11-4 The Binomial Distribution

Determine whether each experiment is a binomial experiment or can be reduced to a binomial experiment. If so, describe a trial, determine the random variable, and state n, p, and q.

1. A study finds that 58% of people have pets. You ask 100 people how many pets they have.

ANSWER:

This experiment cannot be reduced to a binomial experiment because there are more than two possible outcomes.

2. You roll a die 15 times and find the sum of all of the rolls.

ANSWER:

This experiment cannot be reduced to a binomial experiment because there are more than two possible outcomes.

3. A poll found that 72% of students plan on going to the homecoming dance. You ask 30 students if they are going to the homecoming dance.

ANSWER:

This experiment can be reduced to a binomial experiment. Success is yes, failure is no, a trial is asking a student, and the random variable is the number of yeses; $n = 30, p = 0.72, q = 0.28$.

4. Conduct a binomial experiment to determine the probability of drawing an ace or a king from a deck of cards. Then compare the experimental and theoretical probabilities of the experiment.

ANSWER:

Sample answer:
Step 1 A trial is drawing a card from a deck. The simulation will consist of 20 trials.
Step 2 A success is drawing an ace or a king. The probability of success is $\frac{2}{13}$ and the probability of failure is $\frac{11}{13}$.
Step 3 The random variable X represents the number of aces or kings drawn in 20 trials.
Step 4 Use a random number generator. Let 0−1 represent drawing an ace or a king. Let 2−12 represent drawing all other cards. Make a frequency table and record the results as you run the generator.

Outcome	Tally	Frequency
Ace or King	𝍸 |	6
Other Outcomes	𝍸 𝍸 ||||	14

The experimental probability is $\frac{6}{20}$ or 30%. This is greater than the theoretical probability of $\frac{2}{13}$ or about 15.4%.

5. **GAMES** Aiden has earned five spins of the wheel. He will receive a prize each time the spinner lands on WIN. What is the probability that he receives three prizes?

A 4.2%
B 5.8%
C 7.1%
D 8.8%

ANSWER:

D

6. **CCSS PRECISION** A poll at Steve's high school was taken to see if students are in favor of spending class money to expand the junior-senior parking lot. Steve surveyed 6 random students from the population.

Expand the Parking Lot	
favor	85%
oppose	15%

a. Determine the probabilities associated with the number of students that Steve asked who are in favor of expanding the parking lot by calculating the probability distribution.
b. What is the probability that no more than 2 people are in favor of expanding the parking lot?
c. How many students should Steve expect to find who are in favor of expanding the parking lot?

ANSWER:

a. 0 in favor, 0.00001 or 0.001%;
1 in favor, 0.00039 or 0.039%;
2 in favor, 0.00549 or 0.549%;
3 in favor, 0.04145 or 4.145%;
4 in favor, 0.17618 or 17.618%;
5 in favor, 0.39933 or 39.933%;
6 in favor, 0.37715 or 37.715%

b. 0.00589 or 0.589%
c. 5

Determine whether each experiment is a binomial experiment or can be reduced to a binomial experiment. If so, describe a trial, determine the random variable, and state n, p, and q.

7. There is a 35% chance that it rains each day in a given month. You record the number of days that it rains for that month.

ANSWER:

This experiment can be reduced to a binomial experiment. Success is a day that it rains, failure is a day it does not rain, a trial is a day, and the random variable X is the number of days it rains; n = the number of days in the month, $p = 0.35$, $q = 0.65$.

8. A survey found that on a scale of 1 to 10, a movie received a 7.8 rating. A movie theater employee asks 200 patrons to rate the movie on a scale of 1 to 10.

ANSWER:

This experiment cannot be reduced to a binomial experiment because there are more than two possible outcomes.

9. A ball is hidden under one of the hats shown below. A hat is chosen, one at a time, until the ball is found.

ANSWER:

This experiment cannot be reduced to a binomial experiment because the events are not independent. The probability of choosing the hat that covers the ball changes after each selection.

10. **DICE** Conduct a binomial experiment to determine the probability of rolling a 7 with two dice. Then compare the experimental and theoretical probabilities of the experiment.

