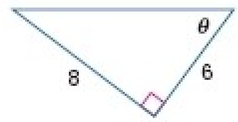
Find the values of the six trigonometric functions for angle *θ*.

1. 

SOLUTION:

Opposite side = 8
Adjacent Side = 6
Let *x* be the hypotenuse.
By the Pythagorean theorem,

$$x = \sqrt{8^2 + 6^2}$$
$$= 10$$

Therefore, hypotenuse = 10.

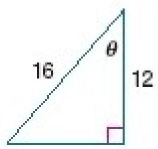
The trigonometric ratios are:

$$\sin\theta = \frac{\text{Opposite Side}}{\text{Hypotenuse}}$$
$$\cos\theta = \frac{\text{Adjacent Side}}{\text{Hypotenuse}}$$
$$\tan\theta = \frac{\text{Opposite Side}}{\text{Adjacent Side}}$$
$$\csc\theta = \frac{\text{Hypotenuse}}{\text{Opposite Side}}$$
$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Adjacent Side}}$$
$$\cot\theta = \frac{\text{Adjacent Side}}{\text{Opposite Side}}$$

Substitute:

$$\sin\theta = \frac{8}{10} \text{ or } \frac{4}{5}$$
$$\cos\theta = \frac{6}{10} \text{ or } \frac{3}{5}$$
$$\tan\theta = \frac{8}{6} \text{ or } \frac{4}{3}$$
$$\csc\theta = \frac{10}{8} \text{ or } \frac{5}{4}$$
$$\sec\theta = \frac{10}{6} \text{ or } \frac{5}{3}$$
$$\cot\theta = \frac{6}{8} \text{ or } \frac{3}{4}$$

12-1 Trigonometric Functions in Right Triangles

2. 

SOLUTION:

Adjacent side = 12
Hypotenuse = 16

$$\text{opposite side } = \sqrt{16^2 - 12^2}$$
$$= 4\sqrt{7}$$

The trigonometric ratios are:

$$\sin\theta = \frac{\text{Opposite Side}}{\text{Hypotenuse}}$$

$$\cos\theta = \frac{\text{Adjacent Side}}{\text{Hypotenuse}}$$

$$\tan\theta = \frac{\text{Opposite Side}}{\text{Adjacent Side}}$$

$$\csc\theta = \frac{\text{Hypotenuse}}{\text{Opposite Side}}$$

$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Adjacent Side}}$$

$$\cot\theta = \frac{\text{Adjacent Side}}{\text{Opposite Side}}$$

Substitute:

$$\sin\theta = \frac{4\sqrt{7}}{16} \text{ or } \frac{\sqrt{7}}{4}$$
$$\cos\theta = \frac{12}{16} \text{ or } \frac{3}{4}$$
$$\tan\theta = \frac{4\sqrt{7}}{12} \text{ or } \frac{\sqrt{7}}{3}$$
$$\csc\theta = \frac{16}{4\sqrt{7}} \text{ or } \frac{4\sqrt{7}}{7}$$
$$\sec\theta = \frac{16}{12} \text{ or } \frac{4}{3}$$
$$\cot\theta = \frac{12}{4\sqrt{7}} \text{ or } \frac{3\sqrt{7}}{7}$$

In a right triangle, $\angle A$ is acute. Find the values of the five remaining trigonometric functions.

3. $\cos A = \frac{4}{7}$

SOLUTION:

$$\cos A = \frac{\text{Adjacent Side}}{\text{Hypotenuse}} = \frac{4}{7}$$

Therefore:

$$\text{Opposite side} = \sqrt{7^2 - 4^2}$$
$$= \sqrt{33}$$

$$\sin A = \frac{\text{Opposite Side}}{\text{Hypotenuse}} = \frac{\sqrt{33}}{7}$$

$$\tan A = \frac{\text{Opposite Side}}{\text{Adjacent Side}} = \frac{\sqrt{33}}{4}$$

$$\csc A = \frac{\text{Hypotenuse}}{\text{Opposite Side}} = \frac{7\sqrt{33}}{33}$$

$$\sec A = \frac{\text{Hypotenuse}}{\text{Adjacent Side}} = \frac{7}{4}$$

$$\cot A = \frac{\text{Adjacent Side}}{\text{Opposite Side}} = \frac{4\sqrt{33}}{4}$$

4. $\tan A = \frac{20}{21}$

SOLUTION:

$$\tan A = \frac{\text{Opposite Side}}{\text{Adjacent Side}} = \frac{20}{21}$$

Therefore:

$$\text{hypotenuse} = \sqrt{21^2 + 20^2} = 29$$

$$\sin A = \frac{\text{Opposite Side}}{\text{Hypotenuse}} = \frac{20}{29}$$

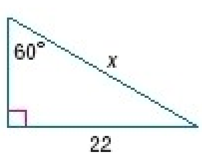
$$\cos A = \frac{\text{Adjacent Side}}{\text{Hypotenuse}} = \frac{21}{29}$$

$$\csc A = \frac{\text{Hypotenuse}}{\text{Opposite Side}} = \frac{29}{20}$$

$$\sec A = \frac{\text{Hypotenuse}}{\text{Adjacent Side}} = \frac{29}{21}$$

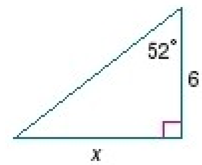
$$\cot A = \frac{\text{Adjacent Side}}{\text{Opposite Side}} = \frac{21}{20}$$

Use a trigonometric function to find the value of *x*. Round to the nearest tenth

5. 

