

## 10-6 The Binomial Theorem

**Expand each binomial.**

1.  $(c + d)^5$

**ANSWER:**

$$c^5 + 5c^4d + 10c^3d^2 + 10c^2d^3 + 5cd^4 + d^5$$

2.  $(g + h)^7$

**ANSWER:**

$$g^7 + 7g^6h + 21g^5h^2 + 35g^4h^3 + 35g^3h^4 + 21g^2h^5 + 7gh^6 + h^7$$

3.  $(x - 4)^6$

**ANSWER:**

$$x^6 - 24x^5 + 240x^4 - 1280x^3 + 3840x^2 - 6144x + 4096$$

4.  $(2y - z)^5$

**ANSWER:**

$$32y^5 - 80y^4z + 80y^3z^2 - 40y^2z^3 + 10yz^4 - z^5$$

5.  $(x + 3)^5$

**ANSWER:**

$$x^5 + 15x^4 + 90x^3 + 270x^2 + 405x + 243$$

6.  $(y - 4z)^4$

**ANSWER:**

$$y^4 - 16y^3z + 96y^2z^2 - 256yz^3 + 256z^4$$

7. **GENETICS** If a woman is equally as likely to have a baby boy or a baby girl, use binomial expansion to determine the probability that 5 of her 6 children are girls. Do not consider identical twins.

**ANSWER:**

$$\frac{3}{32} \text{ or } 0.09375$$

**Find the indicated term of each expression.**

8. fourth term of  $(b + c)^9$

**ANSWER:**

$$84b^6c^3$$

9. fifth term of  $(x + 3y)^8$

**ANSWER:**

$$5670x^4y^4$$

10. third term of  $(a - 4b)^6$

**ANSWER:**

$$240a^4b^2$$

11. sixth term of  $(2c - 3d)^8$

**ANSWER:**

$$-108,864c^3d^5$$

12. last term of  $(5x + y)^5$

**ANSWER:**

$$y^5$$

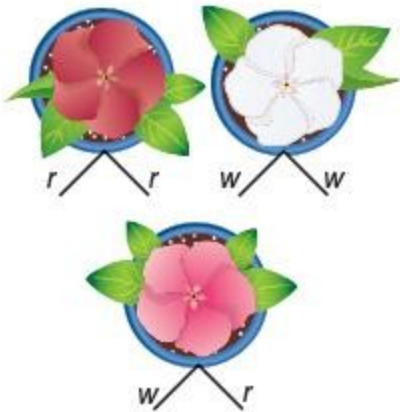
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13. first term of  $(3a + 8b)^5$

ANSWER:

$$243a^5$$

14. **CCSS MODELING** The color of a particular flower is determined by the combination of two genes, also called *alleles*. If the flower has two red alleles  $r$ , the flower is red. If the flower has two white alleles  $w$ , the flower is white. If the flower has one allele of each color, the flower will be pink. In a lab, two pink flowers are mated and eventually produce 1000 offspring. How many of the 1000 offspring will be pink?



ANSWER:

$$500$$

**Expand each binomial.**

15.  $(a - b)^6$

ANSWER:

$$a^6 - 6a^5b + 15a^4b^2 - 20a^3b^3 + 15a^2b^4 - 6ab^5 + b^6$$

16.  $(c - d)^7$

ANSWER:

$$c^7 - 7c^6d + 21c^5d^2 - 35c^4d^3 + 35c^3d^4 - 21c^2d^5 + 7cd^6 - d^7$$

17.  $(x + 6)^6$

ANSWER:

$$x^6 + 36x^5 + 540x^4 + 4320x^3 + 19,440x^2 + 46,656x + 46,656$$

18.  $(y - 5)^7$

ANSWER:

$$y^7 - 35y^6 + 525y^5 - 4375y^4 + 21,875y^3 - 65,625y^2 + 109,375y - 78,125$$

19.  $(2a + 4b)^4$

ANSWER:

$$16a^4 + 128a^3b + 384a^2b^2 + 512ab^3 + 256b^4$$

20.  $(3a - 4b)^5$

ANSWER:

$$243a^5 - 1620a^4b + 4320a^3b^2 - 5760a^2b^3 + 3840ab^4 - 1024b^5$$

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21. **COMMITTEES** If an equal number of men and women applied to be on a community planning committee and the committee needs a total of 10 people, find the probability that 7 of the members will be women. Assume that committee members will be chosen randomly.

