

11-2 Distributions of Data

1. **EXERCISE** The amount of time that James ran on a treadmill for the first 24 days of his workout is shown.

| Time (minutes) | | | | | | | | | | | |
|----------------|----|----|----|----|----|----|----|----|----|----|----|
| 23 | 10 | 18 | 24 | 13 | 27 | 19 | 7 | 25 | 30 | 15 | 22 |
| 10 | 28 | 23 | 16 | 29 | 26 | 26 | 22 | 12 | 23 | 16 | 27 |

- a. Use a graphing calculator to create a histogram. Then describe the shape of the distribution.
 b. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.

ANSWER:

a.



negatively skewed

- b. Sample answer: The distribution is skewed, so use the five-number summary. The times range from 7 to 30 minutes. The median is 22.5 minutes, and half of the data are between 15.5 and 26 minutes.

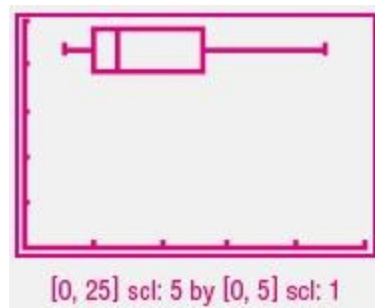
2. **RESTAURANTS** The total number of times that 20 random people either ate at a restaurant or bought fast food in a month are shown.

| Restaurants or Fast Food | | | | | | | | | |
|--------------------------|----|---|----|---|----|----|---|----|----|
| 4 | 7 | 5 | 13 | 3 | 22 | 13 | 6 | 5 | 10 |
| 7 | 18 | 4 | 16 | 8 | 5 | 15 | 3 | 12 | 6 |

- a. Use a graphing calculator to create a box-and-whisker plot. Then describe the shape of the distribution.
 b. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.

ANSWER:

a.



positively skewed

- b. Sample answer: The distribution is skewed, so use the five-number summary. The data range from 3 to 22 times. The median number is 7 times, and half of the data are between 5 and 13 times.

3. **CCSS TOOLS** The total fundraiser sales for the students in two classes at Cantonville High School are shown.

| Mrs. Johnson's Class (dollars) | | | | | |
|--------------------------------|----|----|----|----|----|
| 6 | 14 | 17 | 12 | 38 | 15 |
| 11 | 12 | 23 | 6 | 14 | 28 |
| 16 | 13 | 27 | 34 | 25 | 32 |
| 21 | 24 | 21 | 17 | 16 | |

| Mr. Edmunds' Class (dollars) | | | | | |
|------------------------------|----|----|----|----|----|
| 29 | 38 | 21 | 28 | 24 | 33 |
| 14 | 19 | 28 | 15 | 30 | 6 |
| 31 | 23 | 33 | 12 | 38 | 28 |
| 18 | 34 | 26 | 34 | 24 | 37 |

- a. Use a graphing calculator to create a histogram

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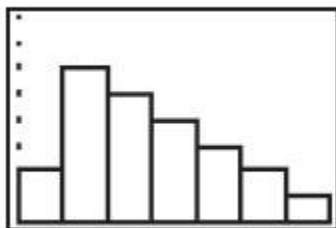
for each data set. Then describe the shape of each distribution.

b. Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.

ANSWER:

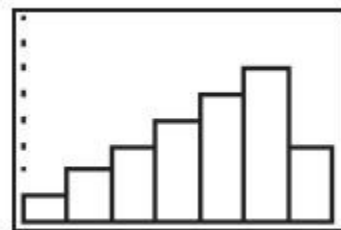
a.

Mrs. Johnson's Class



[5, 40] scl: 5 by [0, 8] scl: 1

Mr. Edmunds' Class



[5, 40] scl: 5 by [0, 8] scl: 1

Mrs. Johnson's class, positively skewed; Mr. Edmunds' class, negatively skewed

b. Sample answer: The distributions are skewed, so use the five-number summaries. The range for both classes is the same. However, the median for Mrs. Johnson's class is 17 and the median for Mr. Edmunds' class is 28. The lower quartile for Mr. Edmunds' class is 20. Since this is greater than the median for Mrs. Johnson's class, this means that 75% of the data from Mr. Edmunds' class is greater than 50% of the data from Mrs. Johnson's class. Therefore, we can conclude that the students in Mr. Edmunds' class had slightly higher sales overall than the students in Mrs. Johnson's class.

