10-4 Infinite Geometric Series

Determine whether each infinite geometric series is *convergent* or *divergent*.

1. 16 – 8 + 4 – …

ANSWER:
convergent

2. 32 – 48 + 72 – …

ANSWER:
divergent

3. 0.5 + 0.7 + 0.98 + …

ANSWER:
divergent

4. 1 + 1 + 1 + …

ANSWER:
divergent

Find the sum of each infinite series, if it exists.

5. 440 + 220 + 110 + …

ANSWER:
880

6. 520 + 130 + 32.5 + …

ANSWER:
$693\frac{1}{3}$

7. $\frac{1}{4}+\frac{3}{8}+\frac{9}{16}+\ldots$

ANSWER:
No sum exists.

8. $\frac{32}{9}+\frac{16}{3}+8+\ldots$

ANSWER:
No sum exists.

9. **CCSS SENSE-MAKING** A certain drug has a half-life of 8 hours after it is administered to a patient. What percent of the drug is still in the patient's system after 24 hours?

ANSWER:
12.5%

Find the sum of each infinite series, if it exists.

10. $\sum_{k=1}^{\infty} 5 \cdot 4^{k-1}$

ANSWER:
No sum exists.

11. $\sum_{k=1}^{\infty} (-2) \cdot (0.5)^{k-1}$

ANSWER:
–4

12. $\sum_{k=1}^{\infty} 3 \cdot \left(\frac{4}{5}\right)^{k-1}$

ANSWER:
15

13. $\sum_{k=1}^{\infty} \frac{1}{2} \cdot \left(\frac{3}{4}\right)^{k-1}$

ANSWER:
2

Write each repeating decimal as a fraction.

14. $0.\overline{35}$

ANSWER:
$\frac{35}{99}$

15. $0.\overline{642}$

ANSWER:
$\frac{214}{333}$

Determine whether each infinite geometric series is *convergent* or *divergent*.

16. 21 + 63 + 189 + …

ANSWER:
divergent

17. 480 + 360 + 270 + …

ANSWER:
convergent

18. $\frac{3}{4} + \frac{9}{8} + \frac{27}{16} + \ldots$

ANSWER:
divergent

19. $\frac{5}{6} + \frac{10}{9} + \frac{40}{27} + \ldots$

ANSWER:
divergent

20. 0.1 + 0.01 + 0.001 + …

ANSWER:
convergent

21. 0.008 + 0.08 + 0.8 + …

ANSWER:
divergent

Find the sum of each infinite series, if it exists.

22. 18 + 21.6 + 25.92 + …

ANSWER:
No sum exists.

23. –3 – 4.2 – 5.88 – …

ANSWER:
No sum exists.

24. $\frac{1}{2}+\frac{1}{6}+\frac{1}{18}+\ldots$

ANSWER:

$\frac{3}{4}$

25. $\frac{12}{5}+\frac{6}{5}+\frac{3}{5}+\ldots$

ANSWER:

$\frac{24}{5}$

26. $21+14+\frac{28}{3}+\ldots$

ANSWER:

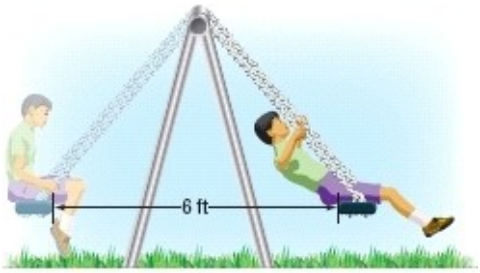
63

27. 32 + 40 + 50 + …

ANSWER:

No sum exists.

28. **SWINGS** If Kerry does not push any harder after his initial swing, the distance traveled per swing will decrease by 10% with each swing. If his initial swing traveled 6 feet, find the total distance traveled when he comes to rest.



ANSWER:

60 ft

Find the sum of each infinite series, if it exists.

29. $\sum_{k=1}^{\infty}\frac{4}{3}\cdot\left(\frac{5}{4}\right)^{k-1}$

ANSWER:

No sum exists.

30. $\sum_{k=1}^{\infty}\frac{1}{4}\cdot 3^{k-1}$

ANSWER:

No sum exists.

