The terminal side of θ in standard position contains each point. Find the exact values of the six trigonometric functions of θ.

1. (1, 2)

SOLUTION:
Find the value of *r*.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{1^2 + 2^2} \\ &= \sqrt{5} \end{aligned}$$

Use $x = 1, y = 2$, and $r = \sqrt{5}$ to write the six trigonometric ratios.

$$\sin\theta = \frac{y}{r} = \frac{2}{\sqrt{5}} \text{ or } \frac{2\sqrt{5}}{5}$$
$$\cos\theta = \frac{x}{r} = \frac{1}{\sqrt{5}} \text{ or } \frac{\sqrt{5}}{5}$$
$$\tan\theta = \frac{y}{x} = \frac{2}{1}$$
$$\csc\theta = \frac{r}{y} = \frac{\sqrt{5}}{2}$$
$$\sec\theta = \frac{r}{x} = \sqrt{5}$$
$$\cot\theta = \frac{x}{y} = \frac{1}{2}$$

2. (−8, −15)

SOLUTION:
Find the value of *r*.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{(-8)^2 + (-15)^2} \\ &= \sqrt{289} \\ &= 17 \end{aligned}$$

Use $x = –8, y = –15$, and $r = 17$ to write the six trigonometric ratios.

$$\sin\theta = \frac{y}{r} = -\frac{15}{17}$$
$$\cos\theta = \frac{x}{r} = -\frac{8}{17}$$
$$\tan\theta = \frac{y}{x} = \frac{15}{8}$$
$$\csc\theta = \frac{r}{y} = -\frac{17}{15}$$
$$\sec\theta = \frac{r}{x} = -\frac{17}{8}$$
$$\cot\theta = \frac{x}{y} = \frac{8}{15}$$

3. (0, −4)

SOLUTION:
Find the value of *r*.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{0^2 + (-4)^2} \\ &= \sqrt{16} \\ &= 4 \end{aligned}$$

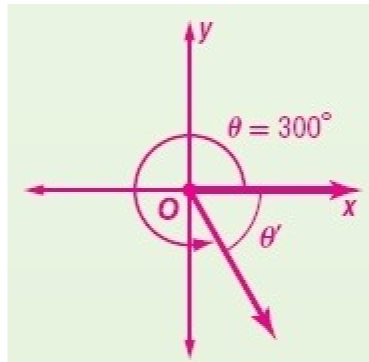
Use $x = 0$, $y = -4$, and $r = 4$ to write the six trigonometric ratios.

$$\sin\theta = \frac{y}{r} = -\frac{4}{4} \text{ or } -1$$
$$\cos\theta = \frac{x}{r} = \frac{0}{4} \text{ or } 0$$
$$\tan\theta = \frac{y}{x} = \frac{-4}{0} \text{ undefined}$$
$$\csc\theta = \frac{r}{y} = -\frac{4}{4} \text{ or } -1$$
$$\sec\theta = \frac{r}{x} = \frac{4}{0} \text{ undefined}$$
$$\cot\theta = \frac{x}{y} = \frac{0}{4} \text{ or } 0$$

Sketch each angle. Then find its reference angle.

4. 300°

SOLUTION:

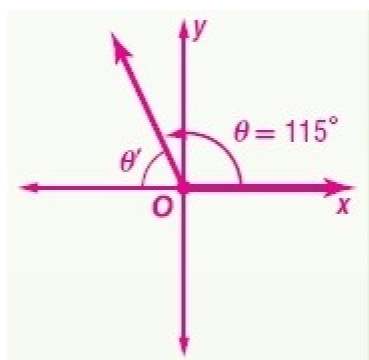


The terminal side of 300° lies in Quadrant IV.

$$\begin{aligned} \theta' &= 360^\circ - \theta \\ &= 360^\circ - 300^\circ \\ &= 60^\circ \end{aligned}$$

5. 115°

SOLUTION:



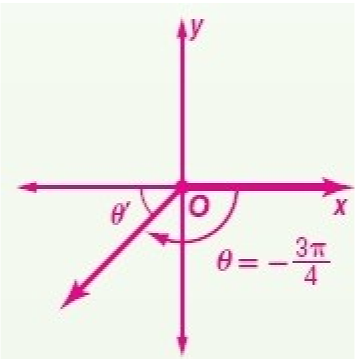
The terminal side of 115° lies in Quadrant II.

$$\begin{aligned} \theta' &= 180^\circ - \theta \\ &= 180^\circ - 115^\circ \\ &= 65^\circ \end{aligned}$$

6. $-\frac{3\pi}{4}$

SOLUTION:

coterminal angle: $-\frac{3\pi}{4}+2\pi=\frac{5\pi}{4}$



The terminal side of $\frac{5\pi}{4}$ lies in Quadrant III.

$$\begin{aligned}\theta' &= \theta-\pi\\ &=\frac{5\pi}{4}-\pi\\ &=\frac{\pi}{4}\end{aligned}$$

Find the exact value of each trigonometric function.

7. $\sin\frac{3\pi}{4}$

SOLUTION:

The terminal side of $\frac{3\pi}{4}$ lies in Quadrant II.

$$\begin{aligned}\theta' &= \pi-\theta\\ &=\pi-\frac{3\pi}{4}\\ &=\frac{\pi}{4}\end{aligned}$$

The sine function is positive in Quadrant II.

$$\begin{aligned}\sin\frac{3\pi}{4} &= \sin\frac{\pi}{4}\\ &=\sin 45^\circ\\ &=\frac{\sqrt{2}}{2}\end{aligned}$$

8. $\tan\frac{5\pi}{3}$

SOLUTION:

The terminal side of $\frac{5\pi}{3}$ lies in Quadrant IV.

$$\begin{aligned}\theta' &= 2\pi - \theta\\ &= 2\pi - \frac{5\pi}{3}\\ &= \frac{\pi}{3}\end{aligned}$$

The tangent function is negative in Quadrant IV.

$$\begin{aligned}\tan\frac{5\pi}{3} &= -\tan\frac{\pi}{3}\\ &= -\tan 60^\circ\\ &= -\sqrt{3}\end{aligned}$$

9. sec 120°

SOLUTION:

The terminal side of 120° lies in Quadrant II.

$$\begin{aligned}\theta' &= 180^\circ - \theta\\ &= 180^\circ - 120^\circ\\ &= 60^\circ\end{aligned}$$

The secant function is negative in Quadrant II.

$$\begin{aligned}\sec 120^\circ &= -\sec 60^\circ\\ &= -2\end{aligned}$$

10. sin 300°

SOLUTION:

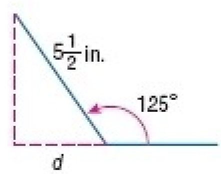
The terminal side of 300° lies in Quadrant IV.

$$\begin{aligned}\theta' &= 360^\circ - \theta\\ &= 360^\circ - 300^\circ\\ &= 60^\circ\end{aligned}$$

The sine function is negative in Quadrant IV.

$$\begin{aligned}\sin 300^\circ &= -\sin 60^\circ\\ &= -\frac{\sqrt{3}}{2}\end{aligned}$$

11. **ENTERTAINMENT** Alejandra opens her portable DVD player so that it forms a 125° angle. The screen is $5\frac{1}{2}$ inches long.



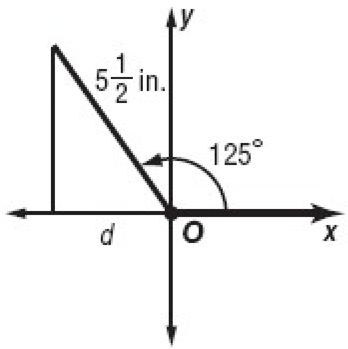
a. Redraw the diagram so that the angle is in standard position on the coordinate plane.

b. Find the reference angle. Then write a trigonometric function that can be used to find the distance to the wall *d* that she can place the DVD player.

c. Use the function to find the distance. Round to the nearest tenth.