4. **RECYCLING** The weekly totals of recycled paper for the junior and senior classes are shown.

| Junior Class (pounds) | | | | | |
|-----------------------|----|----|----|----|----|
| 14 | 24 | 8 | 26 | 19 | 38 |
| 12 | 15 | 12 | 18 | 9 | 24 |
| 12 | 21 | 9 | 15 | 13 | 28 |

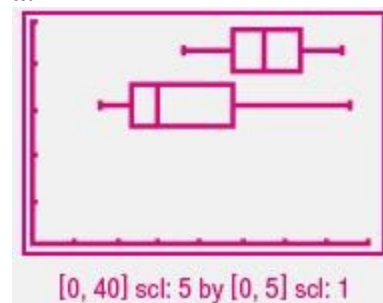
| Senior Class (pounds) | | | | | |
|-----------------------|----|----|----|----|----|
| 25 | 31 | 35 | 20 | 37 | 27 |
| 22 | 32 | 24 | 28 | 18 | 32 |
| 25 | 32 | 22 | 29 | 26 | 35 |

a. Use a graphing calculator to create a box-and-whisker plot for each data set. Then describe the shape of each distribution.

b. Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.

ANSWER:

a.



[0, 40] scl: 5 by [0, 5] scl: 1
junior class, positively skewed; senior class, symmetric

b. Sample answer: One distribution is symmetric and the other is skewed, so use the five-number summaries. The median for the junior class is 15, and the median for the senior class is 27.5. The minimum value for the senior class is 18. This means that all of the weekly totals for the senior class are greater than 50% of the weekly totals for the junior class. Therefore, we can conclude that the senior class's weekly totals were far greater than the junior class's weekly totals.

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For Exercises 5 and 6, complete each step.

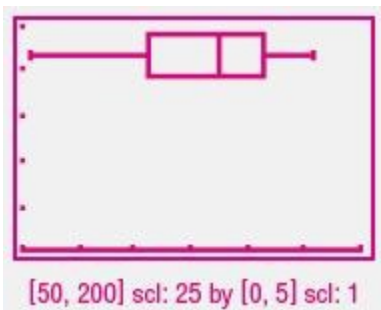
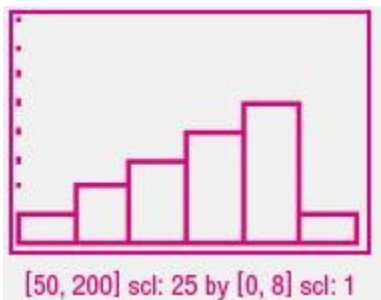
- Use a graphing calculator to create a histogram and a box-and-whisker plot. Then describe the shape of the distribution.
- Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.

5. **FANTASY** The weekly total points of Kevin's fantasy football team are shown.

| Total Points | | | | | | | |
|--------------|-----|-----|-----|-----|-----|-----|-----|
| 165 | 140 | 88 | 158 | 101 | 137 | 112 | 127 |
| 53 | 151 | 120 | 156 | 142 | 179 | 162 | 79 |

ANSWER:

a.



negatively skewed

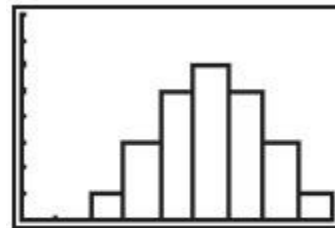
- Sample answer: The distribution is skewed, so use the five-number summary. The points range from 53 to 179. The median is 138.5 points, and half of the data are between 106.5 and 157 points.

6. **MOVIES** The students in one of Mr. Peterson's classes recorded the number of movies they saw over the past month.

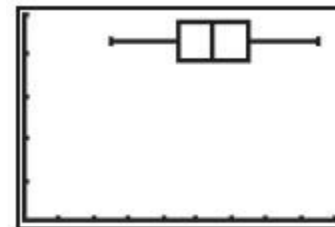
| Movies Seen | | | | | | | | | | | |
|-------------|----|----|----|---|----|----|---|----|----|----|----|
| 14 | 11 | 17 | 9 | 6 | 11 | 7 | 8 | 12 | 13 | 10 | 9 |
| 5 | 11 | 7 | 13 | 9 | 12 | 10 | 9 | 15 | 11 | 13 | 15 |

ANSWER:

a.



