8.25, 8.5, 8.75, …

ANSWER:
14

Write an equation for the *n*th term of each arithmetic sequence.

20. 24, 35, 46, …

ANSWER:
$a_n = 11n + 13$

21. 31, 17, 3, …

ANSWER:
$a_n = -14n + 45$

22. $a_9 = 45$, $d = -3$

ANSWER:
$a_n = -3n + 72$

23. $a_7 = 21, d = 5$

ANSWER:
$a_n = 5n - 14$

24. $a_4 = 12, d = 0.25$

ANSWER:
$a_n = 0.25n + 11$

25. $a_5 = 1.5, d = 4.5$

ANSWER:
$a_n = 4.5n - 21$

26. 9, 2, –5, …

ANSWER:
$a_n = -7n + 16$

27. $a_6 = 22, d = 9$

ANSWER:
$a_n = 9n - 32$

28. $a_8 = -8, d = -2$

ANSWER:
$a_n = -2n + 8$

29. $a_{15} = 7, d = \frac{2}{3}$

ANSWER:
$a_n = \frac{2}{3}n - 3$

30. –12, –17, –22, …

ANSWER:
$a_n = -5n - 7$

31. $a_3 = -\frac{4}{5}, d = \frac{1}{2}$

ANSWER:
$a_n = \frac{1}{2}n - \frac{23}{10}$

32. **CCSS STRUCTURE** José averaged 123 total pins per game in his bowing league this season. He is taking bowling lessons and hopes to bring his average up by 8 pins each new season.

a. Write an equation to represent the *n*th term of the sequence.

b. If the pattern continues, during what season will José average 187 per game?

c. Is it reasonable for this pattern to continue indefinitely? Explain.

ANSWER:
a. $a_n = 115 + 8n$

b. 9th season

c. Sample answer: No; there are a maximum of 300 points in a bowling game, so it would be impossible for the average to continue to climb indefinitely.

Find the arithmetic means in each sequence.

33. 24, ?, ?, ?, ?, –1

ANSWER:
19, 14, 9, 4

34. –6, ?, ?, ?, ?, 49]

ANSWER:
5, 16, 27, 38

35. –28, ?, ?, ?, ?, 7

ANSWER:
–21, –14, –7, 0

36. 84, ?, ?, ?, ?, 39

ANSWER:
75, 66, 57, 48

37. –12, ?, ?, ?, ?, ?, –66

ANSWER:
–21, –30, –39, –48, –57

38. 182, ?, ?, ?, ?, ?, 104

ANSWER:
169, 156, 143, 130, 117

Find the sum of each arithmetic series.

39. the first 100 even natural numbers

ANSWER:
10,100

40. the first 200 odd natural numbers

ANSWER:
40,000

41. the first 100 odd natural numbers

ANSWER:
10,000

42. the first 300 even natural numbers

ANSWER:
90,300

43. –18 + (–15) + (–12) + … + 66

ANSWER:
696

44. –24 + (–18) + (–12) + … + 72

ANSWER:
408

45. $a_1 = -16$, $d = 6$, $n = 24$

ANSWER:
1272

46. $n = 19$, $a_n = 154$, $d = 8$

ANSWER:
1558

47. **CONTESTS** The prizes in a weekly radio contest began at \$150 and increased by \$50 for each week that the contest lasted. If the contest lasted for eleven weeks, how much was awarded in total?

ANSWER:
\$4400

Find the first three terms of each arithmetic series.

48. $n = 32$, $a_n = -86$, $S_n = 224$

ANSWER:
100, 94, 88

49. $a_1 = 48$, $a_n = 180$, $S_n = 1368$

ANSWER:
48, 60, 72

50. $a_1 = 3$, $a_n = 66$, $S_n = 759$

ANSWER:
3, 6, 9

51. $n = 28$, $a_n = 228$, $S_n = 2982$

ANSWER:
–15, –6, 3

52. $a_1 = -72$, $a_n = 453$, $S_n = 6858$

ANSWER:
–72, –57, –42

53. $n = 30$, $a_n = 362$, $S_n = 4770$

ANSWER:
–44, –30, –16

54. $a_1 = 19$, $n = 44$, $S_n = 9350$

ANSWER:
19, 28, 37

55. $a_1 = -33$, $n = 36$, $S_n = 6372$

ANSWER:
–33, –21, –9

56. **PRIZES** A radio station is offering a total of \$8500 in prizes over ten hours. Each hour, the prize will increase by \$100. Find the amounts of the first and last prize.

