10-3 Geometric Sequences and Series

1. **CCSS REGULARITY** Dean is making a family tree for his grandfather. He was able to trace many generations. If Dean could trace his family back 10 generations, starting with his parents how many ancestors would there be?

ANSWER:
2046

Write an equation for the *n*th term of each geometric sequence.

2. 2, 4, 8, …

ANSWER:
$a_n = 2 \cdot 2^{n-1}$

3. 18, 6, 2, …

ANSWER:
$a_n = 18 \cdot \left(\frac{1}{3}\right)^{n-1}$

4. –4, 16, –64, …

ANSWER:
$a_n = (-4) \cdot (-4)^{n-1}$

5. $a_2 = 4$, $r = 3$

ANSWER:
$a_n = \frac{4}{3}(3)^{n-1}$

6. $a_6 = \frac{1}{8}, r = \frac{3}{4}$

ANSWER:
$a_n = \frac{128}{243}\left(\frac{3}{4}\right)^{n-1}$

7. $a_2 = -96$, $r = -8$

ANSWER:
$a_n = 12(-8)^{n-1}$

Find the geometric means of each sequence.

8. 0.25, ___?___, ___?___, ___?___, 64

ANSWER:
1, 4, 16 or –1, 4, –16]

9. 0.20, ___?___, ___?___, ___?___, 125

ANSWER:
1, 5, 25 or –1, 5, –25

10. **GAMES** Miranda arranges some rows of dominoes so that after she knocks over the first one, each domino knocks over two more dominoes when it falls. If there are ten rows, how many dominoes does Miranda use?

ANSWER:
1023

Find the sum of each geometric series.

11. $\sum_{k=1}^{6} 3(4)^{k-1}$

ANSWER:
4095

12. $\sum_{k=1}^{8} 4\left(\frac{1}{2}\right)^{k-1}$

ANSWER:
7.96875

Find a_1 for each geometric series described.

13. $S_n = 85\frac{5}{16}, r = 4, n = 6$

ANSWER:
$\frac{1}{16}$

14. $S_n = 91\frac{1}{12}, r = 3, n = 7$

ANSWER:
$\frac{1}{12}$

15. $S_n = 1020, a_n = 4, r = \frac{1}{2}$

ANSWER:
512

16. $S_n = 121\frac{1}{3}, a_n = \frac{1}{3}, r = \frac{1}{3}$

ANSWER:
81

17. **WEATHER** Heavy rain in Brieanne's town caused the river to rise. The river rose three inches the first day, and each day after rose twice as much as the previous day. How much did the river rise in five days?

ANSWER:
93 in.

Find a_n for each geometric sequence.

18. $a_1 = 2400, r = \frac{1}{4}, n = 7$

ANSWER:
$\frac{75}{128}$ or 0.5859375

19. $a_1 = 800, r = \frac{1}{2}, n = 6$

ANSWER:
25

20. $a_1 = \frac{2}{9}, r = 3, n = 7$

ANSWER:
162

10-3 Geometric Sequences and Series

21. $a_1 = -4, r = -2, n = 8$

ANSWER:
512

22. **BIOLOGY** A certain bacteria grows at a rate of 3 cells every 2 minutes. If there were 260 cells initially, how many are there after 21 minutes?

ANSWER:
864,567

Write an equation for the *n*th term of each geometric sequence.

23. –3, 6, –12, …

ANSWER:
$a_n = (-3)(-2)^{n-1}$

24. 288, –96, 32, …

ANSWER:
$a_n = 288\left(-\frac{1}{3}\right)^{n-1}$

25. –1, 1, –1, …

ANSWER:
$a_n = (-1)(-1)^{n-1}$

26. $\frac{1}{3}, \frac{2}{9}, \frac{4}{27}, \ldots$

ANSWER:
$a_n = \frac{1}{3}\left(\frac{2}{3}\right)^{n-1}$

27. $8, 2, \frac{1}{2}, \ldots$

ANSWER:
$a_n = 8 \cdot \left(\frac{1}{4}\right)^{n-1}$

28. $12, -16, \frac{64}{3}, \ldots$

ANSWER:
$a_n = 12 \cdot \left(-\frac{4}{3}\right)^{n-1}$

29. $a_3 = 28$, $r = 2$

ANSWER:
$a_n = 7(2)^{n-1}$

30. $a_4 = -8$, $r = 0.5$

ANSWER:
$a_n = -64(0.5)^{n-1}$

31. $a_6 = 0.5$, $r = 6$

ANSWER:
$a_n = \frac{1}{15{,}552}(6)^{n-1}$

32. $a_3 = 8, r = \frac{1}{2}$

ANSWER:
$a_n = 32\left(\frac{1}{2}\right)^{n-1}$

33. $a_4 = 24, r = \frac{1}{3}$

ANSWER:
$a_n = 648\left(\frac{1}{3}\right)^{n-1}$

34. $a_4 = 80$, $r = 4$

ANSWER:
$a_n = \frac{5}{4}(4)^{n-1}$

Find the geometric means of each sequence.