[0, 18] scl: 2 by [0, 8] scl: 1



[0, 18] scl: 2 by [0, 5] scl: 1

symmetric

- Sample answer: The distribution is symmetric, so use the mean and standard deviation. The mean number of movies watched was about 10.7 with standard deviation of about 3 movies.

CCSS TOOLS Complete each step.

- Use a graphing calculator to create a histogram for each data set. Then describe the shape of each distribution.
- Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.

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7. **SAT** A group of students took the SAT their sophomore year and again their junior year. Their scores are shown.

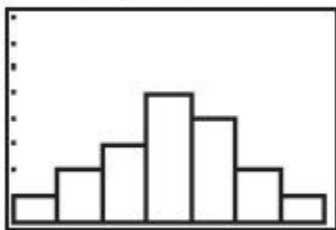
| Sophomore Year Scores | | | | | |
|-----------------------|------|------|------|------|------|
| 1327 | 1663 | 1708 | 1583 | 1406 | 1563 |
| 1637 | 1521 | 1282 | 1752 | 1628 | 1453 |
| 1368 | 1681 | 1506 | 1843 | 1472 | 1560 |

| Junior Year Scores | | | | | |
|--------------------|------|------|------|------|------|
| 1728 | 1523 | 1857 | 1789 | 1668 | 1913 |
| 1834 | 1769 | 1655 | 1432 | 1885 | 1955 |
| 1569 | 1704 | 1833 | 2093 | 1608 | 1753 |

ANSWER:

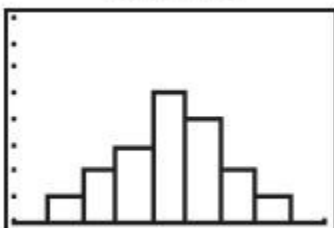
a.

Sophomore Year



[1200, 1900] scl: 100 by [0, 8] scl: 1

Junior Year



[1300, 2200] scl: 100 by [0, 8] scl: 1

both symmetric

- b. Sample answer: The distributions are symmetric, so use the means and standard deviations. The mean score for sophomore year is about 1552.9 with standard deviation of about 147.2. The mean score for junior year is about 1753.8 with standard deviation of about 159.1. We can conclude that the scores and the variation of the scores from the mean both increased from sophomore year to junior year.

8. **INCOME** The total incomes for 18 households in two neighboring cities are shown.

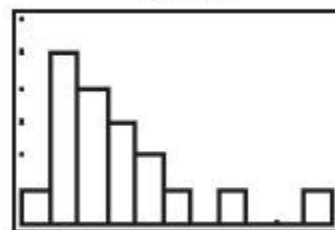
| Yorkshire (thousands of dollars) | | | | | |
|----------------------------------|----|----|----|-----|----|
| 68 | 59 | 61 | 78 | 58 | 66 |
| 56 | 72 | 86 | 58 | 63 | 53 |
| 68 | 58 | 74 | 60 | 103 | 64 |

| Applewood (thousands of dollars) | | | | | |
|----------------------------------|----|----|----|----|----|
| 52 | 55 | 60 | 61 | 55 | 65 |
| 65 | 60 | 45 | 37 | 41 | 71 |
| 50 | 61 | 65 | 66 | 87 | 55 |

ANSWER:

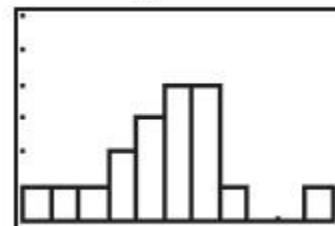
a.

Yorkshire



[50, 105] scl: 5 by [0, 6] scl: 1

Applewood



[35, 90] scl: 5 by [0, 6] scl: 1

Yorkshire, positively skewed; Applewood, negatively skewed

- b. Sample answer: The distributions are skewed, so use the five-number summaries. The median for Yorkshire is 63.5, and the median for Applewood is 60. The lower quartile for Applewood is 52, while the minimum for Yorkshire is 53. This means that 25% of the incomes for Applewood are lower than any of the incomes for Yorkshire. Also, the upper 25% of incomes for Yorkshire is between 72 and 103, while the upper 25% of incomes for Applewood is between 65 and 87. We can conclude that the incomes for the households in Yorkshire are greater than the incomes for the households in Applewood.