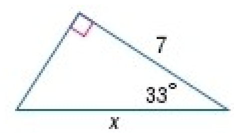
SOLUTION:

$$\sin 60^\circ = \frac{22}{x}$$
$$x = \frac{22}{\sin 60^\circ}$$
$$x \approx 25.4$$

6. 

SOLUTION:

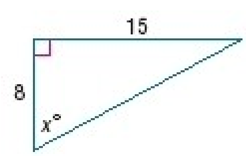
$$\tan 52^\circ = \frac{x}{6}$$
$$x = 6 \tan 52^\circ$$
$$x \approx 7.7$$

7. 

SOLUTION:

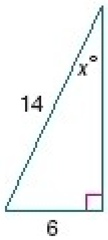
$$\cos 33^\circ = \frac{7}{x}$$
$$x = \frac{7}{\cos 33^\circ}$$
$$x \approx 8.3$$

Find the value of *x*. Round to the nearest tenth.

8. 

SOLUTION:

$$\tan x = \frac{15}{8}$$
$$x = \tan^{-1}\left(\frac{15}{8}\right)$$
$$\approx 61.9$$

9. 

SOLUTION:

$$\sin x = \frac{6}{14}$$
$$x = \sin^{-1}\left(\frac{6}{14}\right)$$
$$x \approx 25.4$$

10. 

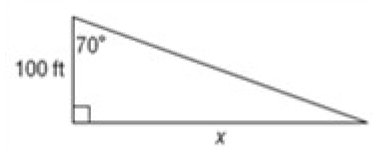
SOLUTION:

$$\cos x = \frac{6}{16}$$
$$x = \cos^{-1}\left(\frac{6}{16}\right)$$
$$\approx 68.0$$

11. **CCSS SENSE-MAKING** Christian found two trees directly across from each other in a canyon. When he moved 100 feet from the tree on his side (parallel to the edge of the canyon), the angle formed by the tree on his side, Christian, and the tree on the other side was 70°. Find the distance across the canyon.

SOLUTION:

Let *x* be the distance across the canyon.

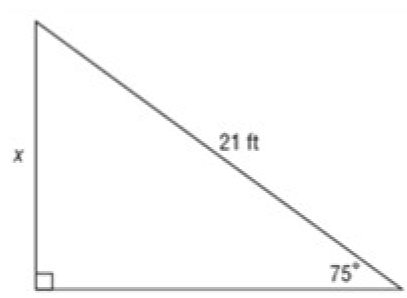


$$\tan 70° = \frac{x}{100}$$
$$x \approx 274.7 \text{ ft}$$

12. **LADDERS** The recommended angle of elevation for a ladder used in fire fighting is 75°. At what height on a building does a 21-foot ladder reach if the recommended angle of elevation is used? Round to the nearest tenth.

SOLUTION:

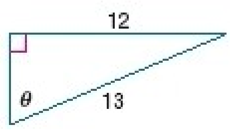
Let *x* be the height of the building.



$$\sin 75° = \frac{x}{21}$$
$$x \approx 20.3 \text{ ft}$$

Find the values of the six trigonometric functions for angle θ.

12-1 Trigonometric Functions in Right Triangles

13. 

SOLUTION:

$$\text{adjacent side} = \sqrt{13^2 - 12^2} = 5$$

The trigonometric ratios are:

$$\sin\theta = \frac{\text{Opposite Side}}{\text{Hypotenuse}}$$

$$\cos\theta = \frac{\text{Adjacent Side}}{\text{Hypotenuse}}$$

$$\tan\theta = \frac{\text{Opposite Side}}{\text{Adjacent Side}}$$

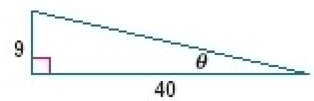
$$\csc\theta = \frac{\text{Hypotenuse}}{\text{Opposite Side}}$$

$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Adjacent Side}}$$

$$\cot\theta = \frac{\text{Adjacent Side}}{\text{Opposite Side}}$$

Substitute:

$$\sin\theta = \frac{12}{13}$$
$$\cos\theta = \frac{5}{13}$$
$$\tan\theta = \frac{12}{5}$$
$$\csc\theta = \frac{13}{12}$$
$$\sec\theta = \frac{13}{5}$$
$$\cot\theta = \frac{12}{5}$$

14. 