ANSWER:

Sample answer:
Step 1 A trial is rolling two dice. The simulation will consist of 25 trials.
Step 2 A success is rolling a 7. The probability of success is $\frac{1}{6}$ and the probability of failure is $\frac{5}{6}$.
Step 3 The random variable X represents the number of times a 7 is rolled in 25 trials.
Step 4 Use a random number generator. Let 0 represent rolling a 7. Let 1−5 represent all other outcomes. Make a frequency table and record the results as you run the generator.

Outcome	Tally	Frequency
Rolling a 7	||||	4
Other Outcomes	𝍸 𝍸 𝍸 𝍸 |	21

The experimental probability is $\frac{4}{25}$ or 16%. This is approximately equal to the theoretical probability of $\frac{1}{6}$ or about 16.7%.

11. **MARBLES** Conduct a binomial experiment to determine the probability of pulling a red marble from the bag. Then compare the experimental and theoretical probabilities of the experiment.

ANSWER:

Sample answer:
Step 1 A trial is pulling out a marble. The simulation will consist of 20 trials.
Step 2 A success is pulling out a red marble. The probability of success is $\frac{5}{12}$ and the probability of failure is $\frac{7}{12}$.
Step 3 The random variable X represents the number of red marbles pulled out in 20 trials.
Step 4 Use a random number generator. Let 0−4 represent pulling out a red marble. Let 5−11 represent all other outcomes. Make a frequency table and record the results as you run the generator.

Outcome	Tally	Frequency
Red Marble	𝍸 𝍸	10
Other Outcomes	𝍸 𝍸	10

The experimental probability is $\frac{10}{20}$ or 50%. This is greater than the theoretical probability of $\frac{5}{12}$ or about 41.7%.

12. **SPINNER** Conduct a binomial experiment to determine the probability of the spinner stopping on an even number. Then compare the experimental and theoretical probabilities of the experiment.

ANSWER:

Sample answer:
Step 1 A trial is spinning the spinner. The simulation will consist of 25 trials.
Step 2 A success is the spinner landing on an even number. The probability of success is $\frac{2}{5}$ and the probability of failure is $\frac{3}{5}$.
Step 3 The random variable X represents the number of times the spinner stops on an even number in 25 trials.
Step 4 Use a random number generator. Let 0−1 represent the spinner stopping on an even number. Let 2−4 represent all other outcomes. Make a frequency table and record the results as you run the generator.

Outcome	Tally	Frequency
Even Number	𝍸 𝍸 |	11
Other Outcomes	𝍸 𝍸 ||||	14

The experimental probability is $\frac{11}{25}$ or 44%. This is slightly greater than the theoretical probability of $\frac{2}{5}$ or 40%.

13. **CARDS** Conduct a binomial experiment to determine the probability of drawing a face card out of a standard deck of cards. Then compare the experimental and theoretical probabilities of the experiment.

ANSWER:

Sample answer:
Step 1 A trial is drawing a card from a deck. The simulation will consist of 20 trials.
Step 2 A success is drawing a face card. The probability of success is $\frac{3}{13}$ and the probability of failure is $\frac{10}{13}$.
Step 3 The random variable X represents the number of face cards drawn in 20 trials.
Step 4 Use a random number generator. Let 0−2 represent drawing a face card. Let 3−12 represent all other outcomes. Make a frequency table and record the results as you run the generator.

Outcome	Tally	Frequency
Face Card	||	2
Other Cards	𝍸 𝍸 𝍸 |||	18

The experimental probability is $\frac{2}{20}$ or 10%. This is less than the theoretical probability of $\frac{3}{13}$ or about 23.1%

14. **PERSONAL MEDIA PLAYERS** According to a recent survey, 85% of high school students own a personal media player. What is the probability that 6 out of 10 random high school students own a personal media player?

ANSWER:

0.04 or 4%

15. **CARS** According to a recent survey, 92% of high school seniors drive their own car. What is the probability that 10 out of 12 random high school students drive their own car?

ANSWER:

0.183 or 18.3%

16. **SENIOR PROM** According to a recent survey, 25% of high school upperclassmen think that the junior-senior prom is the most important event of the school year. What is the probability that 3 out of 15 random high school upperclassmen think this way?

ANSWER:

0.225 or 22.5%

17. **FOOTBALL** A certain football team has won 75.7% of their games. Find the probability that they win 7 of their next 12 games.