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9. **TUITION** The annual tuitions for a sample of public colleges and a sample of private colleges are shown. Complete each step

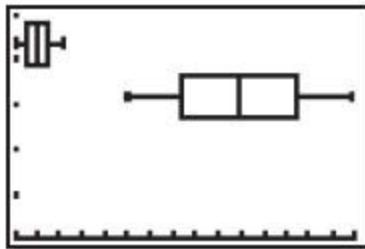
- Use a graphing calculator to create a box-and-whisker plot for each data set. Then describe the shape of each distribution.
- Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.

| Public Colleges (dollars) | | | | | |
|---------------------------|------|------|------|------|------|
| 3773 | 3992 | 3004 | 4223 | 4821 | 3880 |
| 3163 | 4416 | 5063 | 4937 | 3321 | 4308 |
| 4006 | 3508 | 4498 | 3471 | 4679 | 3612 |

| Private Colleges (dollars) | | | | | |
|----------------------------|--------|--------|--------|--------|--------|
| 10,766 | 13,322 | 12,995 | 15,377 | 16,792 | 9147 |
| 15,976 | 11,084 | 17,868 | 7909 | 12,824 | 10,377 |
| 14,304 | 10,055 | 12,930 | 16,920 | 10,004 | 11,806 |

ANSWER:

a.



[3000, 18,000] scl: 1000 by [0, 5] scl: 1

both symmetric

- Sample answer: The distributions are symmetric, so use the means and standard deviations. The mean for the public colleges is \$4037.50 with standard deviation of about \$621.93. The mean for private colleges is about \$12,803.11 with standard deviation of about \$2915.20. We can conclude that not only is the average cost of private schools far greater than the average cost of public schools, but the variation of the costs from the mean is also much greater.

10. **DANCE** The total amount of money that a random sample of seniors spent on prom is shown. Complete each step.

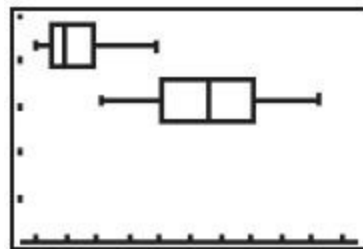
- Use a graphing calculator to create a box-and-whisker plot for each data set. Then describe the shape of each distribution.
- Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.

| Boys (dollars) | | | | | |
|----------------|-----|-----|-----|-----|-----|
| 253 | 288 | 304 | 283 | 348 | 276 |
| 322 | 368 | 247 | 404 | 450 | 341 |
| 291 | 260 | 394 | 302 | 297 | 272 |

| Girls (dollars) | | | | | |
|-----------------|-----|-----|-----|-----|-----|
| 682 | 533 | 602 | 504 | 635 | 541 |
| 489 | 703 | 453 | 521 | 472 | 368 |
| 562 | 426 | 382 | 668 | 352 | 587 |

ANSWER:

a.



[225, 775] scl: 50 by [0, 5] scl: 1

boys, positively skewed; girls, symmetric

- Sample answer: One distribution is skewed and the other is symmetric, so use the five-number summaries. The maximum value for the boys is 450. The lower quartile for the girls is 453. This means that 75% of the data for the girls is higher than any data for the boys. We can conclude that 75% of the girls spent more money on prom than any of the boys.

11. **BASKETBALL** Refer to the beginning of the lesson. The points that Craig scored in the remaining games are shown.

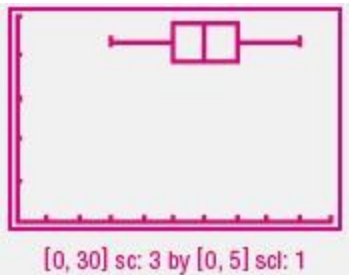
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| Points Scored | | | |
|---------------|----|----|----|
| 18 | 10 | 18 | 21 |
| 9 | 25 | 13 | 17 |
| 17 | 12 | 24 | 19 |
| 20 | 17 | 27 | 21 |

- a. Use a graphing calculator to create a box-and-whisker plot. Describe the center and spread of the data.
- b. Craig scored 0, 2, 1, and 0 points in the first four games. Use a graphing calculator to create a box-and-whisker plot that includes the new data. Then find the mean and median of the new data set.
- c. What effect does adding the scores from the first four games have on the shape of the distribution and on how you should describe the center and spread?