SOLUTION:

$$\text{hypotenuse} = \sqrt{40^2 + 9^2} = 41$$

The trigonometric ratios are:

$$\sin\theta = \frac{\text{Opposite Side}}{\text{Hypotenuse}}$$

$$\cos\theta = \frac{\text{Adjacent Side}}{\text{Hypotenuse}}$$

$$\tan\theta = \frac{\text{Opposite Side}}{\text{Adjacent Side}}$$

$$\csc\theta = \frac{\text{Hypotenuse}}{\text{Opposite Side}}$$

$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Adjacent Side}}$$

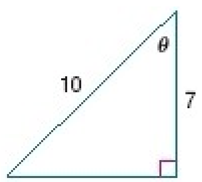
$$\cot\theta = \frac{\text{Adjacent Side}}{\text{Opposite Side}}$$

Substitute:

$$\sin\theta = \frac{9}{41}$$
$$\cos\theta = \frac{40}{41}$$
$$\tan\theta = \frac{9}{40}$$
$$\csc\theta = \frac{41}{9}$$

$\sec\theta = \frac{41}{40}$

$\cot\theta = \frac{40}{9}$

15. 

SOLUTION:

$$\text{opposite side} = \sqrt{10^2 - 7^2} = \sqrt{51}$$

The trigonometric ratios are:

$$\sin\theta = \frac{\text{Opposite Side}}{\text{Hypotenuse}}$$

$$\cos\theta = \frac{\text{Adjacent Side}}{\text{Hypotenuse}}$$

$$\tan\theta = \frac{\text{Opposite Side}}{\text{Adjacent Side}}$$

$$\csc\theta = \frac{\text{Hypotenuse}}{\text{Opposite Side}}$$

$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Adjacent Side}}$$

$$\cot\theta = \frac{\text{Adjacent Side}}{\text{Opposite Side}}$$

Substitute:

$\sin\theta = \frac{\sqrt{51}}{10}$

$\cos\theta = \frac{7}{10}$

$\tan\theta = \frac{\sqrt{51}}{7}$

$\csc\theta = \frac{10}{\sqrt{51}}$ or $\frac{10\sqrt{51}}{51}$

$\sec\theta = \frac{10}{7}$

$\cot\theta = \frac{7}{\sqrt{51}}$ or $\frac{7\sqrt{51}}{51}$

16. 

SOLUTION:

$$\text{hypotenuse} = \sqrt{9^2 + 6^2} = 3\sqrt{13}$$

The trigonometric ratios are:

$$\sin\theta = \frac{\text{Opposite Side}}{\text{Hypotenuse}}$$

$$\cos\theta = \frac{\text{Adjacent Side}}{\text{Hypotenuse}}$$

$$\tan\theta = \frac{\text{Opposite Side}}{\text{Adjacent Side}}$$

$$\csc\theta = \frac{\text{Hypotenuse}}{\text{Opposite Side}}$$

$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Adjacent Side}}$$

$$\cot\theta = \frac{\text{Adjacent Side}}{\text{Opposite Side}}$$

Substitute:

$$\sin\theta = \frac{6}{3\sqrt{13}} \text{ or } \frac{2\sqrt{13}}{13}$$
$$\cos\theta = \frac{9}{3\sqrt{13}} \text{ or } \frac{3\sqrt{13}}{13}$$
$$\tan\theta = \frac{6}{9} = \frac{2}{3}$$
$$\csc\theta = \frac{\sqrt{13}}{2}$$
$$\sec\theta = \frac{\sqrt{13}}{3}$$
$$\cot\theta = \frac{3}{2}$$

In a right triangle, $\angle A$ and $\angle B$ are acute. Find the values of the five remaining trigonometric functions.

17. $\tan A = \frac{8}{15}$

SOLUTION:

$$\tan A = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{8}{15}$$

Therefore:

$$\text{hypotenuse} = \sqrt{15^2 + 8^2} = 17$$

$$\sin A = \frac{\text{Opposite Side}}{\text{Hypotenuse}} = \frac{8}{17}$$

$$\cos A = \frac{\text{Adjacent Side}}{\text{Hypotenuse}} = \frac{15}{17}$$

$$\csc A = \frac{\text{Hypotenuse}}{\text{Opposite Side}} = \frac{17}{8}$$

$$\sec A = \frac{\text{Hypotenuse}}{\text{Adjacent Side}} = \frac{17}{15}$$

$$\cot A = \frac{\text{Adjacent Side}}{\text{Opposite Side}} = \frac{15}{8}$$

18. $\cos A = \frac{3}{10}$

SOLUTION:

$$\cos A = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{3}{10}$$

Therefore:

$$\text{opposite side} = \sqrt{10^2 - 3^2} = \sqrt{91}$$

$$\sin A = \frac{\text{Opposite Side}}{\text{Hypotenuse}} = \frac{\sqrt{91}}{10}$$

$$\tan A = \frac{\text{Opposite Side}}{\text{Adjacent Side}} = \frac{\sqrt{91}}{3}$$

$$\csc A = \frac{\text{Hypotenuse}}{\text{Opposite Side}} = \frac{10\sqrt{91}}{91}$$

$$\sec A = \frac{\text{Hypotenuse}}{\text{Adjacent Side}} = \frac{10}{3}$$

$$\cot A = \frac{\text{Adjacent Side}}{\text{Opposite Side}} = \frac{3\sqrt{91}}{91}$$

