

11-5 The Normal Distribution

A normal distribution has a mean of 416 and a standard deviation of 55.

1. Find the range of values that represent the middle 99.7% of the distribution.

ANSWER:

$$251 < X < 581$$

2. What percent of the data will be less than 361?

ANSWER:

16%

3. **CCSS TOOLS** The number of texts sent per day by a sample of 811 teens is normally distributed with a mean of 38 and a standard deviation of 7.

a. About how many teens sent between 24 and 38 texts?

b. What is the probability that a teen selected at random sent less than 45 texts?

ANSWER:

a. about 386 teens

b. 84%

Find the missing variable. Indicate the position of X in the distribution.

4. z if $\mu = 89$, $X = 81$, and $\sigma = 11.5$

ANSWER:

-0.70; 0.70 standard deviations less than the mean

5. z if $\mu = 13.3$, $X = 17.2$, and $\sigma = 1.9$

ANSWER:

2.05; 2.05 standard deviations greater than the mean

6. X if $z = -1.38$, $\mu = 68.9$, and $\sigma = 6.6$

ANSWER:

59.8; 1.38 standard deviations less than the mean

7. σ if $\mu = 21.1$, $X = 13.7$, and $z = -2.40$

ANSWER:

3.08; 2.40 standard deviations less than the mean

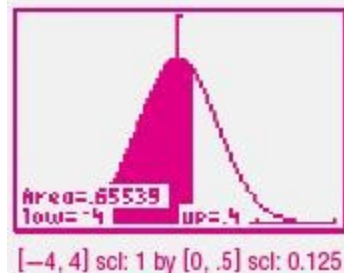
8. **CONCERTS** The number of concerts attended per year by a sample of 925 teens is normally distributed with a mean of 1.8 and a standard deviation of 0.5. Find each probability. Then use a graphing calculator to sketch the area under each curve.

a. $P(X < 2)$

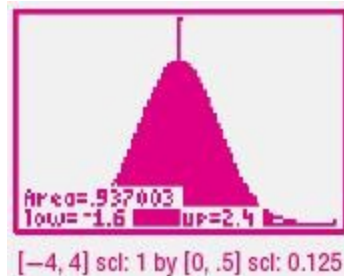
b. $P(1 < X < 3)$

ANSWER:

a. 65.5%



b. 93.7%



A normal distribution has a mean of 29.3 and a standard deviation of 6.7.

9. Find the range of values that represent the outside 5% of the distribution.

ANSWER:

$$X < 15.9 \text{ or } X > 42.7$$

10. What percent of the data will be between 22.6 and 42.7?

ANSWER:

81.5%

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11. **GYMS** The number of visits to a gym per year by a sample of 522 members is normally distributed with a mean of 88 and a standard deviation of 19.

- About how many members went to the gym at least 50 times?
- What is the probability that a member selected at random went to the gym more than 145 times?

ANSWER:

- about 509 members
- 0.15%

Find the missing variable. Indicate the position of X in the distribution.

12. z if $\mu = 3.3$, $X = 3.8$, and $\sigma = 0.2$

ANSWER:

2.5; 2.5 standard deviations greater than the mean

13. z if $\mu = 19.9$, $X = 18.7$, and $\sigma = 0.9$

ANSWER:

-1.33; 1.33 standard deviations less than the mean

14. μ if $z = -0.92$, $X = 44.2$, and $\sigma = 8.3$

ANSWER:

51.8; 0.92 standard deviations less than the mean

15. X if $\mu = 138.8$, $\sigma = 22.5$, and $z = 1.73$

ANSWER:

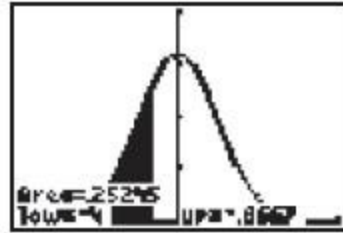
177.7; 1.73 standard deviations greater than the mean

16. **VENDING** A vending machine dispenses about 8.2 ounces of coffee. The amount varies and is normally distributed with a standard deviation of 0.3 ounce. Find each probability. Then use a graphing calculator to sketch the corresponding area under the curve.

- $P(X < 8)$
- $P(X > 7.5)$

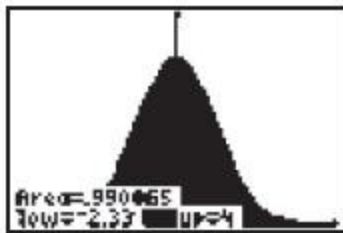
ANSWER:

- 25.2%



$[-4, 4]$ scl: 1 by $[0, .5]$ scl: 0.125

- 99.0%



$[-4, 4]$ scl: 1 by $[0, .5]$ scl: 0.125

17. **CAR BATTERIES** The useful life of a certain car battery is normally distributed with a mean of 113,627 miles and a standard deviation of 14,266 miles. The company makes 20,000 batteries a month.