35. $810, \underline{?}, \underline{?}, \underline{?}, 10$

ANSWER:
270, 90, 30 or –270, 90, –30

36. $640, \underline{?}, \underline{?}, \underline{?}, 2.5$

ANSWER:
160, 40, 10 or –160, 40, –10

37. $\frac{7}{2}, \underline{?}, \underline{?}, \underline{?}, \frac{56}{81}$

ANSWER:
$\frac{7}{3}, \frac{14}{9}, \frac{28}{27}$ or $-\frac{7}{3}, \frac{14}{9}, -\frac{28}{27}$

38. $\frac{729}{64}, \underline{?}, \underline{?}, \underline{?}, \frac{324}{9}$

ANSWER:
$\frac{243}{16}, \frac{81}{4}, 27$ or $-\frac{243}{16}, \frac{81}{4}, -27$

39. Find two geometric means between 3 and 375.

ANSWER:
15, 75

40. Find two geometric means between 16 and –2.

ANSWER:
–8, 4

41. **CCSS PERSEVERANCE** A certain water filtration system can remove 70% of the contaminants each time a sample of water is passed through it. If the same water is passed through the system four times, what percent of the original contaminants will be removed from the water sample?

ANSWER:
99.19%

Find the sum of each geometric series.

42. $a_1 = 36, r = \frac{1}{3}, n = 8$

ANSWER:
53.9918

43. $a_1 = 16, r = \frac{1}{2}, n = 9$

ANSWER:
31.9375

44. $a_1 = 240, r = \frac{3}{4}, n = 7$

ANSWER:
831.855

45. $a_1 = 360, r = \frac{4}{3}, n = 8$

ANSWER:
9707.82

46. **VACUUMS** A vacuum claims to pick up 80% of the dirt every time it is run over the carpet. Assuming this is true, what percent of the original amount of dirt is picked up after the seventh time the vacuum is run over the carpet?

ANSWER:
99.99%

Find the sum of each geometric series.

47. $\sum_{k=1}^{7} 4(-3)^{k-1}$

ANSWER:
2188

48. $\sum_{k=1}^{8} (-3)(-2)^{k-1}$

ANSWER:
255

49. $\sum_{k=1}^{9} (-1)(4)^{k-1}$

ANSWER:
–87, 381

50. $\sum_{k=1}^{10} 5(-1)^{k-1}$

ANSWER:
0

Find a_1 for each geometric series described.

51. $S_n = -2912$, $r = 3$, $n = 6$

ANSWER:
–8

52. $S_n = -10{,}922$, $r = 4$, $n = 7$

ANSWER:
–2

53. $S_n = 1330, a_n = 486, r = \frac{3}{2}$

ANSWER:
64

54. $S_n = 4118, a_n = 128, r = \frac{2}{3}$

ANSWER:
1458

55. $a_n = 1024$, $r = 8$, $n = 5$

ANSWER:
0.25

56. $a_n = 1875$, $r = 5$, $n = 7$

ANSWER:
$\frac{3}{25}$

57. **SCIENCE** One minute after it is released, a gas-filled balloon has risen 100 feet. In each succeeding minute, the balloon rises only 50% as far as it rose in the previous minute. How far will it rise in 5 minutes?

ANSWER:
193.75 ft

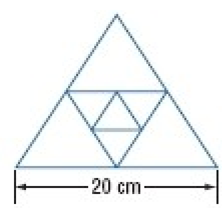
58. **CHEMISTRY** Radon has a half-life of about 4 days. This means that about every 4 days, half of the mass of radon decays into another element. How many grams of radon remain from an initial 60 grams after 4 weeks?

ANSWER:
about 0.46875 g

59. **CCSS REASONING** A virus goes through a computer, infecting the files. If one file was infected initially and the total number of files infected doubles every minute, how many files will be infected in 20 minutes?

ANSWER:
524,288

60. **GEOMETRY** In the figure, the sides of each equilateral triangle are twice the size of the sides of its inscribed triangle. If the pattern continues, find the sum of the perimeters of the first eight triangles.



ANSWER:
about 119.5 cm

61. **PENDULUMS** The first swing of a pendulum travels 30 centimeters. If each subsequent swing travels 95% as far as the previous swing, find the total distance traveled by the pendulum after the 30th swing.