31. $\sum_{k=1}^{\infty}\frac{5}{3}\cdot\left(\frac{3}{7}\right)^{k-1}$

ANSWER:

$\frac{35}{12}$

32. $\sum_{k=1}^{\infty} \frac{2}{3} \cdot \left(\frac{4}{3}\right)^{k-1}$

ANSWER:
No sum exists.

33. $\sum_{k=1}^{\infty} \frac{8}{3} \cdot \left(\frac{5}{6}\right)^{k-1}$

ANSWER:
16

34. $\sum_{k=1}^{\infty} \frac{1}{8} \cdot \left(\frac{1}{12}\right)^{k-1}$

ANSWER:
$\frac{3}{22}$

Write each repeating decimal as a fraction.

35. $0.3\overline{21}$

ANSWER:
$\frac{53}{165}$

36. $0.1\overline{45}$

ANSWER:
$\frac{8}{55}$

37. $2.\overline{18}$

ANSWER:
$\frac{24}{11}$

38. $4.\overline{96}$

ANSWER:
$\frac{164}{33}$

39. $0.12\overline{14}$

ANSWER:
$\frac{601}{4950}$

40. $0.43\overline{36}$

ANSWER:
$\frac{477}{1100}$

41. **FANS** A fan is running at 10 revolutions per second. After it is turned off, its speed decreases at a rate of 75% per second. Determine the number of revolutions completed by the fan after it is turned off.

ANSWER:
$\frac{40}{3}$

42. **CCSS PRECISION** Kamiko deposited $5000 into an account at the beginning of the year. The account earns 8% interest each year.

a. How much money will be in the account after 20 years? (*Hint*: Let $5000(1 + 0.08)^1$ represent the end of the first year.)

b. Is this series *convergent* or *divergent*? Explain.

ANSWER:

a. $23,304.79

b. It is a diverging series. The ratio is 1.08, which is greater than 1.

43. **RECHARGEABLE BATTERIES** A certain rechargeable battery is advertised to recharge back to 99.9% of its previous capacity with every charge. If its initial capacity is 8 hours of life, how many total hours should the battery last?

ANSWER:

8000 hrs

Find the sum of each infinite series, if it exists.

44. $\frac{7}{5}+\frac{21}{20}+\frac{63}{80}+\ldots$

ANSWER:

$\frac{28}{5}$

45. $\frac{15}{4}+\frac{5}{2}+\frac{5}{3}+\ldots$

ANSWER:

$\frac{45}{4}$

46. $-\frac{16}{9}+\frac{4}{3}-1+\ldots$

ANSWER:

$-\frac{64}{63}$

47. $\frac{15}{8}+\frac{5}{2}+\frac{10}{3}+\ldots$

ANSWER:

No sum exists.

48. $\frac{21}{16}+\frac{7}{4}+\frac{7}{3}+\ldots$

ANSWER:

No sum exists.

49. $-\frac{18}{7}+\frac{12}{7}-\frac{8}{7}+\ldots$

ANSWER:

$-\frac{54}{35}$

50. **MULTIPLE REPRESENTATIONS** In this problem, you will use a square of paper that is at least 8 inches on a side.

    a. CONCRETE Let the square be one unit. Cut away one half of the square. Call this piece Term 1. Next, cut away one half of the remaining sheet of paper. Call this piece Term 2. Continue cutting the remaining paper in half and labeling the pieces with a term number as long as possible. List the fractions represented by the pieces.

    b. NUMERICAL If you could cut the squares indefinitely, you would have an infinite series. Find the sum of the series.

    c. VERBAL How does the sum of the series relate to the original square of paper?

    ANSWER:

    a. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

    b. 1

    c. The original square has area 1 unit and the area of all the pieces cannot exceed 1.

51. **PHYSICS** In a physics experiment, a steel ball on a flat track is accelerated, and then allowed to roll freely. After the first minute, the ball has rolled 120 feet. Each minute the ball travels only 40% as far as it did during the preceding minute. How far does the ball travel?

    ANSWER:
    200 ft

52. **PENDULUMS** A pendulum travels 12 centimeters on its first swing and 95% of that distance on each swing thereafter. Find the total distance traveled by the pendulum when it comes to rest.

    ANSWER:
    240 cm

53. **TOYS** If a rubber ball can bounce back to 95% of its original height, what is the total vertical distance that it will travel if it is dropped from an elevation of 30 feet?