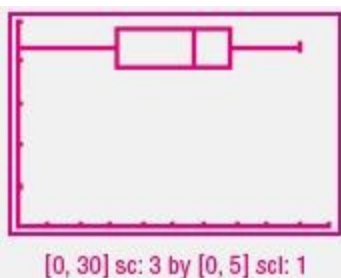
ANSWER:

a.



Sample answer: The distribution is symmetric, so use the mean and standard deviation. The mean of the data is 18 with sample standard deviation of about 5.2 points.

b.



mean: 14.6; median: 17

- c. Sample answer: Adding the scores from the first four games causes the shape of the distribution to go from being symmetric to being negatively skewed. Therefore, the center and spread should be described using the five-number summary.

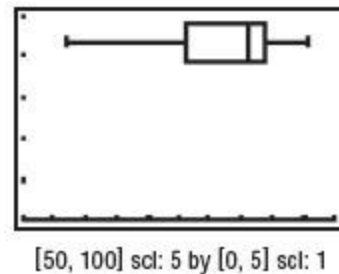
12. **SCORES** Allison's quiz scores are shown.

| Math Quiz Scores | | | | | |
|------------------|----|----|----|----|----|
| 83 | 76 | 86 | 82 | 84 | 57 |
| 86 | 62 | 90 | 96 | 76 | 89 |
| 76 | 88 | 86 | 86 | 92 | 94 |

- a. Use a graphing calculator to create a box-and-whisker plot. Describe the center and spread.
- b. Allison's teacher allows students to drop their two lowest quiz scores. Use a graphing calculator to create a box-and-whisker plot that reflects this change. Then describe the center and spread of the new data set.

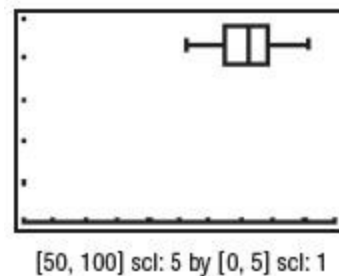
ANSWER:

a.



Sample answer: The distribution is negatively skewed, so use the five-number summary. The scores range from 57 to 96. The median is 86, and half of the data are between 76 and 89.

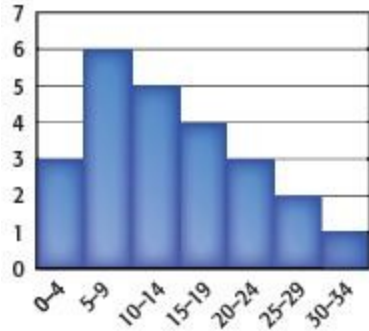
b.



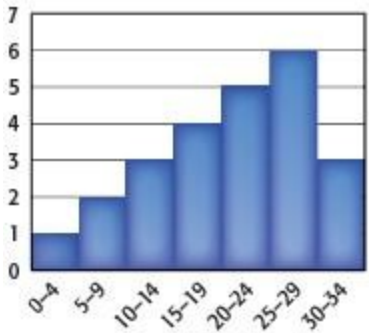
Sample answer: The distribution is symmetric, so use the mean and standard deviation. The mean is about 85.6 with standard deviation of about 5.9.

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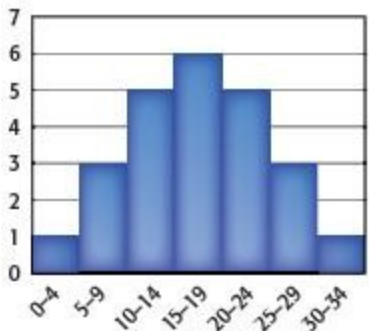
13. **CHALLENGE** Approximate the mean and median for each distribution of data.



a.



b.

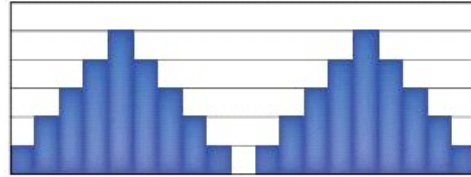


c.