19. tan *B* = 3

SOLUTION:

$$\tan B = \frac{\text{Opposite Side}}{\text{Adjacent Side}} = \frac{3}{1}$$

Therefore:

$$\text{hypotenuse} = \sqrt{3^2 + 1^2}$$
$$= \sqrt{10}$$

$$\sin B = \frac{\text{Opposite Side}}{\text{Hypotenuse}} = \frac{3\sqrt{10}}{10}$$

$$\cos B = \frac{\text{Adjacent Side}}{\text{Hypotenuse}} = \frac{\sqrt{10}}{10}$$

$$\csc B = \frac{\text{Hypotenuse}}{\text{Opposite Side}} = \frac{\sqrt{10}}{3}$$

$$\sec B = \frac{\text{Hypotenuse}}{\text{Adjacent Side}} = \sqrt{10}$$

$$\cot B = \frac{\text{Adjacent Side}}{\text{Opposite Side}} = \frac{1}{3}$$

20. $\sin B = \frac{4}{9}$

SOLUTION:

$$\sin B = \frac{\text{Opposite Side}}{\text{Hypotenuse}} = \frac{4}{9}$$

Therefore:

$$\text{adjacent side} = \sqrt{9^2 - 4^2}$$
$$= \sqrt{65}$$

$$\cos B = \frac{\text{Adjacent Side}}{\text{Hypotenuse}} = \frac{\sqrt{65}}{9}$$

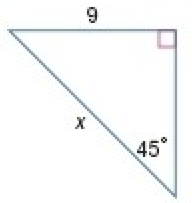
$$\tan B = \frac{\text{Opposite Side}}{\text{Adjacent Side}} = \frac{4\sqrt{65}}{65}$$

$$\csc B = \frac{\text{Hypotenuse}}{\text{Opposite Side}} = \frac{9}{4}$$

$$\sec B = \frac{\text{Hypotenuse}}{\text{Adjacent Side}} = \frac{9\sqrt{65}}{65}$$

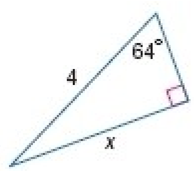
$$\cot B = \frac{\text{Adjacent Side}}{\text{Opposite Side}} = \frac{\sqrt{65}}{4}$$

Use a trigonometric function to find each value of *x*. Round to the nearest tenth.

21. 

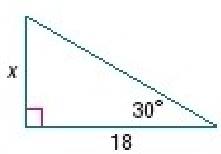
SOLUTION:

$$\sin 45° = \frac{9}{x}$$
$$\frac{1}{\sqrt{2}} = \frac{9}{x}$$
$$x \approx 12.7$$

22. 

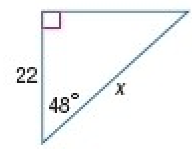
SOLUTION:

$$\sin 64° = \frac{x}{4}$$
$$x \approx 3.6$$

23. 

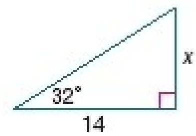
SOLUTION:

$$\tan 30° = \frac{x}{18}$$
$$\frac{1}{\sqrt{3}} = \frac{x}{18}$$
$$x \approx 10.4$$

24. 

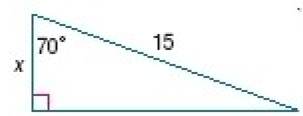
SOLUTION:

$$\cos 48° = \frac{22}{x}$$
$$x \approx 32.9$$

25. 

SOLUTION:

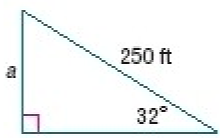
$$\tan 32° = \frac{x}{14}$$
$$x \approx 8.7$$

26. 

SOLUTION:

$$\cos 70° = \frac{x}{15}$$
$$x \approx 5.1$$

27. **PARASAILING** Refer to the beginning of the lesson and the figure shown. Find a, the altitude of a person parasailing, if the tow rope is 250 feet long and the angle formed is 32°. Round to the nearest tenth.



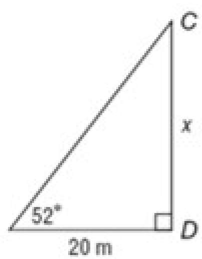
SOLUTION:

$$\sin 32° = \frac{a}{250}$$
$$a \approx 132.5 \text{ ft}$$

28. **CCSS MODELING** Devon wants to build a rope bridge between his treehouse and Cheng's treehouse. Suppose Devon's treehouse is directly behind Cheng's treehouse. At a distance of 20 meters to the left of Devon's treehouse, an angle of 52° is measured between the two treehouses. Find the length of the rope.

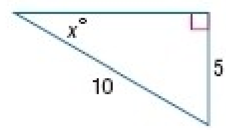
SOLUTION:

Let x be the length of the rope bridge between the houses.



$$\tan 52° = \frac{x}{20}$$
$$x \approx 25.6\ m$$

Find the value of x. Round to the nearest tenth.

29. 

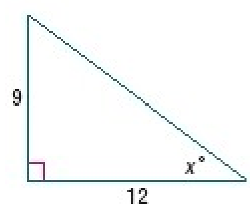
SOLUTION:

$$\sin x = \frac{5}{10} = \frac{1}{2}$$
$$x = \sin^{-1}\left(\frac{1}{2}\right)$$
$$= 30°$$

30. 

