Determine whether each situation describes a *survey*, an *experiment*, or an *observational study*. Then identify the sample, and suggest a population from which it may have been selected.

1. **SCHOOL** A group of high school students is randomly selected and asked to complete the form shown.

ANSWER:

survey; sample: the students in the study; population: the student body

2. **DESIGN** An advertising company wants to test a new logo design. They randomly select 20 participants and watch them discuss the logo.

ANSWER:

observational study; sample: the participants in the study; population: potential customers

CCSS ARGUMENTS Determine whether each situation calls for a *survey*, an *experiment*, or an *observational study*. Explain your reasoning.

3. **LITERACY** A literacy group wants to determine whether high school students that participated in a recent national reading program had higher standardized test scores than high school students that did not participate in the program.

ANSWER:

Observation study; sample answer: The scores of the participants are observed and compared without them being affected by the study.

4. **RETAIL** The research department of a retail company plans to conduct a study to determine whether a dye used on a new T-shirt will begin fading before 50 washes.

ANSWER:

Experiment; sample answer: A sample of dyed shirts will need to be tested, which means that the members of the sample will be affected by the study.

Determine whether each survey question is *biased* or *unbiased*. If biased, explain your reasoning.

5. Which student council candidate's platform do you support?

ANSWER:

unbiased

6. How long have you lived at your current address?

ANSWER:

unbiased

7. **HYBRIDS** A car manufacturer wants to determine what the demand in the U.S. is for hybrid vehicles. State the objective of the survey, suggest a population, and write two unbiased survey questions.

ANSWER:

objective: to determine how many people in the U.S. are interested in purchasing a hybrid vehicle; population: the people surveyed; sample survey questions: Do you currently own a hybrid vehicle? Are you planning on purchasing a hybrid vehicle?

8. Identify any flaws in the experiment design, and describe how they could be corrected.
Experiment: A research company wants to determine whether a new vitamin boosts energy levels and decides to test the vitamin at a college campus. A random sample is taken. The experimental group consists of students who are given the vitamin, and the control group consists of instructors who are given a placebo.

Results: When given a physical test, the experimental group outperformed the control group.
The company concludes that the vitamin is effective.

ANSWER:

Sample answer: The flaw is that the experimental group consists of students, and the control group consists of instructors. On average, college students are younger than their instructors, and thus, are more likely to score higher on a physical test whether given a vitamin or not.

9. **SPORTS** A research company wants to conduct an experiment to test the claim of the protein shake shown. State the objective of the experiment, suggest a population, determine the experimental and control groups, and describe a sample procedure.

ANSWER:

objective: to determine whether the protein shake helps athletes recover from exercise; population: all athletes; experiment group: athletes given the protein shake; control group: athletes given a placebo; sample procedure: The researchers could randomly divide the athletes into two groups: an experimental group given the protein shake and a control group given the placebo. Next, they could have the athletes exercise and then drink the protein shake or placebo. Later, the researchers could interview the athletes to see how they feel.

Determine whether each situation describes a *survey*, an *experiment*, or an *observational study*. Then identify the sample, and suggest a population from which it may have been selected.

10. **FOOD** A grocery store conducts an online study in which customers are randomly selected and asked to provide feedback on their shopping experience.

ANSWER:

survey; sample: customers that take the online survey; population: all customers

11. **GRADES** A research group randomly selects 80 college students, half of whom took a physics course in high school, and compares their grades in a college physics course.

ANSWER:

observational study; sample: physics students selected; population: all college students that take a physics course

12. **HEALTH** A research group randomly chooses 100 people to participate in a study to determine whether eating blueberries reduces the risk of heart disease for adults.

ANSWER:

experiment; sample: adults participating in the study; population: all adults

13. **TELEVISION** A television network mails a questionnaire to randomly selected people across the country to determine whether they prefer watching sitcoms or dramas.

ANSWER:

survey; sample: people that receive the questionnaire; population: all viewers

11-1 Designing a Study

Determine whether each situation calls for a *survey*, an *experiment*, or an *observational study*. Explain your reasoning.