ANSWER:

$$\frac{120}{1024} = \frac{15}{128} \approx 0.117$$

22. **BASEBALL** If a pitcher is just as likely to throw a ball as a strike, find the probability that 11 of his first 12 pitches are balls.

ANSWER:

$$\frac{12}{4096} = \frac{3}{1024} \approx 0.00293$$

Find the indicated term of each expression.

23. third term of  $(x + 2z)^7$

ANSWER:

$$84x^5z^2$$

24. fourth term of  $(y - 3x)^6$

ANSWER:

$$-540y^3x^3$$

25. seventh term of  $(2a - 2b)^8$

ANSWER:

$$7168a^2b^6$$

26. sixth term of  $(4x + 5y)^6$

ANSWER:

$$75,000xy^5$$

27. fifth term of  $(x - 4)^9$

ANSWER:

$$32,256x^5$$

28. fourth term of  $(c + 6)^8$

ANSWER:

$$12,096c^5$$

Expand each binomial.

29.  $\left(x + \frac{1}{2}\right)^5$

ANSWER:

$$x^5 + \frac{5}{2}x^4 + \frac{5}{2}x^3 + \frac{5}{4}x^2 + \frac{5}{16}x + \frac{1}{32}$$

30.  $\left(x - \frac{1}{3}\right)^4$

ANSWER:

$$x^4 - \frac{4}{3}x^3 + \frac{2}{3}x^2 - \frac{4}{27}x + \frac{1}{81}$$

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31.  $\left(2b + \frac{1}{4}\right)^5$

ANSWER:

$$32b^5 + 20b^4 + 5b^3 + \frac{5}{8}b^2 + \frac{5}{128}b + \frac{1}{1024}$$

32.  $\left(3c + \frac{1}{3}\right)^5$

ANSWER:

$$243c^5 + 135c^4 + 30c^3 + \frac{10}{3}c^2 + \frac{5}{27}c + \frac{1}{243}$$

33. **CCSS SENSE-MAKING** In  $\frac{n!}{k!(n-k)!}p^kq^{n-k}$ , let  $p$  represent the likelihood of a success and  $q$  represent the likelihood of a failure.

a. If a place-kicker makes 70% of his kicks within 40 yards, find the likelihood that he makes 9 of his next 10 attempts from within 40 yards.

b. If a quarterback completes 60% of his passes, find the likelihood that he completes 8 of his next 10 attempts.

c. If a team converts 30% of their two-point conversions, find the likelihood that they convert 2 of their next 5 conversions.

ANSWER:

a. 0.121

b. 0.121

c. 0.309

34. **CHALLENGE** Find the sixth term of the expansion of  $(\sqrt{a} + \sqrt{b})^{12}$ . Explain your reasoning.

ANSWER:

$$792a^3b^2\sqrt{ab}$$

35. **REASONING** Explain how the terms of  $(x + y)^n$  and  $(x - y)^n$  are the same and how they are different.

ANSWER:

Sample answer: While they have the same terms, the signs for  $(x + y)^n$  will all be positive, while the signs for  $(x - y)^n$  will alternate.

36. **REASONING** Determine whether the following statement is true or false. Explain your reasoning.

*The eighth and twelfth terms of  $(x + y)^{20}$  have the same coefficients.*

ANSWER:

Sample answer: False; a binomial to the 20<sup>th</sup> power will have 21 terms. The eleventh term will be in the middle and the rest of the terms will be symmetric. The tenth term corresponds with the twelfth term.