- About how many batteries will last between 90,000 and 110,000 miles?
- About how many batteries will last more than 125,000 miles?
- What is the probability that if you buy a car battery at random, it will last less than 100,000 miles?

ANSWER:

- about 7000 batteries
- about 4200 batteries
- 17.0%

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18. **FOOD** The shelf life of a particular snack chip is normally distributed with a mean of 173.3 days and a standard deviation of 23.6 days.
- About what percent of the product lasts between 150 and 200 days?
 - About what percent of the product lasts more than 225 days?
 - What range of values represents the outside 5% of the distribution?

ANSWER:

- 71.0%
 - 1.4%
 - $X > 220.5$ or $X < 126.1$
19. **FINANCIAL LITERACY** The insurance industry uses various factors including age, type of car driven, and driving record to determine an individual's insurance rate. Suppose insurance rates for a sample population are normally distributed.
- If the mean annual cost per person is \$829 and the standard deviation is \$115, what is the range of rates you would expect the middle 68% of the population to pay annually?
 - If 900 people were sampled, how many would you expect to pay more than \$1000 annually?
 - Where on the distribution would you expect a person with several traffic citations to lie? Explain your reasoning.
 - How do you think auto insurance companies use each factor to calculate an individual's insurance rate?

ANSWER:

- between \$714 and \$944
- 62
- Sample answer: I would expect people with several traffic citations to lie to the far right of the distribution where insurance costs are highest, because I think insurance companies would charge them more.
- Sample answer: As the probability of an accident occurring increases, the more an auto insurance company is going to charge. I think auto insurance companies would charge younger people more than older people because they have not been driving as long. I think they would charge more for expensive cars and sports cars and less for cars that have good safety ratings. I think they would charge a person less if they have a good driving record and more if they have had tickets and accidents.

20. **STANDARDIZED TESTS** Nikki took three national standardized tests and scored an 86 on all three. The table shows the mean and standard deviation of each test.

	Math	Science	Social Studies
μ	76	81	72
σ	9.7	6.2	11.6

- Calculate the z-values that correspond to her score on each test.
- What is the probability of a student scoring an 86 or lower on each test?
- On which test was Nikki's standardized score the highest? Explain your reasoning.

ANSWER:

- Math: 1.03; Science: 0.81; Social Studies: 1.21
- Math: 84.8%; Science: 79.1%; Social Studies: 88.7%
- Social Studies; Sample answer: When the distributions are standardized, Nikki's relative scores on each test are 84.8%, 79.1%, and 88.7%. These standardized scores are Nikki's scores in relation to the population of scores for each individual test. Therefore, her 86% on the Social Studies test was better than 88.7% of the other test takers' scores on that test. This is the highest percentage of the three tests, so Nikki performed the best in Social Studies.

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21. **CCSS CRITIQUE** A set of normally distributed tree diameters have mean 11.5 cm, standard deviation 2.5, and range 3.6 to 19.8. Monica and Hiroko are to find the range that represents the middle 68% of the data. Is either of them correct? Explain.

Monica

The data span 16.2 cm. 68% of 16.2 is about 11 cm. Center this 11-cm range around the mean of 11.5 cm. This 68% group will range from about 6 cm to about 17 cm.

Hiroko

The middle 68% span from $\mu + \sigma$ to $\mu - \sigma$. So we move 2.5 cm below 11.5 and then 2.5 cm above 11.5. The 68% group will range from 9 cm to 14 cm.

ANSWER:

Hiroko; Monica's solution would work with a uniform distribution.

22. **CHALLENGE** A case of digital audio players has an average battery life of 8.2 hours with a standard deviation of 0.7 hour. Eight of the players have a battery life greater than 9.3 hours. If the sample is normally distributed, how many players are in the case?

ANSWER:

138

23. **REASONING** The term six sigma process comes from the notion that if one has six standard deviations between the mean of a process and the nearest specification limit, there will be practically no items that fail to meet the specifications. Is this a true assumption? Explain.

ANSWER:

Sample answer: True; according to the Empirical Rule, 99% of the data lie within 3 standard deviations of the mean. Therefore, only 1% will fall outside of three sigma. An infinitesimally small amount will fall outside of six-sigma.

24. **REASONING** *True or false:* According to the Empirical Rule, in a normal distribution, most of the data will fall within one standard deviation of the mean. Explain.

ANSWER:

Sample answer: True; according to the Empirical Rule, 68% of the data lie within 1 standard deviation of the mean.