14. **FASHION** A fashion magazine plans to poll 100 people in the U.S. to determine whether they would be more likely to buy a subscription if given a free issue.

    ANSWER:

    Survey; sample answer: The data will be obtained from opinions given by members of the sample population.

15. **TRAVEL** A travel agency randomly calls 250 U.S. citizens and asks them what their favorite vacation destination is.

    ANSWER:

    Survey; sample answer: The data will be obtained from opinions given by members of the sample population.

16. **FOOD** Chee wants to examine the eating habits of 100 random students at lunch to determine how many students eat in the cafeteria.

    ANSWER:

    Observational study; sample answer: The eating habits of the participants will be observed and compared without them being affected by the study.

17. **ENGINEERING** An engineer is planning to test 50 metal samples to determine whether a new titanium alloy has a higher strength than a different alloy.

    ANSWER:

    Experiment; sample answer: Metal samples will need to be tested, which means that the members of the sample will be affected by the study.

Determine whether each survey question is *biased* or *unbiased*. If biased, explain your reasoning.

18. Do you think that the school needs a new gym and football field?

    ANSWER:

    Biased; sample answer: The question is asking about two issues: whether the school needs a new gym and whether the school needs a new football field.

19. Which is your favorite football team, the Dallas Cowboys or the Pittsburgh Steelers?

    ANSWER:

    Biased; sample answer: The question only gives two options, and thus encourages a certain response.

20. Do you play any extracurricular sports?

    ANSWER:

    unbiased

21. Don't you agree that students should carpool to school?

    ANSWER:

    Biased; sample answer: The question encourages a certain response. The phrase "don't you agree" suggests that the people surveyed should agree.

22. **COLLEGE** A school district wants to conduct a survey to determine the number of juniors in the district who are planning to attend college after high school. State the objective of the survey, suggest a population, and write two unbiased survey questions.

    ANSWER:

    objective: to determine the number of juniors in the district planning to attend college after high school; population: all juniors in the district; sample survey questions: What grade are you in? Do you plan on attending college after graduation?

11-1 Designing a Study

23. Identify any flaws in the experiment design, and describe how they could be corrected.

Experiment: A supermarket chain wants to determine whether shoppers are more likely to buy sunscreen if it is located near the checkout line. The experimental group consists of a group of stores in the midwest in which the sunscreen was moved next to the checkout line, and the control group consists of stores in Arizona in which the sunscreen was not moved.

Results: The Arizona stores sold more sunscreen than the midwest stores. The company concluded that moving the sunscreen closer to the checkout line did not increase sales.

ANSWER:

Sample answer: The flaw is that the experimental group consists of stores in the Midwest, and the control group consists of stores in Arizona. On average, the temperature is higher in Arizona than in the Midwest, and people use more sunscreen. Therefore, the sunscreen sales in stores located in those regions would likely be different and should not be compared in an experiment.

24. **CCSS ARGUMENTS** In chemistry class, Pedro learned that copper objects become dull over time because the copper reacts with air to form a layer of copper oxide. He plans to use the supplies shown below to determine whether a mixture of lemon juice and salt will remove copper oxide from pennies.


2 lemons   1 teaspoon   30 dull pennies   plastic bowl

a. State the objective of the experiment, suggest a population, determine the experimental and control groups, and describe a sample procedure.
b. What factors do you think should be considered when selecting pennies for the experiment? Explain your reasoning.

ANSWER:

a. objective: to determine whether a mixture of salt and lemon juice will remove copper oxide from copper objects; population: all copper objects; experimental group: pennies that are submerged in the mixture; control group: pennies that are submerged in a placebo mixture; sample procedure: Pedro could randomly assign the pennies into two groups, and create the lemon and salt mixture. Next, he could submerge the experimental group of pennies in the mixture for a certain period of time, remove them, and then visually compare the two groups of pennies.
b. Sample answer: The pennies should be chosen so that they have roughly the same amount of copper oxide and are from the same time period. Using these guidelines could eliminate bias regarding the initial condition and composition, so that the conditions of the experimental and control groups are exactly the same at the start of the experiment.

