11-3 Probability Distributions

Identify the random variable in each distribution, and classify it as *discrete* or *continuous*. Explain your reasoning.

1. the number of pages linked to a Web page

   ANSWER:

   The random variable X is the number of pages linked to a Web page. The pages are countable, so X is discrete.

2. the number of stations in a cable package

   ANSWER:

   The random variable X is the number of stations in a cable package. The stations are countable, so X is discrete.

3. the amount of precipitation in a city per month

   ANSWER:

   The random variable X is the amount of precipitation in a city per month. Precipitation can be anywhere within a certain range. Therefore, X is continuous.

4. the number of cars passing through an intersection in a given time interval

   ANSWER:

   The random variable X is the number of cars passing through an intersection. The cars are countable, so X is discrete.

5. X represents the sum of the values of two spins of the wheel.

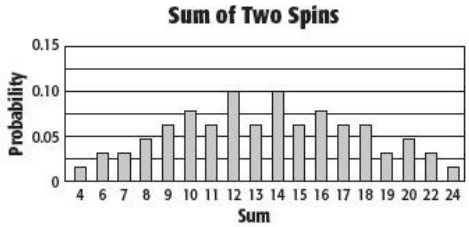
   a. Construct a relative-frequency table showing the theoretical probabilities.
   b. Graph the theoretical probability distribution.
   c. Construct a relative-frequency table for 100 trials.
   d. Graph the experimental probability distribution.
   e. Find the expected value for the sum of two spins of the wheel.
   f. Find the standard deviation for the sum of two spins of the wheel.

ANSWER:

a.

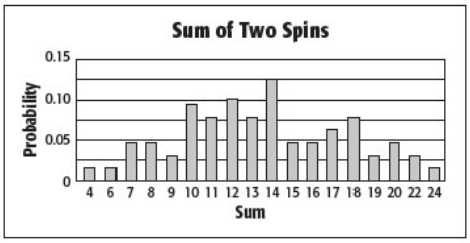
Sum	Frequency	Relative Frequency
4	1	$\frac{1}{64}$
6	2	$\frac{1}{32}$
7	2	$\frac{1}{32}$
8	3	$\frac{3}{64}$
9	4	$\frac{1}{16}$
10	5	$\frac{5}{64}$
11	4	$\frac{1}{16}$
12	7	$\frac{7}{64}$
13	4	$\frac{1}{16}$
14	7	$\frac{7}{64}$
15	4	$\frac{1}{16}$
16	5	$\frac{5}{64}$
17	4	$\frac{1}{16}$
18	4	$\frac{1}{16}$
19	2	$\frac{1}{32}$
20	3	$\frac{3}{64}$
22	2	$\frac{1}{32}$
24	1	$\frac{1}{64}$

b.



c.

Sum	Frequency	Relative Frequency
4	1	$\frac{1}{64}$
6	1	$\frac{1}{64}$
7	3	$\frac{3}{64}$
8	3	$\frac{3}{64}$
9	2	$\frac{1}{32}$
10	6	$\frac{3}{32}$
11	5	$\frac{5}{64}$
12	7	$\frac{7}{64}$
13	5	$\frac{5}{64}$
14	8	$\frac{1}{8}$
15	3	$\frac{3}{64}$
16	3	$\frac{3}{64}$
17	4	$\frac{1}{16}$
18	5	$\frac{5}{64}$
19	2	$\frac{1}{32}$
20	3	$\frac{3}{64}$
22	2	$\frac{1}{32}$
24	1	$\frac{1}{64}$



d.

e. 13.5

f. 4.29

Identify the random variable in each distribution, and classify it as *discrete* or *continuous*. Explain your reasoning.

6. the number of texts received per week

ANSWER:

The random variable X is the number of texts per week. The texts are countable, so X is discrete.

7. the number of diggs (or "likes") for a Web page

ANSWER:

The random variable X is the number of diggs for a web page. The diggs are countable, so X is discrete.

8. the height of a plant after a specific amount of time

ANSWER:

The random variable X is the height of a plant. Height can be anywhere within a certain range. Therefore, X is continuous.