ANSWER:
\$400 and \$1300

Find the sum of each arithmetic series.

57. $\sum_{k=1}^{16}(4k-2)$

ANSWER:
512

58. $\sum_{k=4}^{13}(4k+1)$

ANSWER:
350

59. $\sum_{k=5}^{16}(2k+6)$

ANSWER:
324

60. $\sum_{k=0}^{12}(-3k+2)$

ANSWER:
–208

61. **FINANCIAL LITERACY** Daniela borrowed some money from her parents. She agreed to pay $50 at the end of the first month and $25 more each additional month for 12 months. How much does she pay in total after the 12 months?

ANSWER:
$2250

62. **GRAVITY** When an object is in free fall and air resistance is ignored, it falls 16 feet in the first second, an additional 48 feet during the next second, and 80 feet during the third second. How many total feet will the object fall in 10 seconds?

ANSWER:
1600 ft

Use the given information to write an equation that represents the *n*th term in each arithmetic sequence.

63. The 100th term of the sequence is 245. The common difference is 13.

ANSWER:
$a_n = 13n - 1055$

64. The eleventh term of the sequence is 78. The common difference is –9.

ANSWER:
$a_n = -9n + 177$

65. The sixth term of the sequence is –34. The 23rd term is 119.

ANSWER:
$a_n = 9n - 88$

66. The 25th term of the sequence is 121. The 80th term is 506.

ANSWER:
$a_n = 7n - 54$

67. **CCSS MODELING** The rectangular tables in a reception hall are often placed end-to-end to form one long table. The diagrams below show the number of people who can sit at each of the table arrangements.

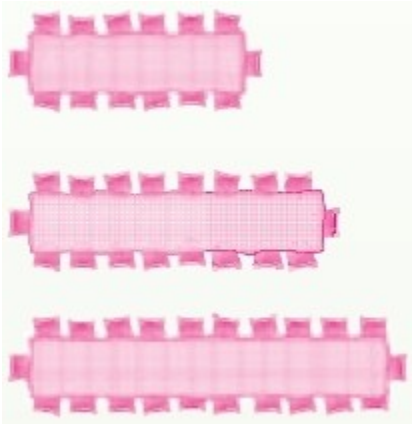
a. Make drawings to find the next three numbers as tables are added one at a time to the arrangement.

b. Write an equation representing the *n*th number in this pattern.

c. Is it possible to have seating for exactly 100 people with such an arrangement? Explain.

ANSWER:

a. 14, 18, 22



b. $p_n = 4n + 2$

c. No; there is no whole number n for which $4n + 2 = 100$.

68. **PERFORMANCE** A certain company pays its employees according to their performance. Belinda is paid a flat rate of $200 per week plus $24 for every unit she completes. If she earned $512 in one week, how many units did she complete?

ANSWER:

13

69. **SALARY** Terry currently earns $28,000 per year. If Terry expects a $4000 increase in salary every year, after how many years will he have a salary of $100,000 per year?

ANSWER:

the 19th year

70. **SPORTS** While training for cross country, Silvia plans to run 3 miles per day for the first week, and then increase the distance by a half mile each of the following weeks.

a. Write an equation to represent the *n*th term of the sequence.

b. If the pattern continues, during which week will she be running 10 miles per day?

c. Is it reasonable for this pattern to continue indefinitely? Explain.

ANSWER:

a. $a_n = 2.5 + 0.5n$

b. 15th wk

c. Sample answer: No; eventually the number of miles per day will become unrealistic.