ANSWER:

- a. Sample answer: mean = 14; median = 10
 b. Sample answer: mean = 20; median = 24
 c. Sample answer: mean = 17; median = 17

14. **CCSS ARGUMENTS** Distributions of data are not always symmetric or skewed. If a distribution has a gap in the middle, like the one shown, two separate clusters of data may result, forming a *bimodal distribution*. How can the center and spread of a bimodal distribution be described?



ANSWER:

Sample answer: Since the distribution has two clusters, an overall summary of center and spread would give an inaccurate depiction of the data. Instead, summarize the center and spread of each cluster individually using its mean and standard deviation.

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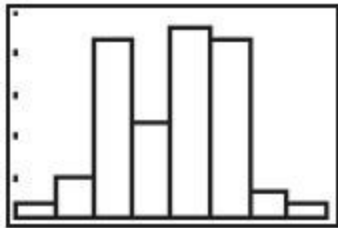
15. **OPEN ENDED** Find a real-world data set that appears to represent a symmetric distribution and one that does not. Describe each distribution. Create a visual representation of each set of data.

ANSWER:

Sample answer: The heights of the players on the Pittsburgh Steelers roster appear to represent a normal distribution.

| Heights of the Players on the 2009 Pittsburgh Steelers Roster (inches) | | | | | | | |
|--|----|----|----|----|----|----|----|
| 75 | 74 | 71 | 70 | 74 | 75 | 77 | 72 |
| 71 | 72 | 70 | 70 | 75 | 78 | 71 | 75 |
| 77 | 71 | 69 | 70 | 77 | 75 | 74 | 73 |
| 77 | 71 | 73 | 76 | 76 | 74 | 72 | 75 |
| 75 | 70 | 70 | 74 | 73 | 76 | 79 | 73 |
| 71 | 69 | 70 | 77 | 77 | 80 | 75 | 77 |
| 67 | 74 | 69 | 76 | 77 | 76 | | |

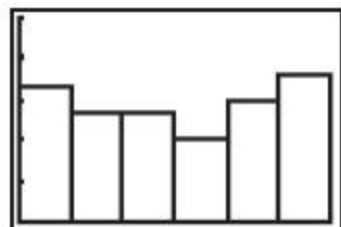
The mean of the data is about 73.61 in. or 6 ft 1.61 in. The standard deviation is about 2.97 in.



[66, 82] scl: 2 by [0, 15] scl: 3

The birth months of the players do not display central tendency.

| Birth Months of the Players on the 2009 Pittsburgh Steelers Roster | | | | | | | |
|--|----|----|----|----|----|----|---|
| 1 | 12 | 10 | 3 | 11 | 1 | 10 | 5 |
| 4 | 8 | 9 | 11 | 1 | 1 | 11 | 5 |
| 8 | 6 | 11 | 4 | 3 | 4 | 8 | 5 |
| 3 | 7 | 2 | 1 | 11 | 4 | 3 | 2 |
| 1 | 1 | 6 | 1 | 6 | 8 | 11 | 9 |
| 3 | 3 | 1 | 6 | 9 | 1 | 9 | 9 |
| 6 | 5 | 10 | 11 | 11 | 12 | | |



[0, 12] scl: 2 by [0, 15] scl: 3

16. **WRITING IN MATH** Explain the difference between positively skewed, negatively skewed, and symmetric sets of data, and give an example of each.

ANSWER:

Sample answer: The distribution for a data set is positively skewed when the majority of the data is on the left of the mean and a tail appears to the right of the mean. An example is when the data set includes the height of everyone in an elementary school, most of the data will be on the left side (the students), while a comparatively small amount will be on the right (the teachers and staff). The distribution for a data set is negatively skewed when the majority of the data is on the right of the mean and a tail appears to the left of the mean. An example is when the batting averages of a baseball lineup are listed, most of the data will be at a certain level while the pitcher will typically be much lower. The distribution for a data set is symmetric when the data are evenly distributed on both sides of the mean. An example is when test scores are calculated for an entire state, most of the students will place in the middle, while some will place above or below.