ANSWER:

0.096 or 9.6%

18. **GARDENING** Peter is planting 24 irises in his front yard. The flowers he bought were a combination of two varieties, blue and white. The flowers are not blooming yet, but Peter knows that the probability of having a blue flower is 75%. What is the probability that 20 of the flowers will be blue?

ANSWER:

0.132 or 13.2%

19. **FOOTBALL** A field goal kicker is accurate 75% of the time from within 35 yards. What is the probability that he makes exactly 7 of his next 10 kicks from within 35 yards?

Range (yd)	Accuracy (%)
0–35	75
35–45	62
45+	20

ANSWER:

0.25 or 25%

20. **BABIES** Mr. and Mrs. Davis are planning to have 3 children. The probability of each child being a boy is 50%. What is the probability that they will have 2 boys?

ANSWER:

0.375 or 37.5%

21. **CCSS SENSE-MAKING** According to a recent survey, 52% of high school students own a laptop. Ten random students are chosen.
a. Determine the probabilities associated with the number of students that own a laptop by calculating the probability distribution.
b. What is the probability that at least 8 of the 10 students own a laptop?
c. How many students should you expect to own a laptop?

ANSWER:

a. 0 own a laptop, 0.0006 or 0.06%; 1 owns a laptop, 0.007 or 0.7%, 2 own a laptop, 0.0343 or 3.43%; 3 own a laptop, 0.0991 or 9.91%; 4 own a laptop, 0.1878 or 18.78%; 5 own a laptop, 0.2441 or 24.41%; 6 own a laptop, 0.2204 or 22.04%; 7 own a laptop, 0.1364 or 13.64%; 8 own a laptop, 0.0554 or 5.54%; 9 own a laptop, 0.0133 or 1.33%; 10 own a laptop, 0.0014 or 0.14%
b. about 7%
c. 5

22. **ATHLETICS** A survey was taken to see the percent of students that participate in sports for their school. Six random students are chosen.

Student Athletics	
0 sports	20%
1 sport	55%
2 sports	20%
3+ sports	5%

a. Determine the probabilities associated with the number of students playing in at least one sport by calculating the probability distribution.
b. What is the probability that no more than 2 of the students participated in a sport?
c. How many students should you expect to have participated in at least one sport?

ANSWER:

a. 0 play at least 1 sport, 0.00006 or 0.006%; 1 plays at least 1 sport, 0.00154 or 0.154%; 2 play at least 1 sport, 0.01536 or 1.536%; 3 play at least 1 sport, 0.08192 or 8.192%; 4 play at least 1 sport, 0.24576 or 24.576%; 5 play at least 1 sport, 0.39322 or 39.322%; 6 play at least 1 sport, 0.26214 or 26.214%
b. 0.01696 or 1.696%
c. 5

23. **CCSS MODELING** An online poll showed that 57% of adults still own vinyl records. Moe surveyed 8 random adults from the population.
a. Determine the probabilities associated with the number of adults that still own vinyl records by calculating the probability distribution.
b. What is the probability that no less than 6 of the people surveyed still own vinyl records?
c. How many people should Moe expect to still own vinyl records?

ANSWER:

a. 0 own vinyl records, 0.001 or 0.1%; 1 owns vinyl records, 0.012 or 1.2%; 2 own vinyl records, 0.058 or 5.8%; 3 own vinyl records, 0.152 or 15.2%; 4 own vinyl records, 0.253 or 25.3%; 5 own vinyl records, 0.268 or 26.8%; 6 own vinyl records, 0.178 or 17.8%; 7 own vinyl records, 0.067 or 6.7%; 8 own vinyl records, 0.011 or 1.1%
b. 0.256 or 25.6%
c. 5

A binomial distribution has a 60% rate of success. There are 18 trials.

24. What is the probability that there will be at least 12 successes?

ANSWER:
0.37 or 37%

25. What is the probability that there will be 12 failures?

ANSWER:
0.0145 or 1.45%

26. What is the expected number of successes?

ANSWER:
11

27. **DECISION MAKING** Six roommates randomly select someone to wash the dishes each day.
a. What is the probability that the same person has to wash the dishes 3 times in a given week?
b. What method can the roommates use to select who washes the dishes each day?

ANSWER:
a. about 7.8%
b. Sample answer: They can roll a six-sided die.