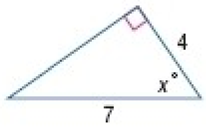
SOLUTION:

$$\tan x = \frac{19}{8}$$
$$x = \tan^{-1}\left(\frac{19}{8}\right)$$
$$\approx 67.2°$$

31. 

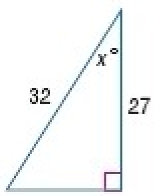
SOLUTION:

$$\tan x = \frac{9}{12} = \frac{3}{4}$$
$$x = \tan^{-1}\left(\frac{3}{4}\right)$$
$$\approx 36.9°$$

32. 

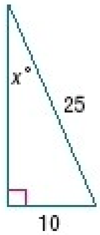
SOLUTION:

$$\cos x = \frac{4}{7}$$
$$x = \cos^{-1}\left(\frac{4}{7}\right) \approx 55.2°$$

33. 

SOLUTION:

$$\cos x = \frac{27}{32}$$
$$x = \cos^{-1}\left(\frac{27}{32}\right) \approx 32.5°$$

34. 

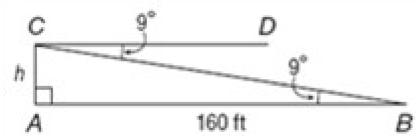
SOLUTION:

$$\sin x = \frac{10}{25} = \frac{2}{5}$$
$$x = \sin^{-1}\left(\frac{2}{5}\right) \approx 23.6°$$

35. **SQUIRRELS** Adult flying squirrels can make glides of up to 160 feet. If a flying squirrel glides a horizontal distance of 160 feet and the angle of descent is 9°, find its change in height.

SOLUTION:

Let *h* be the height.



$$\tan 9° = \frac{h}{160}$$
$$h \approx 25.3 \text{ ft}$$

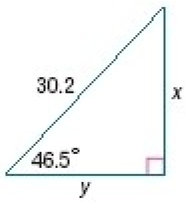
36. **HANG GLIDING** A hang glider climbs at a 20° angle of elevation. Find the change in altitude of the hang glider when it has flown a horizontal distance of 60 feet.

SOLUTION:

Let *x* be the change in altitude.

$$\tan 20° = \frac{x}{60}$$
$$x \approx 21.8 \text{ ft}$$

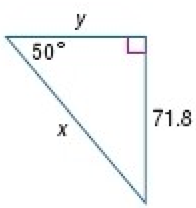
Use trigonometric functions to find the values of x and y. Round to the nearest tenth.

37. 

SOLUTION:

$$\sin 46.5° = \frac{x}{30.2}$$
$$x \approx 21.9$$

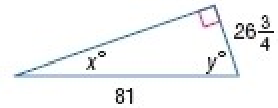
$$\cos 46.5° = \frac{y}{30.2}$$
$$y \approx 20.8$$

38. 

SOLUTION:

$$\sin 50° = \frac{71.8}{x}$$
$$x \approx 93.7$$

$$\tan 50° = \frac{71.8}{y}$$
$$y \approx 60.2$$

39. 

SOLUTION:

$$\sin x = \frac{26\frac{3}{4}}{81}$$
$$x = \sin^{-1}\left(\frac{26\frac{3}{4}}{81}\right)$$
$$\approx 19.3$$

$$\cos y = \frac{26\frac{3}{4}}{81}$$
$$y = \cos^{-1}\left(\frac{26\frac{3}{4}}{81}\right)$$
$$\approx 70.7$$

Solve each equation

40. $\cos A = \frac{3}{19}$

SOLUTION:

$$\cos A = \frac{3}{19}$$
$$A = \cos^{-1}\left(\frac{3}{19}\right)$$
$$\approx 80.9$$

41. $\sin N = \frac{9}{11}$

SOLUTION:

$$\sin N = \frac{9}{11}$$
$$N = \sin^{-1}\left(\frac{9}{11}\right)$$
$$\approx 54.9$$

42. tan *X* = 15

SOLUTION:

$$\tan X = 15$$
$$X = \tan^{-1}(15)$$
$$\approx 86.2$$

43. sin *T* = 0.35

SOLUTION:

$$\sin T = 0.35$$
$$T = \sin^{-1}(0.35)$$
$$\approx 20.5$$

44. tan *G* = 0.125

SOLUTION:

$$\tan G = 0.125$$
$$G = \tan^{-1}(0.125)$$
$$\approx 7.1$$

45. cos *Z* = 0.98

SOLUTION:

$$\cos Z = 0.98$$
$$Z = \cos^{-1}(0.98)$$
$$\approx 11.5$$

46. **MONUMENTS** A monument casts a shadow 24 feet long. The angle of elevation from the end of the shadow to the top of the monument is 50°.

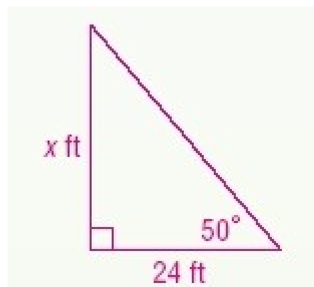
a. Draw and label a right triangle to represent this situation.

b. Write a trigonometric function that can be used to find the height of the monument.

c. Find the value of the function to determine the height of the monument to the nearest tenth.