    ANSWER:
    1170 ft

54. **CARS** During a maintenance inspection, a tire is removed from a car and spun on a diagnostic machine. When the machine is turned off, the spinning tire completes 20 revolutions the first second and 98% of the revolutions each additional second. How many revolutions does the tire complete before it stops spinning?

    ANSWER:
    1000 revolutions

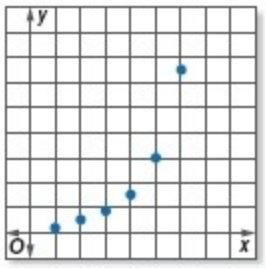
55. **ECONOMICS** A state government decides to stimulate its economy by giving $500 to every adult. The government assumes that everyone who receives the money will spend 80% on consumer goods and that the producers of these goods will in turn spend 80% on consumer goods. How much money is generated for the economy for every $500 that the government provides?

    ANSWER:
    $2500

56. **SCIENCE MUSEUM** An exhibit at a science museum offers visitors the opportunity to experiment with the motion of an object on a spring. One visitor pulls the object down and lets it go. The object travels 1.2 feet upward before heading back the other way. Each time the object changes direction, it decreases its distance by 20% when compared to the previous direction. Find the total distance traveled by the object.

ANSWER:
6 ft

Match each graph with its corresponding description.

57. 

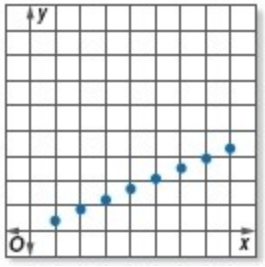
a. converging geometric series

b. diverging geometric series

c. converging arithmetic series

d. diverging arithmetic series

ANSWER:
b

58. 

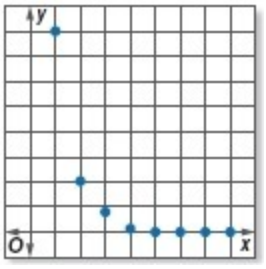
a. converging geometric series

b. diverging geometric series

c. converging arithmetic series

d. diverging arithmetic series

ANSWER:
d

59. 

a. converging geometric series

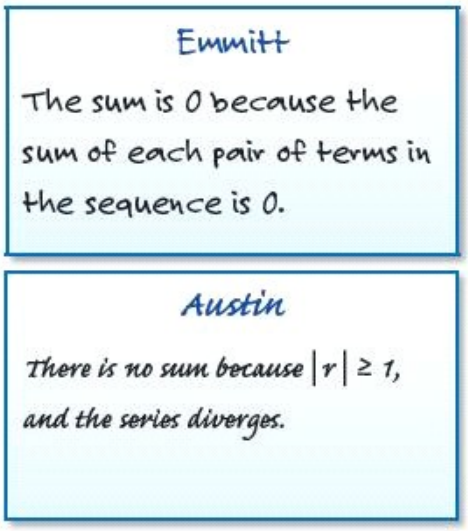
b. diverging geometric series

c. converging arithmetic series

d. diverging arithmetic series

ANSWER:
a

60. **ERROR ANALYSIS** Emmitt and Austin are asked to find the sum of $1 - 1 + 1 - \ldots$. Is either of them correct? Explain your reasoning.



ANSWER:

Sample answer: Austin; the common ratio of the series $r = -1$, so the absolute value of $r = 1$ and the series diverges.

61. **PROOF** Derive the formula for the sum of an infinite geometric series.

ANSWER:

Sample answer:

The sum of a geometric series is $S_n = \frac{a_1 - a_1 r^n}{1 - r}$.

For an infinite series with $|r| < 1$, $r^n \to 0 \text{ as } n \to \infty$.

Thus, $S = \frac{a_1 - a_1(0)}{1 - r}$ or $\frac{a_1}{1 - r}$.

62. **CHALLENGE** For what values of b does $3 + 9b + 27b^2 + 81b^3 + \ldots$ have a sum?

ANSWER:

$-\frac{1}{3} < b < \frac{1}{3}$

63. **REASONING** When does an infinite geometric series have a sum, and when does it not have a sum? Explain your reasoning.

ANSWER:

Sample answer: An infinite geometric series has a sum when the common ratio has an absolute value less than 1. When this occurs, the terms will approach 0 as n approaches infinity. With the future terms almost 0, the sum of the series will approach a limit. When the common ratio is 1 or greater, the terms will keep increasing and approach infinity as n approaches infinity and the sum of the series will have no limit.