28. **DECISION MAKING** A committee of five people randomly selects someone to take the notes of each meeting.
a. What is the probability that a person takes notes less than twice in 10 meetings?
b. What method can the committee use to select the notetaker each meeting?
c. If the method described in part b results in the same person being notetaker for nine straight meetings, would this result cause you to question the method

ANSWER:
a. about 37.6%
b. Sample answer: They can roll a six-sided die. If a six is rolled, they roll again.
c. Sample answer: While it is possible for the same person to be chosen nine straight times, it is rather unlikely. The fairness of the die should be questioned.

Each binomial distribution has *n* trials and *p* probability of success. Determine the most likely number of successes.

29. $n = 8, p = 0.6$

ANSWER:
5

30. $n = 10, p = 0.4$

ANSWER:
4

31. $n = 6, p = 0.8$

ANSWER:
5

32. $n = 12, p = 0.55$

ANSWER:
7

33. $n = 9, p = 0.75$

ANSWER:
7

34. $n = 11, p = 0.35$

ANSWER:
4

35. **SWEEPSTAKES** A beverage company is having a sweepstakes. The probability of winning selected prizes is shown below. If Ernesto purchases 8 beverages, what is the probability that he wins at least one prize?

Odds of Winning	
beverage	1 in 10
CD	1 in 200
hat	1 in 250
MP3 player	1 in 20,000
car	1 in 25,000,000

ANSWER:
0.603 or 60.3%

Each binomial distribution has *n* trials and *p* probability of success. Determine the probability of *s* successes

36. $n = 8, p = 0.3, s \geq 2$

ANSWER:
0.744 or 74.4%

37. $n = 10, p = 0.2, s > 2$

ANSWER:
0.322 or 32.2%

38. $n = 6, p = 0.6, s \leq 4$

ANSWER:
0.767 or 76.7%

39. $n = 9, p = 0.25, s \leq 5$

ANSWER:
0.99 or 99%

40. $n = 10, p = 0.75, s \geq 8$

ANSWER:
0.526 or 52.6%

41. $n = 12, p = 0.1, s < 3$

ANSWER:
0.889 or 88.9%

42. **CHALLENGE** A poll of students determined that 88% wanted to go to college. Eight random students are chosen. The probability that at least x students want to go to college is about 0.752 or 75.2%. Solve for *x*.

ANSWER:
7

43. **WRITING IN MATH** What should you consider when using a binomial distribution to make a decision?

ANSWER:
Sample answer: You should consider the type of situation for which the binomial distribution is being used. For example, if a binomial distribution is being used to predict outcomes regarding an athletic event, the probabilities of success and failure could change due to other variables such as weather conditions or player health. So, binomial distributions should be used cautiously when making decisions involving events that are not completely random.

44. **OPEN ENDED** Describe a real-world setting within your school or community activities that seems to fit a binomial distribution. Identify the key components of your setting that connect to binomial distributions.

ANSWER:
Sample answer: During May and June, lunches are held outside, weather permitting. Also during this time, there has historically been a 15% chance of rain. So, to determine the probability of not having rain for at least 24 of these 28 days, the binomial distribution would use $p = 0.85$, $q = 0.15$, and $n = 28$.

45. **WRITING IN MATH** Describe how binomial distributions are connected to Pascal's triangle.

ANSWER:
Sample answer: A full binomial distribution can be determined by expanding the binomial, which itself utilizes Pascal's triangle.

46. **WRITING IN MATH** Explain the relationship between a binomial experiment and a binomial distribution.

ANSWER:
Sample answer: A binomial distribution shows the probabilities of the outcomes of a binomial experiment.

47. **EXTENDED RESPONSE** Carly is taking a 10-question multiple-choice test in which each question has four choices. If she guesses on each question, what is the probability that she will get
a. 7 questions correct?
b. 9 questions correct?
c. 0 questions correct?
d. 3 questions correct?

ANSWER:

a. 0.003 or 0.3%
b. 0.00003 or 0.003%
c. 0.056 or 5.6%
d. 0.25 or 25%

48. What is the maximum point of the graph of the equation $y = -2x^2 + 16x + 5$?
A (–4, –59)
B (–4, –91)
C (4, 37)
D (4, 101)

ANSWER:

C

49. **GEOMETRY** On a number line, point X has coordinate –8 and point Y has coordinate 4. Point P is $\frac{2}{3}$ of the way from X to Y. What is the coordinate of P?
F –4
G –2
H 0
J 2

ANSWER:

H

50. **SAT/ACT** The cost of 4 CDs is d dollars. At this rate, what is the cost, in dollars, of 36 CDs?
A $9d$
B $144d$
C $\frac{9d}{4}$
D $\frac{d}{36}$
E $\frac{36}{d}$

ANSWER:

A

Identify the random variable in each distribution, and classify it as *discrete* or *continuous*. Explain your reasoning.