SOLUTION:

a.



b. Let *x* be the height of the monument.

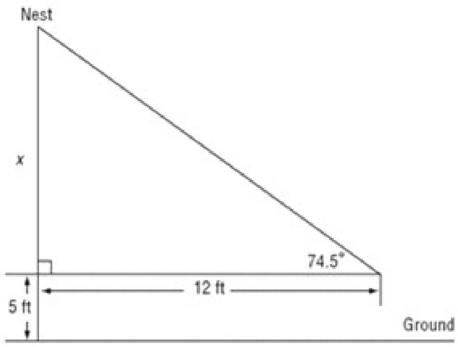
$$\tan 50° = \frac{x}{24}$$

c. Solve for *x*.

$$\tan 50° = \frac{x}{24}$$
$$x \approx 28.6 \text{ ft}$$

47. **NESTS** Tabitha's eyes are 5 feet above the ground as she looks up to a bird's nest in a tree. If the angle of elevation is 74.5° and she is standing 12 feet from the tree's base, what is the height of the bird's nest? Round to the nearest foot.

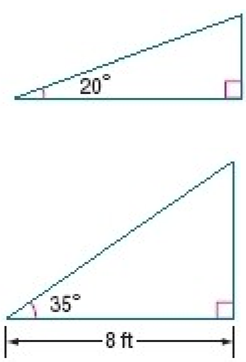
SOLUTION:



$$\tan 74.5 = \frac{x}{12}$$
$$x \approx 43 \text{ ft}$$

Therefore, the height of the nest = 43 + 5 = 48 ft.

48. **RAMPS** Two bicycle ramps each cover a horizontal distance of 8 feet. One ramp has a 20° angle of elevation, and the other ramp has a 35° angle of elevation, as shown at the right.



a. How much taller is the second ramp than the first? Round to the nearest tenth.

b. How much longer is the second ramp than the first? Round to the nearest tenth.

SOLUTION:

a. Let x be the height of the first ramp and y be the height of the second ramp.

$$\tan 20° = \frac{x}{8}$$
$$x \approx 2.9$$
$$\tan 35° = \frac{y}{8}$$
$$y \approx 5.6$$

$$y - x = 5.6 - 2.9$$
$$= 2.7$$

Therefore, the second ramp is 2.7 ft taller than the first ramp.

b. Let l be the length of the first ramp and m the length of the second ramp.

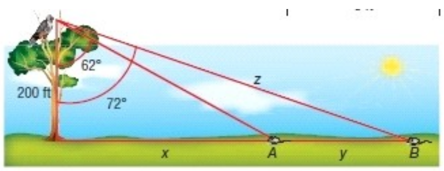
$$\cos 20° = \frac{8}{l}$$
$$l \approx 8.5$$
$$\cos 35° = \frac{8}{m}$$
$$m \approx 9.8$$
$$m - l = 9.8 - 8.5 = 1.3$$

Therefore, the second ramp is 1.3 ft longer than the first ramp.

49. **FALCONS** A falcon at a height of 200 feet sees two mice A and B, as shown in the diagram.

a. What is the approximate distance z between the falcon and mouse B?

b. How far apart are the two mice?



SOLUTION:

a.

$$\cos 72° = \frac{200}{z}$$
$$z \approx 647.2 \text{ ft}$$

b.

$$\tan 62° = \frac{x}{200}$$
$$x \approx 376.1$$

Also:

$$\tan 72° = \frac{x+y}{200} = \frac{376.1+y}{200}$$
$$y \approx 239.4$$

Therefore, the two mice are about 239.4 ft apart.

In $\triangle ABC$, $\angle C$ is a right angle. Use the given measurements to find the missing side lengths and missing angle measures of $\triangle ABC$. Round to the nearest tenth if necessary.

50. $m\angle A = 36°, a = 12$

SOLUTION:

$$A + B + C = 180°$$
$$36° + B + 90° = 180°$$
$$B = 54°$$

$$\tan 36° = \frac{12}{b}$$
$$b \approx 16.5$$
$$\sin 36° = \frac{12}{c}$$
$$c \approx 20.4$$

51. $m\angle B = 31°, b = 19$

SOLUTION:

$$A + B + C = 180°$$
$$A = 59°$$
$$\tan 31° = \frac{19}{a}$$
$$a \approx 31.6$$
$$\sin 31° = \frac{19}{c}$$
$$c \approx 36.9$$

52. $a = 8$, $c = 17$

SOLUTION:

$$\sin A = \frac{8}{17}$$
$$A = \sin^{-1}\left(\frac{8}{17}\right) \approx 28.1$$

$$\cos B = \frac{8}{17}$$
$$B = \cos^{-1}\left(\frac{8}{17}\right) \approx 61.9$$

$$b = \sqrt{17^2 - 8^2} = 15$$

53. $\tan A = \frac{4}{5}, a = 6$

SOLUTION:

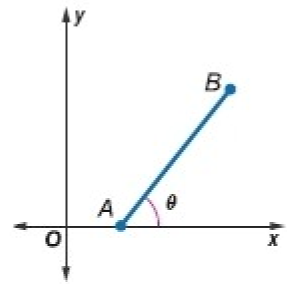
$$\tan A = \frac{4}{5}$$
$$A = \tan^{-1}\left(\frac{4}{5}\right) \approx 38.7$$

$$A + B + C = 180°$$
$$38.7° + B + 90° \approx 180°$$
$$B \approx 51.3°$$

$$\tan A = \frac{a}{b}$$
$$\tan 38.7 \approx \frac{6}{b}$$
$$b \approx 7.5$$

$$\sin 38.7° = \frac{6}{c}$$
$$c \approx 9.6$$

54. **CHALLENGE** A line segment has endpoints $A(2, 0)$ and $B(6, 5)$, as shown in the figure. What is the measure of the acute angle θ formed by the line segment and the x-axis? Explain how you found the measure.



SOLUTION:

About 51.3°; if a right triangle is drawn with $\overline{AB}$ as the hypotenuse, then the side opposite angle θ is 5 and the side adjacent angle θ is 4. $\tan A = \frac{5}{4}$, so $A \approx$ 51.3°.

55. **CCSS ARGUMENTS** Determine whether the following statement is true or false. Explain your reasoning.

For any acute angle, the sine function will never have a negative value.

SOLUTION:

True. $\sin\theta = \frac{\text{opp}}{\text{hyp}}$ and the values of the opposite side and the hypotenuse of an acute triangle are positive, so the value of the sine function is positive.

56. **OPEN ENDED** In right triangle ABC, sin A = sin C. What can you conclude about ΔABC? Justify your reasoning.

SOLUTION:

$\sin A = \sin C$

So:

$$\frac{\text{sideopp } A}{\text{hyp}} = \frac{\text{sideopp } C}{\text{hyp}}$$

Therefore:

$\text{sideopp } A = \text{sideopp } C$

Since the two sides have the same measure, the triangle is isosceles.

57. **WRITING IN MATH** A roof has a slope of $\frac{2}{3}$. Describe the connection between the slope and the angle of elevation θ that the roof makes with the horizontal. Then use an inverse trigonometric function to find θ.

SOLUTION:

Sample answer: The slope describes the ratio of the vertical rise to the horizontal run of the roof. The vertical rise is opposite the angle that the roof makes with the horizontal. The horizontal run is the adjacent side. So, the tangent of the angle of elevation equals the ratio of the rise to the run, or the slope of the roof; $\theta \approx 33.7°$.

58. **EXTENDED RESPONSE** Your school needs 5 cases of yearbooks. Neighborhood Yearbooks lists a case of yearbooks at $153.85 with a 10% discount on an order of 5 cases. Yearbooks R Us lists a case of yearbooks at $157.36 with a 15% discount on 5 cases.

a. Which company would you choose?

b. What is the least amount that you would have to spend for the yearbooks?

SOLUTION:

a. Calculate the price of 5 cases of Neighborhood Yearbooks.

$5 \times \$153.85 = \$769.25$

After 10% discount:

$10\%(\$769.25) = \$76.93$

So:

$\$769.25 - \$76.93 = \$692.32$

Calculate the price of 5 cases of Yearbooks R Us yearbooks.

$5 \times \$157.36 = \$786.8$

After 15% discount:

$15\%(\$786.8) = \$118.02$

So:

$\$786.8 - \$118.02 = \$668.78$

Since $668.78 < $786.8, it will be profitable to buy books from Yearbooks R Us.

b. The least amount is $668.78.

59. **SHORT RESPONSE** As a fundraiser, the marching band sold T-shirts and hats. They sold a total of 105 items and raised $1170. If the cost of a hat was $10 and the cost of a T-shirt was $15, how many T-shirts were sold?

SOLUTION:

Let x be the number of T-shirts sold. Total number of items sold is 105.

Therefore, number of hats sold is $105 - x$.

So:

$(105 - x)10 + 15x = 1170$

Solve for x:

$5x = 120$

$x = 24$

Number of T-shirts sold is 24.

60. A hot dog stand charges price x for a hot dog and price y for a drink. Two hot dogs and one drink cost $4.50. Three hot dogs and two drinks cost $7.25.

Which matrix could be multiplied by $\begin{bmatrix} 4.50 \\ 7.25 \end{bmatrix}$ to find x and y?

A $\begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix}$

B $\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$

C $\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

D $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

SOLUTION:

The system of equations represents the given situation:

$2x + y = 4.50$

$3x + 2y = 7.25$

Rewrite the system of equations.

$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4.50 \\ 7.25 \end{bmatrix}$.

To find the values of x and y, multiply the equation by inverse of matrix $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$.

Inverse matrix of $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ is $\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$.

Option B is the correct answer.

61. **SAT/ACT** The length and width of a rectangle are in the ratio of 5:12. If the rectangle has an area of 240 square centimeters, what is the length, in centimeters, of its diagonal?