9. the number of files infected by a computer virus

ANSWER:

The random variable X is the number of files infected by a computer virus. The files are countable, so X is discrete.

10. **CCSS PERSEVERANCE** A contestant has won a prize on a game show. The frequency table at the right shows the number of winners for 3200 hypothetical players.

Prize, X	Winners
$100	1120
$250	800
$500	480
$1000	320
$2500	256
$5000	128
$7500	64
$10,000	32

a. Construct a relative-frequency table showing the theoretical probability.
b. Graph the theoretical probability distribution.
c. Construct a relative-frequency table for 50 trials.
d. Graph the experimental probability distribution.

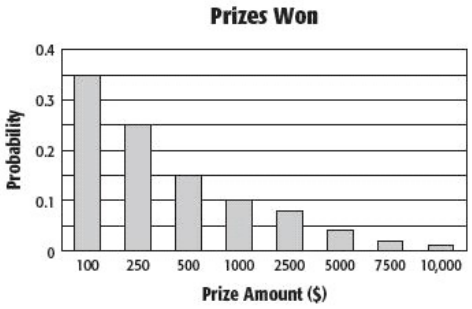
e. Find the expected value.
f. Find the standard deviation.

ANSWER:

a.

Prize, X	$P(X)$
$100	0.35
$250	0.25
$500	0.15
$1000	0.10
$2500	0.08
$5000	0.04
$7500	0.02
$10,000	0.01

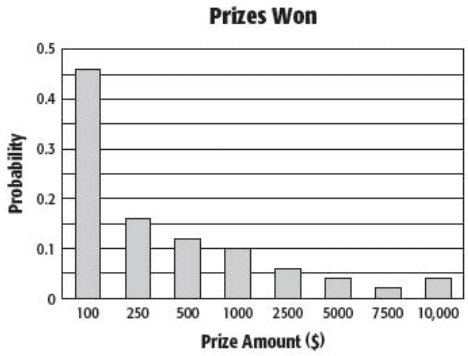
b.



c.

Prize, X	Frequency	Relative Frequency
$100	23	0.46
$250	8	0.16
$500	6	0.12
$1000	5	0.10
$2500	3	0.06
$5000	2	0.04
$7500	1	0.02
$10,000	2	0.04

d.



e. $922.50
f. 1711.91

11. **SNOW DAYS** The following probability distribution lists the probable number of snow days per school year at North High School. Use this information to determine the expected number of snow days per year.

Number of Snow Days Per Year									
Days	0	1	2	3	4	5	6	7	8
Probability	0.1	0.1	0.15	0.15	0.25	0.1	0.08	0.05	0.02

ANSWER:
3.34

12. **CARDS** In a standard deck of 52 cards, there are 4 different suits.
a. If jacks = 11, queens = 12, kings = 13, and aces = 1, what is the expected value of a card that is drawn from a standard deck?
b. If you are dealt 7 cards with replacement, what is the expected number of spades?

ANSWER:
a. 7
b. 1.75

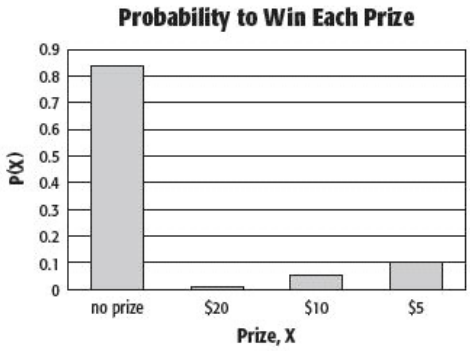
13. **RAFFLES** The table shows the probability distribution for a raffle if 100 tickets are sold for $1 each. There is 1 prize for $20, 5 prizes for $10, and 10 prizes for $5.

Distribution of Prizes				
Prize	no prize	$20	$10	$5
Probability	0.84	0.01	0.05	0.10

a. Graph the theoretical probability distribution.
b. Find the expected value.
c. Interpret the results you found in part b. What can you conclude about the raffle?