ANSWER:
about 471 cm

62. **PHONE CHAINS** A school established a phone chain in which every staff member calls two other staff members to notify them when the school closes due to weather. The first round of calls begins with the superintendent calling both principals. If there are 94 total staff members and employees at the school, how many rounds of calls are there?

ANSWER:
7

63. **TELEVISIONS** High Tech Electronics advertises a weekly installment plan for the purchase of a popular brand of high definition television. The buyer pays \$5 at the end of the first week, \$5.50 at the end of the second week, \$6.05 at the end of the third week, and so on got one year. Assume 1 year = 52 weeks)

a. What will the payments be at the end of the 10th, 20th, and 40th weeks?

b. Find the total cost of the TV.

c. Why is the cost found in part **b** not entirely accurate?

ANSWER:
a. \$11.79, \$30.58, \$205.72

b. \$7052.15

c. Each payment made is rounded to the nearest penny, so the sum of the payments will actually be more than the sum found in part b.

64. **PROOF** Derive the General Sum Formula using theAlternate Sum Formula.

ANSWER:

$S_n = \dfrac{a_1 - a_1 r^n}{1-r}$	General sum formula
$= \dfrac{a_1 - a_1 r^{n-1} \cdot r}{1-r}$	$r \cdot r^{n-1} = r^n$
$a_n = a_1 \cdot r^{n-1}$	Formula for *n*th term
$S_n = \dfrac{a_1 - a_n r}{1-r}$	Substitution

65. **PROOF** Derive a sum formula that does not include a_1.

ANSWER:

$S_n = \dfrac{a_1 - a_n r}{1-r}$	Alternate sum formula
$a_n = a_1 \cdot r^{n-1}$	Formula for nth term
$\dfrac{a_n}{r^{n-1}} = a_1$	Divide both sides by r^{n-1}
$S_n = \dfrac{\frac{a_n}{r^{n-1}} - a_n r}{1-r}$	Substitution.
$= \dfrac{\frac{a_n}{r^{n-1}} - \frac{a_n r \cdot r^{n-1}}{r^{n-1}}}{1-r}$	Multiply by $= \dfrac{\frac{r^{n-1}}{r^{n-1}}}{1}$.
$= \dfrac{\frac{a_n(1-r^n)}{r^{n-1}}}{1-r}$	Simplify.
$= \dfrac{a_n(1-r^n)}{r^{n-1}(1-r)}$	Divide by $(1-r)$.
$= \dfrac{a_n(1-r^n)}{r^{n-1}-r^n}$	Simplify.

66. **OPEN ENDED** Write a geometric series for which $r = \dfrac{3}{4}$ and $n = 6$.

ANSWER:
Sample answer:

$$256 + 192 + 144 + 108 + 81 + \frac{243}{4}$$

67. **REASONING** Explain how $\sum_{k=1}^{10} 3(2)^{k-1}$ needs to be altered to refer to the same series if k = 1 changes to k = 0. Explain your reasoning.

ANSWER:
Sample answer: $n - 1$ needs to change to n, and the 10 needs to change to a 9. When this happens, the terms for both series will be identical (a_1 in the first series will equal a_0 in the second series, and so on), and the series will be equal to each other.

68. **PROOF** Prove the formula for the nth term of a geometric sequence.

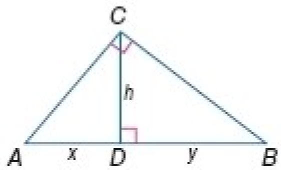
ANSWER:
Sample answer: Let a_n = the nth term of the sequence and r = the common ratio.

$a_2 = a_1 \cdot r$ Definition of the second term of a geometric sequence
$a_3 = a_2 \cdot r$ Definition of the third term of a geometric sequence
$a_3 = a_1 \cdot r \cdot r$ Substitution
$a_3 = a_1 \cdot r^2$ Associative Property of Multiplication
$a_3 = a_1 \cdot r^{3-1}$ $3 - 1 = 2$
$a_n = a_1 \cdot r^{n-1}$ $n = 3$

69. **CHALLENGE** The fifth term of a geometric sequence is $\frac{1}{27}$th of the eighth term. If the ninth term is 702, what is the eighth term?

ANSWER:
234

70. **CHALLENGE** Use the fact that h is the geometric mean between x and y in the figure to find h^4 in terms of x and y.



ANSWER:
x^2y^2

71. **OPEN ENDED** Write a geometric series with 6 terms and a sum of 252.

ANSWER:
Sample answer: 4 + 8 + 16 + 32 + 64 + 128

72. **WRITING IN MATH** How can you classify a sequence? Explain your reasoning.

ANSWER:
Sample answer: A series is arithmetic if every pair of consecutive terms shares a common difference. A series is geometric if every pair of consecutive terms shares a common ratio. If the series displays both qualities, then it is both arithmetic and geometric. If the series displays neither quality, then it is neither geometric nor arithmetic.

