

10-5 Recursion and Iteration

Find the first five terms of each sequence described.

1. $a_1 = 16$, $a_{n+1} = a_n + 4$

ANSWER:

16, 20, 24, 28, 32

2. $a_1 = -3$, $a_{n+1} = a_n + 8$

ANSWER:

-3, 5, 13, 21, 29

3. $a_1 = 5$, $a_{n+1} = 3a_n + 2$

ANSWER:

5, 17, 53, 161, 485

4. $a_1 = -4$, $a_{n+1} = 2a_n - 6$

ANSWER:

-4, -14, -34, -74, -154

Write a recursive formula for each sequence.

5. 3, 8, 18, 38, 78, ...

ANSWER:

$$a_{n+1} = 2a_n + 2; a_1 = 3$$

6. 5, 14, 41, 122, 365, ...

ANSWER:

$$a_{n+1} = 3a_n - 1; a_1 = 5$$

7. **FINANCING** Ben financed a \$1500 rowing machine to help him train for the college rowing team. He could only make a \$100 payment each month, and his bill increased by 1% due to interest at the end of each month.

a. Write a recursive formula for the balance owed at the end of each month.

b. Find the balance owed after the first four months.

c. How much interest has accumulated after the first six months?

ANSWER:

a. $a_n = 1.01a_{n-1} - 100$; $a_1 = 1500$

b. \$1415, \$1329.15, \$1242.44, \$1154.87

c. \$77.08

Find the first three iterates of each function for the given initial value.

8. $f(x) = 5x + 2$, $x_0 = 8$

ANSWER:

42, 212, 1062

9. $f(x) = -4x + 2$, $x_0 = 5$

ANSWER:

-18, 74, -294

10. $f(x) = 6x + 3$, $x_0 = -4$

ANSWER:

-21, -123, -735

10-5 Recursion and Iteration

11. $f(x) = 8x - 4$, $x_0 = -6$

ANSWER:

-52, -420, -3364

CCSS PERSEVERANCE Find the first five terms of each sequence described.

12. $a_1 = 10$, $a_{n+1} = 4a_n + 1$

ANSWER:

10, 41, 165, 661, 2645

13. $a_1 = -9$, $a_{n+1} = 2a_n + 8$

ANSWER:

-9, -10, -12, -16, -24

14. $a_1 = 12$, $a_{n+1} = a_n + n$

ANSWER:

12, 13, 15, 18, 22

15. $a_1 = -4$, $a_{n+1} = 2a_n + n$

ANSWER:

-4, -7, -12, -21, -38

16. $a_1 = 6$, $a_{n+1} = 3a_n - n$

ANSWER:

6, 17, 49, 144, 428

17. $a_1 = -2$, $a_{n+1} = 5a_n + 2n$

ANSWER:

-2, -8, -36, -174, -862

18. $a_1 = 7$, $a_2 = 10$, $a_{n+2} = 2a_n + a_{n+1}$

ANSWER:

7, 10, 24, 44, 92

19. $a_1 = 4$, $a_2 = 5$, $a_{n+2} = 4a_n - 2a_{n+1}$

ANSWER:

4, 5, 6, 8, 8

20. $a_1 = 4$, $a_2 = 3x$, $a_n = a_{n-1} + 4a_{n-2}$

ANSWER:

4, 3x, 3x + 16, 15x + 16, 27x + 80

21. $a_1 = 3$, $a_2 = 2x$, $a_n = 4a_{n-1} - 3a_{n-2}$

ANSWER:

3, 2x, 8x - 9, 26x - 36, 80x - 117

22. $a_1 = 2$, $a_2 = x + 3$, $a_n = a_{n-1} + 6a_{n-2}$

ANSWER:

2, x + 3, x + 15, 7x + 33, 13x + 123

23. $a_1 = 1$, $a_2 = x$, $a_n = 3a_{n-1} + 6a_{n-2}$

ANSWER:

1, x, 3x + 6, 15x + 18, 63x + 90

10-5 Recursion and Iteration

Write a recursive formula for each sequence.

24. 16, 10, 7, 5.5, 4.75, ...

ANSWER:

$$a_{n+1} = 0.5a_n + 2; a_1 = 16$$

25. 32, 12, 7, 5.75, ...

ANSWER:

$$a_{n+1} = 0.25a_n + 4; a_1 = 32$$

26. 4, 15, 224, 50,175, ...

ANSWER:

$$a_{n+1} = (a_n)^2 - 1; a_1 = 4$$

27. 1, 2, 9, 730, ...

ANSWER:

$$a_{n+1} = (a_n)^3 + 1; a_1 = 1$$

28. 9, 33, 129, 513, ...

ANSWER:

$$a_{n+1} = 4a_n - 3; a_1 = 9$$

29. 480, 128, 40, 18, ...

ANSWER:

$$a_{n+1} = 0.25a_n + 8; a_1 = 480$$

30. 393, 132, 45, 16, ...