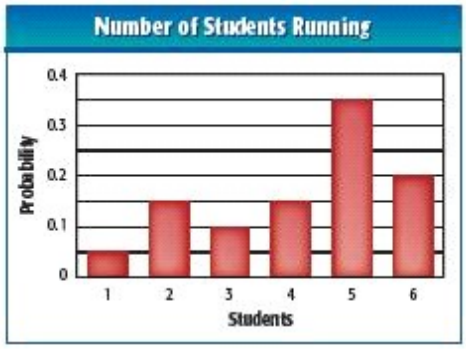
ANSWER:

a.



b. $1.20
c. Sample answer: The expected value is positive, so a person buying a ticket can expect to win $0.20 even after the cost of the ticket is considered. Thus, a person would want to participate in this raffle. On the other hand, this raffle is guaranteed to lose money for the organizers and they should change the distribution of prizes or not do the raffle.

14. **CCSS TOOLS** Based on previous data, the probability distribution of the number of students running for class president is shown.



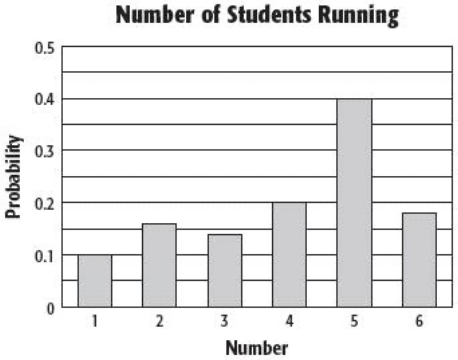
a. Determine the expected number of students who will run. Interpret your results.
b. Construct a relative-frequency table for 50 trials.
c. Graph the experimental probability distribution.

ANSWER:

a. 4.2; Sample answer: The expected number is 4.2, so we can expect there to be 4 students running. We cannot have 0.2 people, so we round to the nearest whole number.
b. Sample answer:

Number of Students, *X*	Frequency	Relative Frequency
1	5	0.10
2	8	0.16
3	7	0.14
4	1	0.02
5	20	0.40
6	9	0.18

c. Sample answer:



15. **BASKETBALL** The distribution below lists the probability of the number of major upsets in the first round of a basketball tournament each year.

Number of Upsets Per Year									
Upsets	0	1	2	3	4	5	6	7	8
Probability	$\frac{1}{32}$	$\frac{1}{16}$	$\frac{3}{32}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{5}{16}$	$\frac{1}{8}$	$\frac{3}{32}$	$\frac{1}{32}$

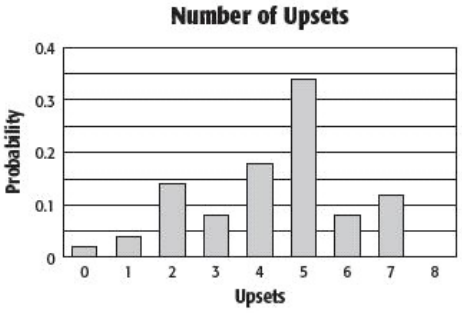
a. Determine the expected number of upsets. Interpret your results.
b. Find the standard deviation.
c. Construct a relative-frequency table for 50 trials.
d. Graph the experimental probability distribution.

ANSWER:

a. 4.34; Sample answer: The expected number is 4.34, so we can expect there to be 4 upsets. We cannot have 0.34 upsets, so we round to the nearest whole number.
b. 1.90
c.

Number of Upsets, *X*	Frequency	Relative Frequency
0	1	0.02
1	2	0.04
2	7	0.14
3	4	0.08
4	9	0.18
5	17	0.34
6	4	0.08
7	6	0.12
8	0	0

d.



16. **RAFFLES** The French Club sold 500 raffle tickets for $1 each. The first prize ticket will win $100, 2 second prize tickets will each win $10, and 5 third prize tickets each win $5.
a. What is the expected value of a single ticket?
b. Calculate the standard deviation of the probability distribution.
c. DECISION MAKING The Glee Club is offering a raffle with a similar expected value and a standard deviation of 2.2. In which raffle should you participate? Explain your reasoning.