25. **OPEN ENDED** Find a set of real-world data that appears to be normally distributed. Calculate the range of values that represent the middle 68%, the middle 95%, and the middle 99.7% of the distribution.

ANSWER:

Sample answer: The scores per team in each game of the first round of the 2010 NBA playoffs. The mean is 96.56 and the standard deviation is 11.06. The middle 68% of the distribution is $85.50 < X < 107.62$. The middle 95% is $74.44 < X < 118.68$. The middle 99.7% is $63.38 < X < 129.74$.

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26. **WRITING IN MATH** Describe the relationship between the z -value, the position of an interval of X in the normal distribution, the area under the normal curve, and the probability of the interval occurring. Use an example to explain your reasoning.

ANSWER:

Sample answer: The z -value represents the position of a value X in a normal distribution. A z -value of 1.43 means that the corresponding data value X is 1.43 standard deviations to the right of the mean in the distribution. An interval of all of the values greater than X in the distribution will be represented by the area under the curve from $z = 1.43$ to $z = 4$. This area is equivalent to the probability of the interval occurring (a random data value falling within the interval).

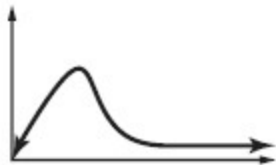
27. The lifetimes of 10,000 light bulbs are normally distributed. The mean lifetime is 300 days, and the standard deviation is 40 days. How many light bulbs will last between 260 and 340 days?

- A 2500
- B 3400
- C 5000
- D 6800

ANSWER:

D

28. Which description best represents the graph?

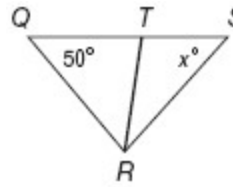


- F negatively skewed
- G no correlation
- H normal distribution
- J positively skewed

ANSWER:

J

29. **SHORT RESPONSE** In the figure below, $RT = TS$ and $QR = QT$. What is the value of x ?



ANSWER:

32.5

30. **SAT/ACT** The integer 99 can be expressed as a sum of n consecutive positive integers. The value of n could be which of the following?

- I. 2
- II. 3
- III. 6

A I only

B II only

C III only

D I and II only

E I, II, and III

ANSWER:

E

31. **SNOW** There is a 25% chance that it snows each day during a given week. Find the probability that it snows 3 out of the next 7 days.

ANSWER:

17.3%

Identify the random variable in each distribution, and classify it as *discrete* or *continuous*. Explain your reasoning.

32. the number of pages in a newspaper

ANSWER:

The random variable X is the number of stations in a cable package. The stations are finite and countable, so X is discrete.

33. the amount of precipitation in a city per month

ANSWER:

The random variable X is the amount of precipitation in a city per month. Precipitation can be anywhere within a certain range. Therefore, X is continuous.

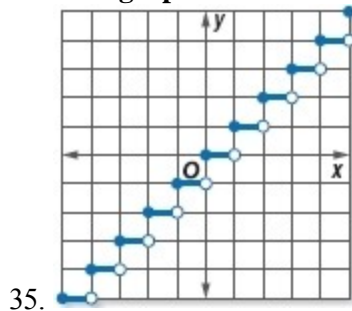
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34. **BRIDGES** The Sydney Harbour Bridge connects the Sydney central business district to northern metropolitan Sydney. It has an arch in the shape of a parabola that opens downward. Write an equation of a parabola to model the arch, assuming that the origin is at the surface of the water, beneath the vertex of the arch.
Refer to Page 778.

ANSWER:

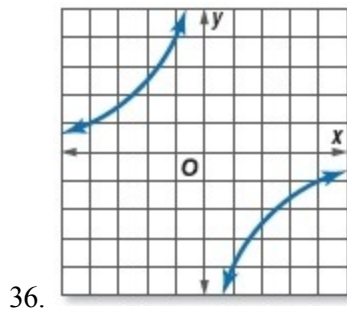
about $y = -0.00065x^2 + 440$

Identify the type of function represented by each graph.



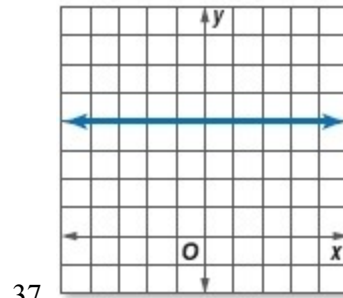
ANSWER:

greatest integer



ANSWER:

inverse variation or rational



ANSWER:

constant

38. Calculate the standard deviation of the population of data.

13	18	17	21	16	9	11	28	8	10
7	19	16	16	12	19	21	11	8	13

ANSWER:

5.28