ANSWER:

$$a_{n+1} = \frac{a_n}{3} + 1; a_1 = 393$$

31. 68, 104, 176, 320, ...

ANSWER:

$$a_{n+1} = 2a_n - 32; a_1 = 84$$

32. **FINANCIAL LITERACY** Mr. Edwards and his company deposit \$20,000 into his retirement account at the end of each year. The account earns 8% interest before each deposit.

a. Write a recursive formula for the balance in the account at the end of each year.

b. Determine how much is in the account at the end of each of the first 8 years.

ANSWER:

a. $a_n = 1.08a_{n-1} + 20000$

b.

$$a_1 = \$20,000,$$

$$a_2 = \$41,600,$$

$$a_3 = \$64,928,$$

$$a_4 = \$90,122.24,$$

$$a_5 = \$117,332.02,$$

$$a_6 = \$146,718.58,$$

$$a_7 = \$178,456.07,$$

$$a_8 = \$212,732.56$$

10-5 Recursion and Iteration

Find the first three iterates of each function for the given initial value.

$$33.f(x) = 12x + 8, x_0 = 4$$

ANSWER:

56, 680, 8168

$$34.f(x) = -9x + 1, x_0 = -6$$

ANSWER:

55, -494, 4447

$$35.f(x) = -6x + 3, x_0 = 8$$

ANSWER:

-45, 273, -1635

$$36.f(x) = 8x + 3, x_0 = -4$$

ANSWER:

-29, -229, -1829

$$37.f(x) = -3x^2 + 9, x_0 = 2$$

ANSWER:

-3, -18, -963

$$38.f(x) = 4x^2 + 5, x_0 = -2$$

ANSWER:

21, 1769, 12,517,449

$$39.f(x) = 2x^2 - 5x + 1, x_0 = 6$$

ANSWER:

43, 3484, 24,259,093

$$40.f(x) = -0.25x^2 + x + 6, x_0 = 8$$

ANSWER:

-2, 3, 6.75

$$41.f(x) = x^2 + 2x + 3, x_0 = \frac{1}{2}$$

ANSWER:

4.25, 29.5625, 936.0664

$$42.f(x) = 2x^2 + x + 1, x_0 = -\frac{1}{2}$$

ANSWER:

1, 4, 37

10-5 Recursion and Iteration

43. **FRACTALS** Consider the figures at the right. The number of blue triangles increases according to a specific pattern.



- Write a recursive formula for the number of blue triangles in the sequence of figures.
- How many blue triangles will be in the sixth figure?

ANSWER:

- $a_n = 3a_{n-1}; a_1 = 1$
- 243

44. **FINANCIAL LITERACY** Miguel's monthly car payment is \$234.85. The recursive formula $b_n = 1.005b_{n-1} - 234.85$ describes the balance left on the loan after n payments. Find the balance of the \$10,000 loan after each of the first eight payments.

ANSWER:

\$9815.15, \$9629.38, \$9442.67, \$9255.04, \$9066.46, \$8876.94, \$8686.48, \$8495.06

45. **CONSERVATION** Suppose a lake is populated with 10,000 fish. A year later, 80% of the fish have died or been caught, and the lake is replenished with 10,000 new fish. If the pattern continues, will the lake eventually run out of fish? If not, will the population of the lake converge to any particular value? Explain.

ANSWER:

No; the population of fish will reach 12,500. Each year, 20% of 12,500 or 2500 fish, plus 10,000 additional fish, yields 12,500 fish.

46. **GEOMETRY** Consider the pattern.



- Write a sequence of the total number of triangles in the first six figures.
- Write a recursive formula for the number of triangles.
- How many triangles will be in the tenth figure?

ANSWER:

- 1, 4, 10, 19, 31, 46
- $a_n = a_{n-1} + 3(n-1)$
- 136

47. **SPREADSHEETS** Consider the sequence with $x_0 = 20,000$ and $f(x) = 0.3x + 5000$.

- Enter x_0 in cell A1 of your spreadsheet. Enter “ $= (0.3)*(A1) + 5000$ ” in cell A2. What answer does it provide?
- Copy cell A2, highlight cells A3 through A70, and paste. What do you notice about the sequence?
- How do spreadsheets help analyze recursive sequences?

ANSWER:

- 11,000
- It converges to 7142.857.
- Sample answer: They make it easier to analyze recursive sequences because they can produce the first 100 terms instantaneously; it would take a long time to calculate the terms by hand.

10-5 Recursion and Iteration

48. **VIDEO GAMES** The final monster in Helena's video game has 100 health points. During the final battle, the monster regains 10% of its health points after every 10 seconds. If Helena can inflict damage to the monster that takes away 10 health points every 10 seconds without getting hurt herself, will she ever kill the monster? If so, when?

ANSWER:

Yes; between 250 and 260 seconds

49. **CCSS CRITIQUE** Marcus and Armando are finding the first three iterates of $f(x) = 5x - 3$ for an initial value of $x_0 = 4$. Is either of them correct? Explain.