ANSWER:

a. -0.71
b. 4.53
c. Sample answer: The standard deviation of the probability distribution for the Glee Club raffle is about half the standard deviation for the French Club raffle, so the Glee Club raffle is less risky. Since they have similar expected values, the riskier raffle will also have the potential to win more, so both raffles have good and bad qualities. It is up to the individual participant to decide which one to choose.

17. **DECISION MAKING** Carmen is thinking about investing $10,000 in two different investment funds. The expected rates of return and the corresponding probabilities for each fund are listed below. Compare the two investments using the expected value and standard deviation. Which investment would you advise Carmen to choose, and why?

Fund A
30% chance of a $1900 profit
30% chance of a $600 profit
15% chance of a $200 loss
25% chance of a $500 loss

Fund B
40% chance of a $1600 profit
10% chance of a $900 profit
10% chance of a $300 loss
40% chance of a $400 loss

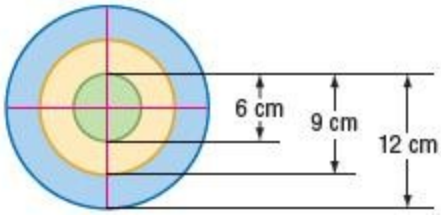
ANSWER:

Sample answer: The expected value of Funds A and B is $595 and $540, respectively. The standard deviation for Fund A is about 951.6, while the standard deviation for Fund B is about 941.5. Since the standard deviations are about the same, the funds have about the same amount of risk. Therefore, with a higher expected value, Fund A is the better investment.

18. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate geometric probability.
a. Tabular The spinner shown has a radius of 2.5 inches. Copy and complete the table below.

Color	Probability	Sector Area	Total Area	$\frac{\text{Sector Area}}{\text{Total Area}}$
red				
orange				
yellow				
green				
blue				

b. Verbal Make a conjecture about the relationship between the ratio of the area of the sector to the total area and the probability of the spinner landing on each color.
c. Analytical Consider the dartboard shown.



Predict the probability of a dart landing in each area of the board. Assume that any dart thrown will land on the board and is equally likely to land at any point on the board.
d. Tabular Construct a relative-frequency table for throwing 100 darts.
e. Graphical Graph the experimental probability distribution.

ANSWER:

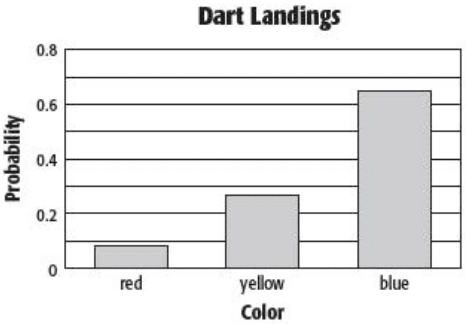
a.

Color	Probability	Sector Area	Total Area	$\frac{\text{Sector Area}}{\text{Total Area}}$
red	$\frac{1}{6}$	3.27 in^2	19.63 in^2	0.166
orange	$\frac{1}{6}$	3.27 in^2	19.63 in^2	0.166
yellow	$\frac{1}{6}$	3.27 in^2	19.63 in^2	0.166
green	$\frac{1}{4}$	4.91 in^2	19.63 in^2	0.25
blue	$\frac{1}{4}$	4.91 in^2	19.63 in^2	0.25

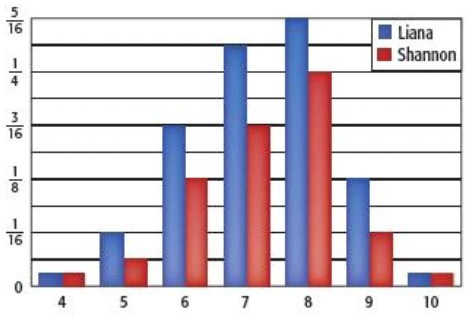
b. Sample answer: The probability is equal to the ratio of the sector area to the total area.
c. green: $\frac{1}{9}$; yellow: $\frac{1}{3}$; blue: $\frac{5}{9}$
d.