25. **REPORTS** The graph shown is from a report on the average number of minutes 8- to 18-year-olds in the U.S. spend on cell phones each day.
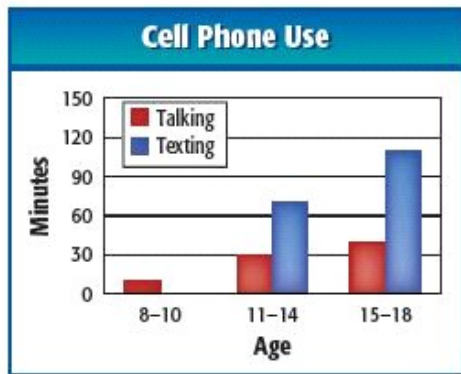
a. Describe the sample and suggest a population.
b. What type of sample statistic do you think was calculated for this report?
c. Describe the results of the study for each age group.
d. Who do you think would be interested in this type of report? Explain your reasoning.

ANSWER:

a. sample: the 8- to 18-year-olds surveyed; population: all 8- to 18-year-olds in the U.S.
b. average time
c. Sample answer: The 8- to 10-year-old group talked for about 10 minutes a day and did not text at all. The 11- to 14-year old group talked for about 30 minutes a day and texted for about 70 minutes a day. The 15- to 18-year-old group talked for about 40 minutes a day and texted for about 110 minutes a day.
d. Sample answer: A cell phone company might use a report like this to determine which age group to target in their ads.

26. **CCSS PERSEVERANCE** In 1936, the Literary Digest reported the results of a statistical study used to predict whether Alf Landon or Franklin D. Roosevelt would win the presidential election that year. The sample consisted of 2.4 million Americans, including subscribers to the magazine, registered automobile owners, and telephone users. The results concluded that Landon would win 57% of the popular vote. The actual election results are shown.

a. Describe the type of study performed, the sample taken, and the population.
b. How do the predicted and actual results compare?
c. Do you think that the survey was biased? Explain your reasoning.

ANSWER:

a. survey; sample: the 2.4 million people polled, population: all U.S. citizens of voting age in 1936
b. According to the predicted results, Landon should have won 57% of the popular vote. However, in the actual election, Roosevelt won 60.8% of the popular vote.
c. Yes; sample answer: The people sampled could afford magazine subscriptions, automobiles, and telephones, suggesting that they were wealthier than the average American citizen. The sampling method did not represent citizens that could not afford these things, and therefore, was not representative of the entire population.

27. **MULTIPLE REPRESENTATIONS** The results of two experiments concluded that Product A is 70% effective and Product B is 80% effective.
a. NUMERICAL To simulate the experiment for Product A, use the random number generator on a graphing calculator to generate 30 integers between 0 and 9. Let 0–6 represent an effective outcome and 7–9 represent an ineffective outcome.

b. TABULAR Copy and complete the frequency table shown using the results. from part **a**. Then use the data to calculate the probability that Product A was effective. Repeat to find the probability for Product B.

Product A	
Number	**Frequency**
0–6	
7–9	

c. ANALYTICAL Compare the probabilities that you found in part **b**. Do you think that the difference in the effectiveness of each product is significant enough to justify selecting one product over the other? Explain.

d. LOGICAL Suppose Product B costs twice as much as Product A. Do you think the probability of the product's effectiveness justifies the price difference to a consumer? Explain.

ANSWER:

a. See students' work.

b. Sample answer for Product A: ≈63.3%

Product A	
Number	**Frequency**
0–6	‖‖ ‖‖ ‖‖ ‖‖
7–9	‖‖ ‖‖	

Sample answer for Product B: ≈76.7%

Product B	
Number	**Frequency**
0–7	‖‖ ‖‖ ‖‖ ‖‖ ‖‖
8–9	‖‖ ‖‖

c. Sample answer: Yes; the probability that Product B is effective is 14.4% higher than that of Product A.

d. Sample answer: It depends on what the product is and how it is being used. For example, if the product is a pencil sharpener, then the lower price may be more important than the effectiveness, and therefore, might not justify the price difference. However, if the product is a life-saving medicine, the effectiveness may be more important than the price, and therefore, might justify the price difference.