51. the number of customers at an amusement park

ANSWER:

The random variable X is the number of customers at an amusement park. The customers are finite and countable, so X is discrete.

52. the running time of a movie

ANSWER:

The random variable X is the running time of a movie. The time can be anywhere within a certain range, so X is continuous.

53. the number of hot dogs sold at a sporting event

ANSWER:

The random variable X is the number of hot dogs sold at a sporting event. The hot dogs are finite and countable, so X is discrete.

54. the distance between two cities

ANSWER:

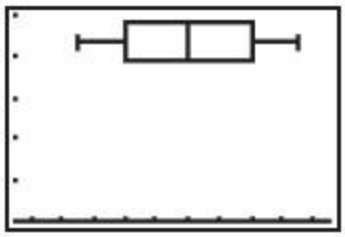
The random variable X is the distance between two cities. The distance can be anywhere within a certain range, so X is continuous.

55. **FINANCIAL LITERACY** The prices of entrees offered by a restaurant are shown.

Prices (dollars)			
11.25	14.75	9.00	17.25
19.75	9.75	20.25	15.50
16.50	21.50	10.25	22.75
12.75	18.50	23.00	13.50

a. Use a graphing calculator to create a box-and-whisker plot. Then describe the shape of the distribution.
b. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.

ANSWER:
a.



[5, 25] scl: 2 by [0, 5] scl: 1
symmetric

b. Sample answer: The distribution is symmetric, so use the mean and standard deviation. The mean is about \$16.02 with standard deviation of about \$4.52.

Find the missing value for each arithmetic sequence.

56. $a_5 = 12,\ a_{16} = 133,\ d = ?$

ANSWER:
11

57. $a_9 = -34,\ a_{22} = 44,\ d = ?$

ANSWER:
6

58. $a_4 = 18,\ a_n = 95,\ d = 7,\ n = ?$

ANSWER:
15

59. $a_8 = ?,\ a_{19} = 31,\ d = 8$

ANSWER:
–57

60. $a_6 = ?,\ a_{20} = 64,\ d = 7$

ANSWER:
–34

61. $a_7 = -28,\ a_n = 76,\ d = 8,\ n = ?$

ANSWER:
20

62. **ASTRONOMY** The table shows the closest and farthest distances of Venus and Jupiter from the center of the Sun in millions of miles.

Planet	Closest	Farthest
Venus	66.8	67.7
Jupiter	460.1	507.4

a. Write an equation for the orbit of each planet. Assuming that the center of the orbit is the origin and the center of the Sun is a focus that lies on the *x*-axis.
b. Which planet has an orbit that is closer to a circle?

ANSWER:
a.
Venus: $\frac{x^2}{4522.5625} + \frac{y^2}{4522.36} = 1$
Jupiter: $\frac{x^2}{234{,}014.06} + \frac{y^2}{233{,}454.74} = 1$
b. Venus

Write an equivalent exponential or logarithmic function.

63. $e^{-x} = 5$

ANSWER:
$-x = \ln 5$

64. $e^2 = 6x$

ANSWER:
$2 = \ln 6x$

65. $\ln e = 1$

ANSWER:

$e^1 = e$

66. $\ln 5.2 = x$

ANSWER:

$e^x = 5.2$

67. $e^{x+1} = 9$

ANSWER:

$x + 1 = \ln 9$

68. $e^{-1} = x^2$

ANSWER:

$-1 = \ln x^2$

69. $\ln \frac{7}{3} = 2x$

ANSWER:

$e^{2x} = \frac{7}{3}$

70. $\ln e^x = 3$

ANSWER:

$e^3 = e^x$

71. **MUSIC** Tina owns 11 pop, 6 country, 16 rock, and 7 rap CDs. Find each probability if she randomly selects 4 CDs.
a. P(2 rock)
b. P(1 rap)
c. P(1 rock and 2 country)

ANSWER:

a. 0.36
b. 0.42
c. 0.05