Marcus

$$f(4) = 5(4) - 3 \text{ or } 17$$
$$f(17) = 5(17) - 3 \text{ or } 82$$

The first three iterates are
4, 17, and 82.

Armando

$$f(4) = 5(4) - 3 \text{ or } 17$$
$$f(17) = 5(17) - 3 \text{ or } 82$$
$$f(82) = 5(82) - 3 \text{ or } 407$$

The first three iterates are
17, 82, and 407.

ANSWER:

Armando; Marcus included x_0 with the iterates and only showed the first 2 iterates.

50. **CHALLENGE** Find a recursive formula for 5, 23, 98, 401, ...

ANSWER:

$$a_{n+1} = 4a_n + 3n; a_1 = 5$$

51. **REASONING** Is the statement "If the first three terms of a sequence are identical, then the sequence is not recursive" sometimes, always, or never true? Explain your reasoning.

ANSWER:

Sample answer: Sometimes; the recursive formula could involve the first three terms. For example, 2, 2, 2, 8, 20, ... is recursive with $a_n + 3 = a_n + a_{n+1} + 2a_{n+2}$.

52. **OPEN ENDED** Write a function for which the first three iterates are 9, 19, and 39.

ANSWER:

Sample answer: $f(x) = 2x + 1, x_0 = 4$

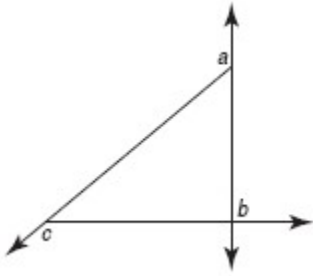
53. **WRITING IN MATH** Why is it useful to represent a sequence with an explicit or recursive formula?

ANSWER:

Sample answer: In a recursive sequence, each term is determined by one or more of the previous terms. A recursive formula is used to produce the terms of the recursive sequence.

10-5 Recursion and Iteration

54. **GEOMETRY** In the figure shown, $a + b + c = ?$



- A 180°
- B 270°
- C 360°
- D 450°

ANSWER:

C

55. **EXTENDED RESPONSE** Bill launches a model rocket from ground level. The rocket's height h in meters is given by the equation $h = -4.9t^2 + 56t$, where t is the time in seconds after the launch.

- a. What is the maximum height the rocket will reach?
- b. How long after it is launched will the rocket reach its maximum height? Round to the nearest tenth of a second.
- c. How long after it is launched will the rocket land? Round to the nearest tenth of a second.

ANSWER:

- a. 160 m
- b. 5.7 s
- c. 11.4 s

56. Which of the following is true about the graphs of $y = 3(x - 4)^2 + 5$ and $y = 3(x + 4)^2 + 5$?

- F Their vertices are maximums.
- G The graphs have the same shape with different vertices.
- H The graphs have different shapes with different vertices.
- J One graph has a vertex that is a maximum, while the other graph has a vertex that is a minimum.

ANSWER:

G

57. Which factors could represent the length times the width?

$$A = 16x^4 - 25y^2$$

- A $(4x - 5y)(4x - 5y)$
- B $(4x + 5y)(4x - 5y)$
- C $(4x^2 - 5y)(4x^2 + 5y)$
- D $(4x^2 + 5y)(4x^2 + 5y)$

ANSWER:

C

10-5 Recursion and Iteration

Write each repeating decimal as a fraction.

58. $0.\overline{7}$

ANSWER:

$$\frac{7}{9}$$

59. $5.\overline{126}$

ANSWER:

$$5\frac{14}{111}$$

60. $6.\overline{259}$

ANSWER:

$$6\frac{7}{27}$$

61. **SPORTS** Adrahan is training for a marathon, about 26 miles. He begins by running 2 miles. Then, when he runs every other day, he runs one and a half times the distance he ran the time before.

a. Write the first five terms of a sequence describing his training schedule.

b. When will he exceed 26 miles in one run?

c. When will he have run 100 total miles?

ANSWER:

a. 2, 3, 4.5, 6.75, 10.125

b. the eighth session

c. during the ninth session

State whether the events are *independent* or *dependent*.

62. tossing a penny and rolling a number cube

ANSWER:

independent

63. choosing first and second place in an academic competition

ANSWER:

dependent

Find each product.

64. $(y + 4)(y + 3)$

ANSWER:

$$y^2 + 7y + 12$$

65. $(x - 2)(x + 6)$

ANSWER:

$$x^2 + 4x - 12$$

66. $(a - 8)(a + 5)$

ANSWER:

$$a^2 - 3a - 40$$

67. $(4h + 5)(h + 7)$

ANSWER:

$$4h^2 + 33h + 35$$

10-5 Recursion and Iteration

68. $(9p - 1)(3p - 2)$

ANSWER:

$$27p^2 - 21p + 2$$

69. $(2g + 7)(5g - 8)$

ANSWER:

$$10g^2 + 19g - 56$$