37. **OPEN ENDED** Write a power of a binomial for which the second term of the expansion is  $6x^4y$ .

ANSWER:

Sample answer:  $\left(x + \frac{6}{5}y\right)^5$

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38. **WRITING IN MATH** Explain how to write out the terms of Pascal's triangle.

**ANSWER:**

Sample answer: The first row is a 1. The second row is two 1s. Each new row begins and ends with 1. Each coefficient is the sum of the two coefficients above it in the previous row.

39. **PROBABILITY** A desk drawer contains 7 sharpened red pencils, 5 sharpened yellow pencils, 3 unsharpened red pencils, and 5 unsharpened yellow pencils. If a pencil is taken from the drawer at random, what is the probability that it is yellow, given that it is one of the sharpened pencils?

A  $\frac{5}{12}$

B  $\frac{7}{20}$

C  $\frac{5}{8}$

D  $\frac{1}{5}$

**ANSWER:**

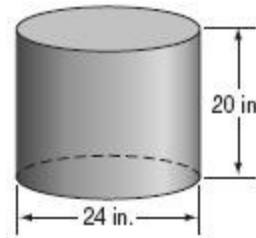
A

40. **GRIDDED RESPONSE** Two people are 17.5 miles apart. They begin to walk toward each other along a straight line at the same time. One walks at the rate of 4 miles per hour, and the other walks at the rate of 3 miles per hour. In how many hours will they meet?

**ANSWER:**

2.5

41. **GEOMETRY** Christie has a cylindrical block that she needs to paint for an art project.



What is the surface area of the cylinder in square inches rounded to the nearest square inch?

F 1960

G 2413

H 5127

J 6634

**ANSWER:**

G

42. Which of the following is a linear function?

A  $y = \frac{x+3}{x+2}$

B  $y = (3x+2)^2$

C  $y = \frac{x+3}{2}$

D  $y = |3x| + 2$

**ANSWER:**

C

## 10-6 The Binomial Theorem

Find the first five terms of each sequence.

43.  $a_1 = -2$ ,  $a_{n+1} = a_n + 5$

ANSWER:

-2, 3, 8, 13, 18

44.  $a_1 = 3$ ,  $a_{n+1} = 4a_n - 10$

ANSWER:

3, 2, -2, -18, -82

45.  $a_1 = 4$ ,  $a_{n+1} = 3a_n - 6$

ANSWER:

4, 6, 12, 30, 84

Find the sum of each infinite geometric series, if it exists.

46.  $-6 + 3 - \frac{3}{2} + \dots$

ANSWER:

-4

47.  $\frac{3}{4} + \frac{1}{4} + \frac{1}{12} + \dots$

ANSWER:

$1\frac{1}{8}$

48.  $\sqrt{3} + 3 + \sqrt{27} + \dots$

ANSWER:

No sum exists.

49. **TRAVEL** A trip between two towns takes 4 hours under ideal conditions. The first 150 miles of the trip is on an interstate, and the last 130 miles is on a highway with a speed limit that is 10 miles per hour less than on the interstate.

a. If  $x$  represents the speed limit on the interstate, write expressions for the time spent at that speed and for the time spent on the other highway.

b. Write and solve an equation to find the speed limits on the two highways.

ANSWER:

a.  $\frac{150}{x}$ ;  $\frac{130}{x-10}$

b.  $\frac{150}{x} + \frac{130}{x-10} = 4$ ; 75 mph, 65 mph

State whether each statement is true or false when  $n = 1$ . Explain.

50.  $\frac{(n+1)(n+1)}{2} = 2$

ANSWER:

true;  $\frac{(1+1)(1+1)}{2} = 2$

51.  $3n + 5$  is even.

ANSWER:

true;  $3(1) + 5 = 8$ , which is even

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52.  $n^2 - 1$  is odd.

**ANSWER:**

false;  $1^2 - 1 = 0$ , which is not odd