F 24

G 26

H 28

J 30

K 32

SOLUTION:
Let *x* be apart.

Therefore, the length and the width of the rectangle are 5*x* and 12*x*.

So:

$$5x(12x) = 240$$
$$60x^2 = 240$$
$$x = 2$$

Therefore, the length and the width of the rectangle are 10 cm and 24 cm.

The diagonal of the rectangle is $\sqrt{24^2 + 10^2}$ or 26 cm.

Option G is the correct answer.

Identify the null and alternative hypotheses for each statement. Then identify the statement that represents the claim.

62. Jack thinks that it takes less than 10 minutes to ride his bike from his home to the store.

SOLUTION:
less than 10 minutes: $\mu < 10$,
not less than 10 minutes: $\mu \geq 10$

The claim is $\mu < 10$, and it is the alternative hypothesis because it does not include equality. The null hypothesis is $\mu \geq 10$, which is the complement of $\mu < 10$.

H_0: $\mu \geq 10$
H_a: $\mu < 10$ (claim)

63. A deli sign says that one 12-inch turkey sandwich contains three ounces of meat.

SOLUTION:
contains 3 ounces: $\mu = 3$
does not contain 3 ounces: $\mu \neq 3$

The claim is $\mu = 3$, and it is the null hypothesis because it does include equality. The alternative hypothesis is $\mu \neq 3$, which is the complement of $\mu = 3$.

H_0: $\mu = 3$ (claim)
H_a: $\mu \neq 3$

64. Mrs. Thomas takes at least 15 minutes to prepare a cake.

SOLUTION:
at least 15 minutes: $\mu \geq 15$
less than 15 minutes: $\mu < 15$

The claim is $\mu \geq 15$, and it is the null hypothesis because it does include equality. The alternative hypothesis is $\mu < 15$, which is the complement of $\mu \geq 15$.

$H_0: \mu \geq 15$ (claim)
$H_a: \mu < 15$

65. **SWIMMING POOL** The number of visits to a community swimming pool per year by a sample of 425 members is normally distributed with a mean of 90 and a standard deviation of 15.

a. About what percent of the members went to the pool at least 45 times?

b. What is the probability that a member selected at random went to the pool more than 120 times?

c. What percent of the members went to the pool between 75 and 105 times?

SOLUTION:

66. **POLLS** A polling company wants to estimate how many people are in favor of a new environmental law. The polling company polls 20 people. The probability that a person is in favor of the law is 0.5.

a. What is the probability that exactly 12 people are in favor of the new law?

b. What is the expected number of people in favor of the law?

SOLUTION:
a. Use the binomial distribution.
The probability of x successes in n independent trials is given by $P(x) = C(n, x)\, s^x f^{n-x}$
Here, s = 0.5. So, f = 0.5.

$$P(x = 12) = C(20, 12)\,(0.5)^{12}\,(0.5)^8 \approx 0.12$$

b. The probability that a person is in favor of the law is 0.5.
So, out of 20 people, there will be 20 (0.5) or 10 people in favor of the law.

Find each product. Include the appropriate units with your answer.

67. $4.3\text{miles}\left(\frac{5280\text{ feet}}{1\text{mile}}\right)$

SOLUTION:
1 mile = 5280 feet

Therefore:

$$4.3 \times 5280\left(\frac{5280}{1 \times 5280}\right)\text{ ft} = 22,704\text{ ft}$$

68. $8\text{gallons}\left(\dfrac{8\text{pints}}{1\text{gallon}}\right)$

SOLUTION:
1 gallon = 8 pints

Therefore:
$8\times 8\left(\dfrac{8}{8}\right)\text{pt} = 64\text{ pt}$

69. $\left(\dfrac{5\text{dollars}}{3\text{meters}}\right)21\text{meters}$

SOLUTION:
$$\left(\frac{5\text{dollars}}{3\text{meters}}\right)21\text{meters} = \frac{5\text{dollars}}{3\text{ meters}}\cdot 21\text{ meters} = 35\text{ dollars}$$

70. $\left(\dfrac{18\text{cubic inches}}{5\text{seconds}}\right)24\text{seconds}$

SOLUTION:
$$\left(\frac{18\text{cubic inches}}{5\text{seconds}}\right)24\text{seconds} = \frac{18\text{cubic inches}}{5\text{ seconds}}\cdot 24\text{ seconds} = 86.4\text{in}^3$$

71. $65\text{degrees}\left(\dfrac{10\text{centimeters}}{3\text{degrees}}\right)$

SOLUTION:
$$65\text{degrees}\left(\frac{10\text{centimeters}}{3\text{degrees}}\right) = 65\text{ degrees}\cdot\frac{10\text{centimeters}}{3\text{ degrees}} = 216\frac{2}{3}\text{ cm}$$

72. $\left(\dfrac{7\text{ liters}}{30\text{minutes}}\right)10\text{minutes}$

SOLUTION:
$$\left(\frac{7\text{ liters}}{30\text{minutes}}\right)10\text{minutes} = \left(\frac{7\text{ liters}}{30\text{ minutes}}\right)10\text{ minutes} = 2\frac{1}{3}L$$