SOLUTION:

a.



b. Reference angle: 180° – 125° = 55°
The trigonometric function that can be used to find the distance to the wall is $\cos 55^\circ = \frac{d}{5\frac{1}{2}}$.

c.

$$\cos 55^\circ = \frac{d}{5\frac{1}{2}}$$
$$5\frac{1}{2}\cos 55^\circ = d$$
$$d = 3.2 \text{ in}$$

The terminal side of *θ* in standard position contains each point. Find the exact values of the six trigonometric functions of *θ*.

12. (5, 12)

SOLUTION:
Find the value of *r*.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{5^2 + 12^2} \\ &= \sqrt{169} \\ &= 13 \end{aligned}$$

Use $x = 5$, $y = 12$, and $r = 13$ to write the six trigonometric ratios.

$$\sin\theta = \frac{y}{r} = \frac{12}{13}$$
$$\cos\theta = \frac{x}{r} = \frac{5}{13}$$
$$\tan\theta = \frac{y}{x} = \frac{12}{5}$$
$$\csc\theta = \frac{r}{y} = \frac{13}{12}$$
$$\sec\theta = \frac{r}{x} = \frac{13}{5}$$
$$\cot\theta = \frac{x}{y} = \frac{5}{12}$$

13. (−6, 8)

SOLUTION:
Find the value of *r*.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{(-6)^2 + 8^2} \\ &= \sqrt{100} \\ &= 10 \end{aligned}$$

Use $x = -6$, $y = 8$, and $r = 10$ to write the six trigonometric ratios.

$$\sin\theta = \frac{y}{r} = \frac{4}{5}$$
$$\cos\theta = \frac{x}{r} = -\frac{3}{5}$$
$$\tan\theta = \frac{y}{x} = -\frac{4}{3}$$
$$\csc\theta = \frac{r}{y} = \frac{5}{4}$$
$$\sec\theta = \frac{r}{x} = -\frac{5}{3}$$
$$\cot\theta = \frac{x}{y} = -\frac{3}{4}$$

14. (3, 0)

SOLUTION:
Find the value of *r*.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{3^2 + 0^2} \\ &= \sqrt{9} \\ &= 3 \end{aligned}$$

Use $x = 3$, $y = 0$, and $r = 3$ to write the six trigonometric ratios.

$$\sin\theta = \frac{y}{r} = 0$$
$$\cos\theta = \frac{x}{r} = 1$$
$$\tan\theta = \frac{y}{x} = 0$$
$$\csc\theta = \frac{r}{y} = \cancel{\frac{3}{0}}\ \text{undefined}$$
$$\sec\theta = \frac{r}{x} = 1$$
$$\cot\theta = \frac{x}{y} = \cancel{\frac{3}{0}}\ \text{undefined}$$

15. (0, −7)

SOLUTION:
Find the value of r.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{0^2 + (-7)^2} \\ &= \sqrt{49} \\ &= 7 \end{aligned}$$

Use $x = 0, y = –7$, and $r = 7$ to write the six trigonometric ratios.

$$\sin\theta = \frac{y}{r} = -1$$

$$\cos\theta = \frac{x}{r} = 0$$

$$\tan\theta = \frac{y}{x} = \cancel{\frac{-7}{0}} \text{ undefined}$$

$$\csc\theta = \frac{r}{y} = -1$$

$$\sec\theta = \frac{r}{x} = \cancel{\frac{7}{0}} \text{ undefined}$$

$$\cot\theta = \frac{x}{y} = 0$$

16. (4, −2)

SOLUTION:
Find the value of r.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{4^2 + (-2)^2} \\ &= \sqrt{20} \\ &= 2\sqrt{5} \end{aligned}$$