17. **DISTRIBUTIONS** Which of the following is a characteristic of a negatively skewed distribution?
A The majority of the data are on the left of the mean.
B The mean and median are approximately equal.
C The mean is greater than the median.
D The mean is less than the median.

ANSWER:

D

18. **SHORT RESPONSE** The average of the test scores of a class of c students is 80, and the average test scores of a class of d students is 85. When the scores of both classes are combined, the average score is 82. What is the value of $\frac{c}{d}$?

ANSWER:

$$\frac{3}{2}$$

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19. **SAT/ACT** What is the multiplicative inverse of $2i$?

F $-2i$

G -2

H $\frac{-i}{2}$

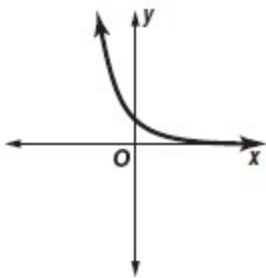
J $\frac{1}{2}$

K $\frac{i}{2}$

ANSWER:

H

20. Which equation best represents the graph?



A $y = 4x$

B $y = x^2 + 4$

C $y = 4^{-x}$

D $y = -4x$

ANSWER:

C

Determine whether each survey question is biased. Explain your reasoning.

21. What toppings do you prefer on your pizza?

ANSWER:

unbiased

22. What is your favorite class, and what teacher gives the easiest homework?

ANSWER:

Biased; sample answer: The question combines two issues.

23. Don't you hate how high gas prices are?

ANSWER:

Biased; sample answer: The question encourages a certain response. The phrase "Don't you hate" encourages you to agree that gas prices are too high.

24. **PARTIES** Suppose each time a new guest arrives at a party, he or she shakes hands with each person already at the party. Prove that after n guests have arrived, a total of $\frac{n(n-1)}{2}$ handshakes have taken place.

ANSWER:

Step 1: After the first guest has arrived, no

handshakes have taken place. $\frac{1(1-1)}{2} = 0$, so the

formula is correct for $n = 1$.

Step 2: Assume that after k guests have arrived, a

total of $\frac{k(k-1)}{2}$ handshakes have taken place, for

some positive integer k .

Step 3: When the $(k + 1)$ st guest arrives, he or she shakes hands with the k guests already there, so the total number of handshakes that have then taken

place is $\frac{k(k-1)}{2} + k$.

$$\begin{aligned} \frac{k(k-1)}{2} + k &= \frac{k(k-1) + 2k}{2} \\ &= \frac{k[(k-1) + 2]}{2} \\ &= \frac{k(k+1)}{2} \text{ or } \frac{(k+1)k}{2} \end{aligned}$$

The last expression is the formula to be proved, where $n = k + 1$. Thus, the formula is true for $n = k + 1$. Therefore, the total number of handshakes is

$\frac{n(n-1)}{2}$ for all positive integers n .

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25. **ASTRONOMY** The orbit of Pluto can be modeled by the equation $\frac{x^2}{39.5^2} + \frac{y^2}{38.3^2} = 1$, where the units are astronomical units. Suppose a comet is following a path modeled by the equation $x = y^2 + 20$.
- Find the point(s) of intersection of the orbits of Pluto and the comet.
 - Will the comet necessarily hit Pluto? Explain.
 - Where do the graphs of $y = 2x + 1$ and $2x^2 + y^2 = 11$ intersect?
 - What are the coordinates of the points that lie on the graphs of both $x^2 + y^2 = 25$ and $2x^2 + 3y^2 = 66$?

ANSWER:

- $(39.2, \pm 4.4)$
- No; the comet and Pluto may not be at either point of intersection at the same time.
- $\left(-\frac{5}{3}, -\frac{7}{3}\right), (1, 3)$
- $(3, \pm 4), (-3, \pm 4)$

Determine whether each situation involves a permutation or a combination. Then find the number of possibilities.

26. the winner of the first, second, and third runners-up in a contest with 8 finalists
- ANSWER:**
permutation; 336
27. selecting two of eight employees to attend a business seminar
- ANSWER:**
combination; 28
28. an arrangement of the letters in the word *MATH*
- ANSWER:**
permutation; 24
29. placing an algebra book, a geometry book, a chemistry book, an English book, and a health book on a shelf
- ANSWER:**
permutation; 120