71. **MUTIPLE REPRESENTATIONS** Consider $\sum_{k=1}^{x}(2k+2)$.

a. TABULAR Make a table of the partial sums of

the series for $1 \le k \le 10$.

b. GRAPHICAL Graph (k, partial sum).

c. GRAPHICAL Graph $f(x) = x^2 + 3x$ on the same grid.

d. VERBAL What do you notice about the two graphs?

e. ANALYTICAL What conclusions can you make about the relationship between quadratic functions and the sum of arithmetic series?

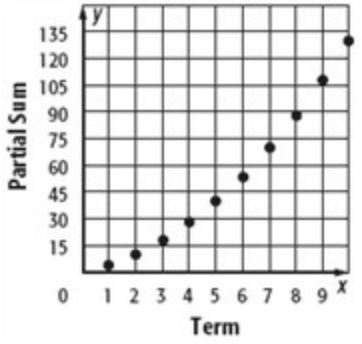
f. ALGEBRAIC Find the arithmetic series that relates to $g(x) = x^2 + 8x$.

ANSWER:

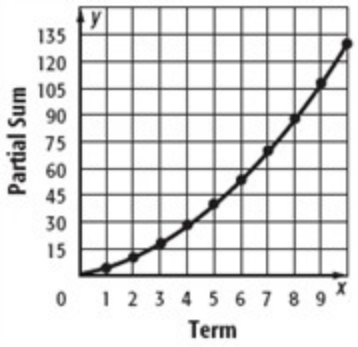
a.

n	S_n
1	4
2	10
3	18
4	28
5	40
6	54
7	70
8	88
9	108
10	130

b.



c.



d. Sample answer: The graphs cover the same range. The domain of the series is the natural numbers, while the domain of the quadratic function is all real numbers, $0 \le x \le 10$.

e. Sample answer: For every partial sum of an arithmetic series, there is a corresponding quadratic function that shares the same range.

f. $\sum_{k=1}^{x} 2k + 7$

Find the value of x.

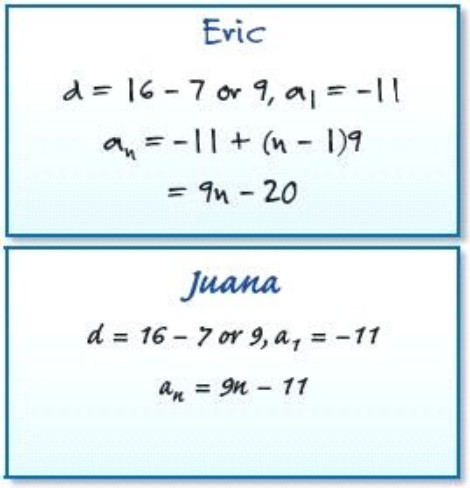
72. $\sum_{k=3}^{x} (6k - 5) = 928$

ANSWER:
18

73. $\sum_{k=5}^{x} (8k + 2) = 1032$

ANSWER:
16

74. **CCSS CRITIQUE** Eric and Juana are determining the formula for the *n*th term for the sequence –11, –2, 7, 16, … . Is either of them correct? Explain your reasoning.



ANSWER:
Sample answer: Eric; Juana missed the step of multiplying d by $n - 1$.

75. **REASONING** If a is the third term in an arithmetic sequence, b is the fifth term, and c is the eleventh term, express c in terms of a and b.

ANSWER:
$4b - 3a$

76. **CHALLENGE** There are three arithmetic means between a and b in an arithmetic sequence. The average of the arithmetic means is 16. What is the average of a and b?

ANSWER:
16

77. **CHALLENGE** Find S_n for $(x + y) + (x + 2y) + (x + 3y) + \ldots$.

ANSWER:
$$S_n = nx + y\left(\frac{n^2 + n}{2}\right)$$

78. **OPEN ENDED** Write an arithmetic series with 8 terms and a sum of 324.

ANSWER:
Sample answer: 9 + 18 + 27 + … + 72

79. **WRITING IN MATH** Compare and contrast arithmetic sequences and series.