Color	Frequency	Relative Frequency
red	8	0.08
yellow	27	0.27
blue	65	0.65

e.



19. **CCSS CRITIQUE** Liana and Shannon each created a probability distribution for the sum of two spins on the spinner. Is either of them correct? Explain your reasoning.



ANSWER:

Sample answer: Liana; Shannon didn't consider every scenario in determining the total probability. For example, in calculating the probability of a sum of 5, she considered spinning a 3 then a 2, but not a 2 then a 3.

20. **REASONING** Determine whether the following statement is true or false. Explain.
If you roll a die 10 times, you will roll the expected value at least twice.

ANSWER:

Sample answer: False; the expected value is 3.5 which is not a possible outcome of a single roll.

21. **OPEN ENDED** Create a discrete probability distribution that shows five different outcomes and their associated probabilities.

ANSWER:

Sample answer: A spinner with 5 equal-sided areas shaded red, blue, yellow, green, and brown.

Color	red	blue	yellow	green	brown
Probability	0.2	0.2	0.2	0.2	0.2

22. **REASONING** Determine whether the following statement is true or false. Explain.
Random variables that can take on an infinite number of values are continuous.

ANSWER:

Sample answer: False; a random variable X representing the number of Web sites on the Internet at any given time is infinite and countable. However, there cannot be 1.3 Web sites, so it is also discrete.

23. **OPEN ENDED** Provide examples of a discrete probability distribution and a continuous probability distribution. Describe the differences between them.

ANSWER:

Sample answer: A discrete probability distribution can be the uniform distribution of the roll of a die. In this type of distribution, there are only a finite number of possibilities. A continuous probability distribution can be the distribution of the lives of 400 batteries. In this distribution, there are an infinite number of possibilities.

24. **WRITING IN MATH** Compare and contrast two investments that have identical expected values and significantly different standard deviations.

ANSWER:

Sample answer: Since the investments have identical expected values, an investor would expect to earn the same amount of money on each investment. However, since they have significantly different standard deviations, the investment with the higher standard deviation is much riskier than the other investment. The greater standard deviation indicates a greater variability, so the riskier investment will have an opportunity to earn more money than the other investment, but it will also have an opportunity to lose more as well.

25. **GRIDDED RESPONSE** The height $f(x)$ of a bouncing ball after x bounces is represented by $f(x) = 140(0.8)^x$. How many times higher is the first bounce than the fifth bounce?

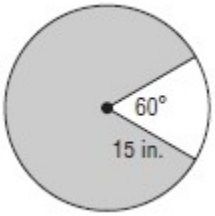
ANSWER:
2.4

26. **PROBABILITY** Andres has a bag that contains 4 red, 6 yellow, 2 blue, and 4 green marbles. If he reaches into the bag and removes a marble without looking, what is the probability that it will not be yellow?

A $\frac{1}{8}$
B $\frac{1}{4}$
C $\frac{3}{8}$
D $\frac{5}{8}$

ANSWER:
D

27. **GEOMETRY** Find the area of the shaded portion of the figure to the nearest square inch.



F 79
G 94
H 589
J 707

ANSWER:
H

28. **SAT/ACT** If x and y are positive integers, which of the following expressions is equivalent to $\frac{(5^x)^y}{5^x} = ?$

A 1^y
B ± 1
C 5^y
D 5^{xy-1}
E 5^{xy-x}

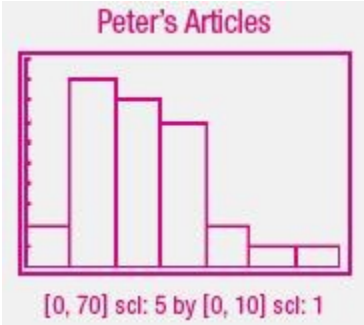
ANSWER:
E

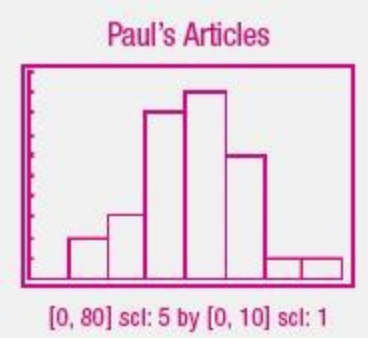
29. **ARTICLES** Peter and Paul each write articles for an online magazine. Their employer keeps track of the number of likes received by each article.
a. Use a graphing calculator to create a histogram for each data set. Then describe the shape of each distribution.
b. Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.