Use $x = 4, y = –2$, and $r = 2\sqrt{5}$ to write the six trigonometric ratios.

$$\sin\theta = \frac{y}{r} = \frac{-2}{2\sqrt{5}} = -\frac{\sqrt{5}}{5}$$

$$\cos\theta = \frac{x}{r} = \frac{4}{2\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\tan\theta = \frac{y}{x} = -\frac{1}{2}$$

$$\csc\theta = \frac{r}{y} = -\sqrt{5}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{5}}{2}$$

$$\cot\theta = \frac{x}{y} = -2$$

17. (−9, −3)

SOLUTION:
Find the value of r.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{(-9)^2 + (-3)^2} \\ &= \sqrt{90} \\ &= 3\sqrt{10} \end{aligned}$$

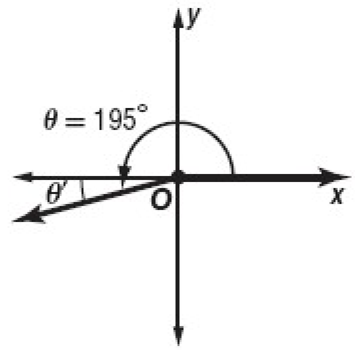
Use $x = -9$, $y = -3$, and $r = 3\sqrt{10}$ to write the six trigonometric ratios.

$$\sin\theta = \frac{y}{r} = \frac{-3}{3\sqrt{10}} = -\frac{\sqrt{10}}{10}$$
$$\cos\theta = \frac{x}{r} = \frac{-9}{3\sqrt{10}} = -\frac{3\sqrt{10}}{10}$$
$$\tan\theta = \frac{y}{x} = \frac{1}{3}$$
$$\csc\theta = \frac{r}{y} = -\sqrt{10}$$
$$\sec\theta = \frac{r}{x} = -\frac{\sqrt{10}}{3}$$
$$\cot\theta = \frac{x}{y} = 3$$

Sketch each angle. Then find its reference angle.

18. 195°

SOLUTION:

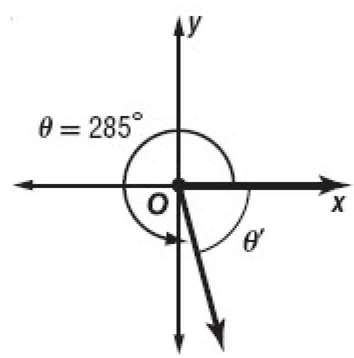


The terminal side of 195° lies in Quadrant III.

$$\begin{aligned} \theta' &= \theta - 180^\circ \\ &= 195^\circ - 180^\circ \\ &= 15^\circ \end{aligned}$$

19. 285°

SOLUTION:



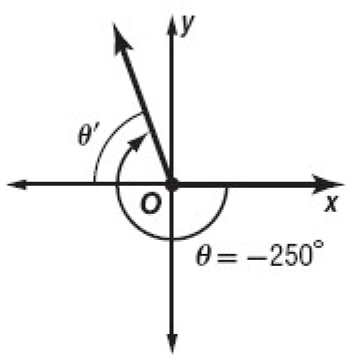
The terminal side of 285° lies in Quadrant IV.

$$\begin{aligned} \theta' &= 360^\circ - \theta \\ &= 360^\circ - 285^\circ \\ &= 75^\circ \end{aligned}$$

20. −250°

SOLUTION:

coterminal angle: $-250° + 360° = 110°$

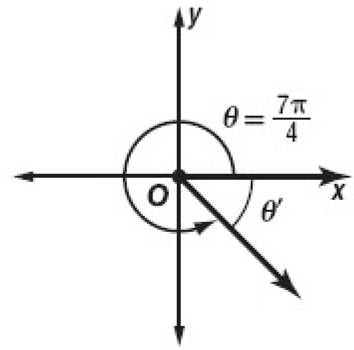


The terminal side of 110° lies in Quadrant II.