REASONING Determine whether each statement is *true* or *false* . If false, explain.

28. To save time and money, population parameters are used to estimate sample statistics.

ANSWER:

False; sample answer: A sample statistic is used to estimate a population parameter.

29. Observational studies and experiments can both be used to study cause-and-effect relationships.

ANSWER:

true

30. **OPEN ENDED** Design an observational study. Identify the objective of the study, define the population and sample, collect and organize the data, and calculate a sample statistic.

ANSWER:

Sample answer:
objective: Determine the average amount of time that students spend studying at the library.
population: All students that study at the library.
sample: 30 randomly selected students studying at the library during a given week.

Study Time (minutes)				
38	16	45	41	63
18	20	17	8	15
41	28	55	19	15
30	11	20	79	24
78	24	26	32	19

mean: ≈26.1 min

31. **CHALLENGE** What factors should be considered when determining whether a given statistical study is reliable?

    ANSWER:

    Sample answer: the sampling method used, the type of sample that was selected, the type of study performed, the survey question(s) that were asked or procedures that were used

32. **WRITING IN MATH** Research each of the following sampling methods. Then describe each method and discuss whether using the method could result in bias.
    a. convenience sample
    b. self-selected sample
    c. stratified sample
    d. systematic sample

    ANSWER:

    a. Sample answer: In a convenience sample, members are selected based on the convenience of the researcher. One example is handing a survey to shoppers as they walk out of the mall. This method could result in bias if the members of the population who are readily available to be sampled are not representative of the entire population.
    b. Sample answer: In a self-selected sample, members volunteer to be in the sample. This method could result in bias if certain groups of people in the population choose not to volunteer.
    c. Sample answer: In a stratified sample, the population is first divided into similar, nonoverlapping groups, and members are then randomly selected from each group. This method could result in bias if the entire population is not represented when divided into groups or if the members are not randomly selected from each group.
    d. Sample answer: In a systematic sample, a rule is used to select the members. This method could result in bias if the rule does not include everyone in the population.

33. **GEOMETRY** In $\triangle ABC$, $BC > AB$. Which of the following must be true?
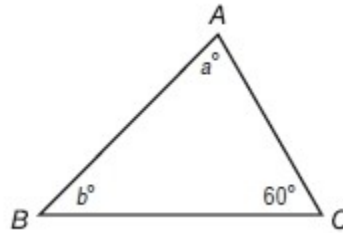
    A $AB = BC$
    B $AC > AB$
    C $a > 60$
    D $a = b$

    ANSWER:

    C

34. **SHORT RESPONSE** What is the solution set of $4^{4x^2-2x-4} = 4^{-2}$?

    ANSWER:

    $\{1, -0.5\}$

35. **SAT/ACT** A pie is divided evenly between 3 boys and a girl. If one boy gives one half of his share to the girl and a second boy keeps two thirds of his share and gives the rest to the girl, what portion will the girl have in all?

    F $\dfrac{5}{24}$
    G $\dfrac{11}{24}$
    H $\dfrac{1}{2}$
    J $\dfrac{13}{24}$
    K $\dfrac{13}{12}$

    ANSWER:

    G

36. Which equation represents a hyperbola?

    A $y^2 = 49 - x^2$
    B $y = 49 - x^2$
    C $y = 49x^2$
    D $y = \dfrac{49}{x}$

    ANSWER:

    D

37. Prove that the statement $9n - 1$ is divisible by 8 is true for all natural numbers.

    ANSWER:

    Step 1: $9^1 - 1 = 8$, which is divisible by 8. The statement is true for $n = 1$.

    Step 2: Assume that $9^k - 1$ is divisible by 8 for some positive integer k. This means that $9^k - 1 = 8r$ for some whole number r.