Peter's Articles
16, 22, 19, 31, 24, 8, 40, 19, 33, 18, 36, 21, 55, 3, 16, 44, 22, 39, 12, 18, 13, 20, 67, 31, 13, 38, 31, 22, 26, 28

Paul's Articles
41, 38, 29, 33, 36, 55, 51, 19, 49, 56, 28, 52, 49, 19, 38, 33, 42, 61, 72, 55, 48, 39, 37, 43, 48, 45, 52, 43, 34, 29

ANSWER:
a.





Peter's articles, positively skewed; Paul's articles, symmetric

b. Sample answer: One of the distributions is symmetric and the other is skewed, so use the five-number summaries. The range for Peter's articles is 64, and the range for Paul's articles is 53. However, the upper quartile for Peter's is 33, while the lower quartile for Paul's is 34. This means that 75% of Paul's articles have more likes (and are more popular) than 75% of Peter's articles. Therefore, we can conclude that Paul's articles are more popular overall.

Determine whether the situation calls for a *survey*, an *observational study*, or an *experiment*. Explain your reasoning.

30. You want to test a drug that reverses male pattern baldness.

ANSWER:

Sample answer: This situation calls for an experiment because the treatment will be tested on a sample group, which means that the members of the sample will be affected by the study.

31. You want to find voters' opinions on recent legislation.

ANSWER:

Sample answer: This situation calls for a survey because the data will be collected from responses from members of a sample of the population.

Find the first five terms of each geometric sequence described.

32. $a_1 = 0.125$, $r = 1.5$

ANSWER:

0.125, 0.1875, 0.28125, 0.421875, 0.632813

33. $a_1 = 0.5$, $r = 2.5$

ANSWER:

0.5, 1.25, 3.125, 7.8125, 19.53125

34. $a_1 = 4$, $r = 0.5$

ANSWER:

4, 2, 1, 0.5, 0.25

35. $a_1 = 12$, $r = \frac{1}{3}$

ANSWER:

12, 4, $\frac{4}{3}, \frac{4}{9}, \frac{4}{27}$

36. $a_1 = 21$, $r = \frac{2}{3}$

ANSWER:

21, 14, $\frac{28}{3}, \frac{56}{9}, \frac{112}{27}$

37. $a_1 = 80$, $r = \frac{5}{4}$

ANSWER:

80, 100, 125, $\frac{625}{4}, \frac{3125}{16}$

38. **COMMUNICATION** A microphone is placed at the focus of a parabolic reflector to collect sound for the television broadcast of a football game. Write an equation for the cross section, assuming that the focus is at the origin, the focus is 6 inches from the vertex, and the parabola opens to the right.

ANSWER:

$x = \frac{1}{24}y^2 - 6$

Solve each equation. Check your solutions.

39. $\log_9 x = \frac{3}{2}$

ANSWER:

27

40. $\log_{\frac{1}{10}} x = -3$

ANSWER:
1000

41. $\log_b 9 = 2$

ANSWER:
3

Expand each power.

42. $(a - b)^3$

ANSWER:
$a^3 - 3a^2b + 3ab^2 - b^3$

43. $(m + n)^4$

ANSWER:
$m^4 + 4m^3n + 6m^2n^2 + 4mn^3 + n^4$

44. $(r + n)^8$

ANSWER:
$r^8 + 8r^7n + 28r^6n^2 + 56r^5n^3 + 70r^4n^4 + 56r^3n^5$
$+ 28r^2n^6 + 8rn^7 + n^8$