64. **CCSS ARGUMENTS** Determine whether the following statement is sometimes, always, or never true. Explain your reasoning.
If the absolute value of a term of any geometric series is greater than the absolute value of the previous term, then the series is divergent.

ANSWER:

Sample answer: Sometimes; the statement is true for all *infinite* geometric series.

65. **OPEN ENDED** Write an infinite series with a sum that converges to 9.

ANSWER:

Sample answer:

$3 + 2 + \frac{4}{3} + \ldots$

66. **OPEN ENDED** Write 3 – 6 + 12 – … using sigma notation in two different ways.

ANSWER:

$\sum_{k=1}^{\infty} 3(-2)^{k-1}; \sum_{k=0}^{\infty} 3(-2)^{k}$

67. **WRITING IN MATH** Explain why an arithmetic series is always divergent.

ANSWER:

An arithmetic series has a common difference, so each term will eventually become more positive or more negative, but never approach 0. With the terms not approaching 0, the sum will never reach a limit and the series cannot converge.

68. **SAT/ACT** What is the sum of an infinite geometric series with a first term of 27 and a common ratio of $\frac{2}{3}$?

A 18

B 34

C 41

D 65

E 81

ANSWER:

E

69. Adelina, Michelle, Masao, and Brandon each simplified the same expression at the board. Each student's work is shown below. The teacher said that while two of them had a correct answer, only one of them had arrived at the correct conclusion using correct steps.

Adelina's work

$$x^2x^{-5} = \frac{x^2}{x^{-5}}$$
$$= x^7, x \neq 0$$

Masao's work

$$x^2x^{-5} = \frac{x^2}{x^5}$$
$$= \frac{1}{x^3}, x \neq 0$$

Michelle's work

$$x^2x^{-5} = \frac{x^2}{x^{-5}}$$
$$= x^{-3}, x \neq 0$$

Brandon's work

$$x^2x^{-5} = \frac{x^2}{x^5}$$
$$= x^3, x \neq 0$$

Which is a completely accurate simplification?

F Adelina's work

G Michelle's work

H Masao's work

J Brandon's work

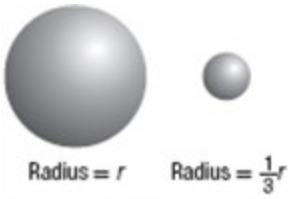
ANSWER:

H

70. **GRIDDED RESPONSE** Evaluate $\log_8 60$ to the nearest hundredth.

ANSWER:
1.97

71. **GEOMETRY** The radius of a large sphere was multiplied by a factor of $\frac{1}{3}$ to produce a smaller sphere.



How does the volume of the smaller sphere compare to the volume of the larger sphere?

A The volume of the smaller sphere is $\frac{1}{9}$ as large.

B The volume of the smaller sphere is $\frac{1}{\pi^3}$ as large.

C The volume of the smaller sphere is $\frac{1}{27}$ as large.

D The volume of the smaller sphere is $\frac{1}{3}$ as large.

ANSWER:
C

72. **GAMES** An audition is held for a TV game show. At the end of each round, one half of the prospective contestants are eliminated from the competition. On a particular day, 524 contestants begin the audition.

a. Write an equation for finding the number of contestants who are left after *n* rounds.

b. Using this method, will the number of contestants who are to be eliminated always be a whole number? Explain.

ANSWER:

a. $a_n = 524\left(\frac{1}{2}\right)^n$

b. No; at the beginning of the third round there will be 131 contestants and one-half of that is 65.5.

73. **CLUBS** A quilting club consists of 9 members. Every week, each member must bring one completed quilt square.

a. Find the first eight terms of the sequence that describes the total number of squares that have been made after each meeting.

b. One particular quilt measures 72 inches by 84 inches and is being designed with 4-inch squares. After how many meetings will the quilt be complete?

ANSWER:
a. 9, 18, 27, 36, 45, 54, 63, 72

b. 42 meetings

Find each function value.

74. $f(x) = 5x - 9,\ f(6)$

ANSWER:
21

75. $g(x) = x^2 - x$, $g(4)$

ANSWER:
12

76. $h(x) = x^2 - 2x - 1$, $h(3)$

ANSWER:
2