    Step 3:

    $$9^k - 1 = 8r$$
    $$9^k = 8r + 1$$
    $$9^{k+1} = 72r + 9$$
    $$9^{k+1} - 1 = 72r + 8$$
    $$9^{k+1} - 1 = 8(9r + 1)$$

    Since r is a whole number, $9r + 1$ is a whole number. Thus, $9^{k+1} - 1$ is divisible by 8, so the statement is true for $n = k + 1$. Therefore, $9^n - 1$ is divisible by 8 for all positive integers n.

38. **INTRAMURALS** Ofelia is taking ten shots in the intramural free-throw shooting competition. How many sequences of hits and misses are there that result in her making eight shots and missing two?

    ANSWER:
    45

    Solve each system of equations.

39. $y = x + 3$
    $y = 2x^2$

    ANSWER:
    $\left(\dfrac{3}{2}, \dfrac{9}{2}\right), (-1, 2)$

40. $x^2 + y^2 = 36$
    $y = x + 2$

    ANSWER:
    $\left(-1 + \sqrt{17}, 1 + \sqrt{17}\right), \left(-1 - \sqrt{17}, 1 - \sqrt{17}\right)$

41. $y^2 + x^2 = 9$
    $y = 7 - x$

    ANSWER:
    no solution

42. $y + x^2 = 3$
    $x^2 + 4y^2 = 36$

    ANSWER:
    $(0, 3) \text{ and } \left(\pm\dfrac{\sqrt{23}}{2}, -\dfrac{11}{4}\right)$

43. $x^2 + y^2 = 64$
    $x^2 + 64y^2 = 64$

    ANSWER:
    $(\pm 8, 0)$

44. $y^2 = x^2 - 25$
    $x^2 - y^2 = 7$

    ANSWER:
    no solution

    Find the distance between each pair of points with the given coordinates.

45. $(9, -2), (12, -14)$

    ANSWER:
    $3\sqrt{17}$ units

46. $(-4, -10), (-3, -11)$

    ANSWER:
    $\sqrt{2}$ units

47. $(1, -14), (-6, 10)$

    ANSWER:
    25 units

48. $(-4, 9), (1, -3)$

    ANSWER:
    13 units

49. (2.3, –1.2), (–4.5, 3.7)

    ANSWER:

    $\sqrt{70.25}$ units

50. (0.23, 0.4), (0.68, –0.2)

    ANSWER:

    0.75 unit

    Simplify. Assume that no variable equals 0.

51. $(5cd^2)(-c^4d)$

    ANSWER:

    $-5c^5\text{d}^{\,3}$

52. $(7x^3y^{-5})(4xy^3)$

    ANSWER:

    $\dfrac{28x^4}{y^2}$

53. $\dfrac{a^2n^6}{an^5}$

    ANSWER:

    an

54. $\left(n^4\right)^4$

    ANSWER:

    n^{16}

55. $\dfrac{-y^5z^7}{y^2z^5}$

    ANSWER:

    $-y^3z^2$

56. $\left(-2r^2t\right)^3\left(3rt^2\right)$

    ANSWER:

    $-24r^7t^5$

Write a quadratic equation with the given root (s). Write the equation in the form $ax^2 + bx + c = 0$, where a, b, and c are integers.

57. –3, 9

    ANSWER:

    $x^2 - 6x - 27 = 0$

58. $-\dfrac{1}{3}, -\dfrac{3}{4}$

    ANSWER:

    $12x^2 + 13x + 3 = 0$

59. 4, –5

    ANSWER:

    $x^2 + x - 20 = 0$

60. **TESTS** Ms. Bonilla's class of 30 students took a biology test. If 20 of her students had an average of 83 on the test and the other students had an average score of 74, what was the average score of the whole class?

    ANSWER:

    80

61. **DRIVING** During a 10-hour trip, Kwan drove 4 hours at 60 miles per hour and 6 hours at 65 miles per hour. What was her average rate, in miles per hour, for the entire trip?

    ANSWER:

    63