ANSWER:
Sample answer: An arithmetic sequence is a list of terms such that any pair of successive terms has a common difference. An arithmetic series is the sum of the terms of an arithmetic sequence.

80. **PROOF** Prove the formula for the *n*th term of an arithmetic sequence.

ANSWER:
Sample answer:
Let a_n = the nth term of the sequence and d = the common difference
$a_2 = a_1 + d$ Definition of the second term of an arithmetic sequence
$a_3 = a_2 + d$ Definition of the third term of an arithmetic sequence
$a_3 = (a_1 + d) + d$ Substitution
$a_3 = a_1 + 2d$ Associative Property of Addition
$a_3 = a_1 + (3 - 1)d$ $3 - 1 = 2$
$a_n = a_1 + (n - 1)d$ $n = 3$

81. **PROOF** Derive a sum formula that does not include a_1.

ANSWER:

$S_n = (a_1 + a_n) \cdot \left(\frac{n}{2}\right)$ General sum formula

$a_n = a_1 + (n - 1)d$ Formula for nth term

$a_n - (n - 1)d = a_1$ Subtract $(n - 1)d$ from both sides

.

$S_n = [a_n - (n-1)d + a_n] \cdot \left(\frac{n}{2}\right)$ Substitution

$S_n = [2a_n - (n-1)d] \cdot \left(\frac{n}{2}\right)$ Simplify.

82. **PROOF** Derive the Alternate Sum Formula using the General Sum Formula.

ANSWER:

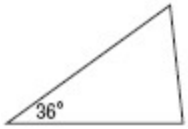
$S_n = (a_1 + a_n) \cdot \left(\frac{n}{2}\right)$ General sum formula

$a_n = a_1 + (n - 1)d$ Formula for nth term

$S_n = [a_1 + a_1 + (n-1)d] \cdot \left(\frac{n}{2}\right)$ Substitution

$S_n = [2a_1 + (n-1)d] \cdot \left(\frac{n}{2}\right)$ Simplify.

83. **SAT/ACT** The measures of the angles of a triangle form an arithmetic sequence. If the measure of the smallest angle is 36°, what is the measure of the largest angle?



A 54°

B 75°

C 84°

D 90°

E 97°

ANSWER:
C

84. The area of a triangle is $\frac{1}{2}q^2 - 8$ and the height is q + 4. Which expression best describes the triangle's length?

F $(q + 1)$

G $(q + 2)$

H $(q - 3)$

J $(q - 4)$

ANSWER:
J

85. The expression $1+\sqrt{2}+\sqrt[3]{3}$ is equivalent to

A $\sum_{k=1}^{3} k^{\frac{1}{k}}$

B $\sum_{k=1}^{3} k^{k}$

C $\sum_{k=1}^{3} k^{-k}$

D $\sum_{k=1}^{3} \sqrt{k}$

ANSWER:
A

86. **SHORT RESPONSE** Trevor can type a 200-word essay in 6 hours. Minya can type the same essay in $4\frac{1}{2}$ hours. If they work together, how many hours will it take them to type the essay?

ANSWER:
$2\frac{4}{7}$

Determine whether each sequence is arithmetic. Write *yes* or *no*.

87. –6, 4, 14, 24, …

ANSWER:
yes

88. $2, \frac{7}{5}, \frac{4}{5}, \frac{1}{5}, \ldots$

ANSWER:
yes

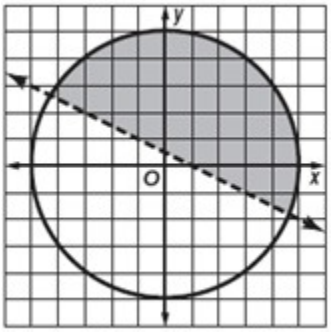
89. 10, 8, 5, 1, ...

ANSWER:
no

Solve each system of inequalities by graphing.