73. Which of the following is closest to $\sqrt[3]{7.32}$?

A 1.8

B 1.9

C 2.0

D 2.1

ANSWER:
B

74. The first term of a geometric series is 5, and the common ratio is −2. How many terms are in the series if its sum is -6825?

F 5

G 9

H 10

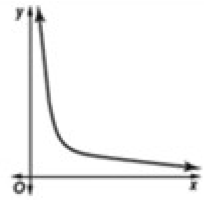
J 12

ANSWER:
J

75. **SHORT RESPONSE** Danette has a savings account. She withdraws half of the contents every year. After 4 years, she has $2000 left. How much did she have in the savings account originally?

ANSWER:
$32,000

76. **SAT/ACT** The curve below could be part of the graph of which function?



A $y = \sqrt{x}$

B $y = x^2 - 5x + 4$

C $y = -x + 20$

D $y = \log x$

E $xy = 4$

ANSWER:
E

77. **MONEY** Elena bought a high-definition LCD television at the electronics store. She paid $200 immediately and $75 each month for a year and a half. How much did Elena pay in total for the TV?

ANSWER:
$1550

Determine whether each sequence is *arithmetic, geometric,* or *neither*. Explain your reasoning.

78. $\frac{1}{10}, \frac{3}{5}, \frac{7}{20}, \frac{17}{20}, \ldots$

ANSWER:
Neither; there is no common difference or ratio.

79. $-\frac{7}{25}, -\frac{13}{50}, -\frac{6}{25}, -\frac{11}{50}, \ldots$

ANSWER:

Arithmetic; the common difference is $\frac{1}{50}$.

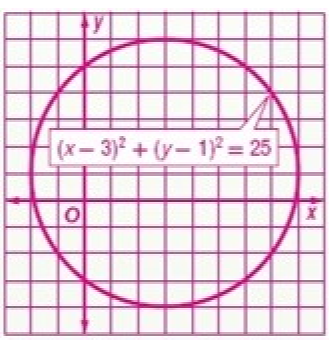
80. $-\frac{22}{3}, -\frac{68}{9}, -\frac{208}{27}, -\frac{632}{81}, \ldots$

ANSWER:
Neither; there is no common difference or ratio.

Find the center and radius of each circle. Then graph the circle.

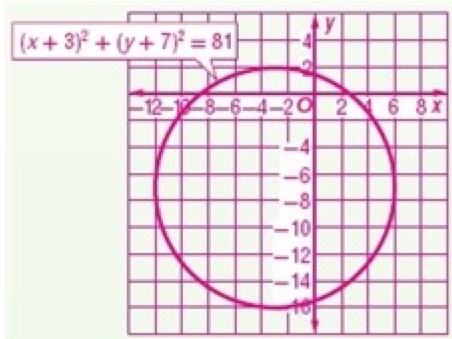
81. $(x - 3)^2 + (y - 1)^2 = 25$

ANSWER:
(3, 1), 5 units



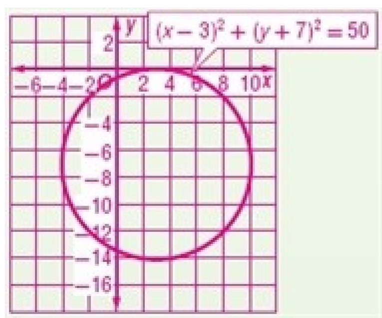
82. $(x + 3)^2 + (y + 7)^2 = 81$

ANSWER:
(–3, –7), 9 units



83. $(x - 3)^2 + (y + 7)^2 = 50$

ANSWER:
$(3, -7), 5\sqrt{2}$ units



84. Suppose y varies jointly as x and z. Find y when x = 9 and z = –5, if y = –90 when z = 15 and x = –6.

ANSWER:
–45

85. **SHOPPING** A certain store found that the number of customers who will attend a sale can be modeled by $N = 125\sqrt[3]{100Pt}$, where N is the number of customers expected, P is the percent of the sale discount, and t is the number of hours the sale will last. Find the number of customers the store should expect for a sale that is 50% off and will last four hours.

ANSWER:
731 customers

Evaluate each expression if $a = -2$, $b = \frac{1}{3}$, and $c = -12$.

86. $\frac{3ab}{c}$

ANSWER:
$\frac{1}{6}$

87. $\frac{a-c}{a+c}$

ANSWER:
$-\frac{5}{7}$

88. $\frac{a^3-c}{b^2}$

ANSWER:
36

89. $\frac{c+3}{ab}$

ANSWER:
$\frac{27}{2}$