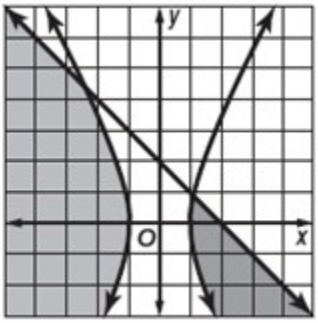
90. $x+2y>1$
$x^2+y^2 \le 25$

ANSWER:



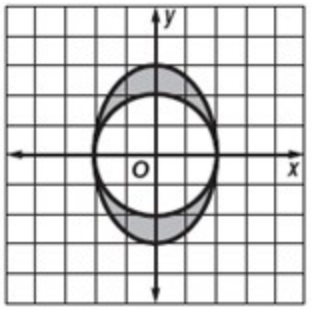
91. $x+y \le 2$
$4x^2-y^2 \ge 4$

ANSWER:



92. $x^2 + y^2 \geq 4$
$4y^2 + 9x^2 \leq 36$

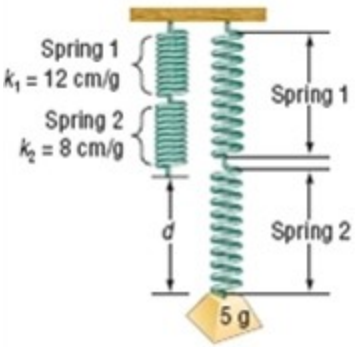
ANSWER:



93. **PHYSICS** The distance a spring stretches is related to the mass attached to the spring. This is represented by $d = km$, where d is the distance, m is the mass, and k is the spring constant. When two springs with spring constants k_1 and k_2 are attached in a series, the resulting spring constant k is found by the equation $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$.

a. If one spring with constant of 12 centimeters per gram is attached in a series with another spring with constant of 8 centimeters per gram, find the resultant spring constant.

b. If a 5-gram object is hung from the series of springs, how far will the springs stretch? Is this answer reasonable in this context?



ANSWER:

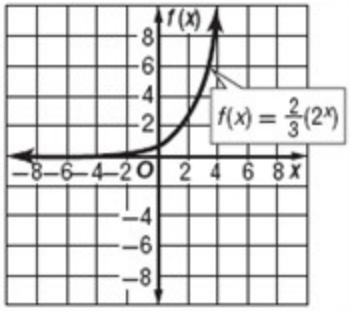
a. 4.8 cm/g

b. 24 cm; The answer is reasonable. The object would stretch the first spring 60 cm and would stretch the second spring 40 cm. The object would have to stretch the combined springs less than it would stretch either of the springs individually.

Graph each function. State the domain and range.

94. $f(x) = \frac{2}{3}(2^x)$

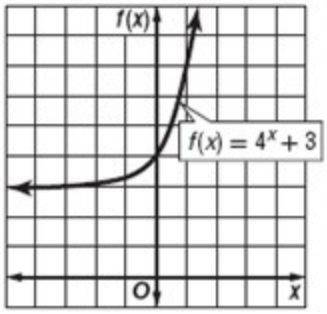
ANSWER:



D = {all real numbers}, R = $\{f(x) \mid f(x) > 0\}$

95. $f(x) = 4^x + 3$

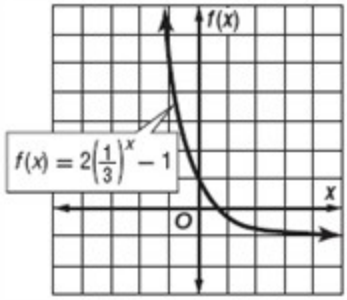
ANSWER:



D = {all real numbers}, R = $\{f(x) \mid f(x) > 3\}$

96. $f(x) = 2\left(\frac{1}{3}\right)^x - 1$

ANSWER:



D = {all real numbers}, R = $\{f(x) \mid f(x) > -1\}$

Solve each equation. Round to the nearest ten-thousandth.

97. $5^x = 52$

ANSWER:
2.4550

98. $4^{3p} = 10$

ANSWER:
0.5537

99. $3^{n+2} = 14.5$

ANSWER:
0.4341

100. $16^{d-4} = 3^{3-d}$

ANSWER:
3.7162