$$\begin{aligned}\theta' &= 180° - \theta \\ &= 180° - 110° \\ &= 70°\end{aligned}$$

21. $\frac{7\pi}{4}$

SOLUTION:



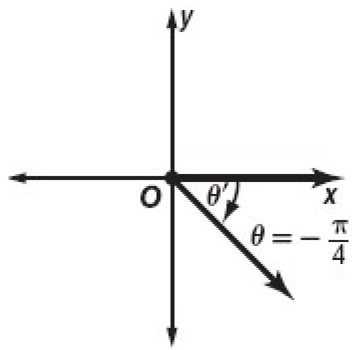
The terminal side $\frac{7\pi}{4}$ lies in Quadrant IV.

$$\begin{aligned}\theta' &= 2\pi - \theta \\ &= 2\pi - \frac{7\pi}{4} \\ &= \frac{\pi}{4}\end{aligned}$$

22. $-\frac{\pi}{4}$

SOLUTION:

coterminal angle: $-\frac{\pi}{4}+2\pi=\frac{7\pi}{4}$



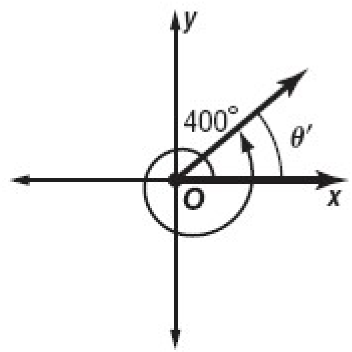
The terminal side $\frac{7\pi}{4}$ lies in Quadrant IV.

$$\theta' = 2\pi - \theta$$
$$= 2\pi - \frac{7\pi}{4}$$
$$= \frac{\pi}{4}$$

23. 400°

SOLUTION:

coterminal angle: $400^\circ - 360^\circ = 40^\circ$



The terminal side of 40° lies in Quadrant I.

$$\theta' = \theta$$
$$= 40^\circ$$

Find the exact value of each trigonometric function.

24. sin 210°

SOLUTION:

The terminal side of 210º lies in Quadrant III.

$$\theta' = \theta - 180^\circ$$
$$= 210^\circ - 180^\circ$$
$$= 30^\circ$$

The sine function is negative in Quadrant III.

$$\sin 210^\circ = -\sin 30^\circ$$
$$= -\frac{1}{2}$$

25. tan 315°

SOLUTION:
The terminal side of 315º lies in Quadrant IV.

$$\begin{aligned}\theta' &= 360^\circ - \theta \\ &= 360^\circ - 315^\circ \\ &= 45^\circ\end{aligned}$$

The tangent function is negative in Quadrant IV.

$$\begin{aligned}\tan 315^\circ &= -\tan 45^\circ \\ &= -1\end{aligned}$$

26. cos 150°

SOLUTION:
The terminal side of 150º lies in Quadrant II.

$$\begin{aligned}\theta' &= 180^\circ - \theta \\ &= 180^\circ - 150^\circ \\ &= 30^\circ\end{aligned}$$

The cosine function is negative in Quadrant II.

$$\begin{aligned}\cos 150^\circ &= -\cos 30^\circ \\ &= -\frac{\sqrt{3}}{2}\end{aligned}$$

27. csc 225°

SOLUTION:
The terminal side of 225º lies in Quadrant III.

$$\begin{aligned}\theta' &= \theta - 180^\circ \\ &= 225^\circ - 180^\circ \\ &= 45^\circ\end{aligned}$$

The cosecant function is negative in Quadrant III.

$$\begin{aligned}\csc 225^\circ &= -\csc 45^\circ \\ &= -\sqrt{2}\end{aligned}$$

28. $\sin \frac{4\pi}{3}$

SOLUTION:
The terminal side $\frac{4\pi}{3}$ lies in Quadrant III.

$$\begin{aligned}\theta' &= \theta - \pi \\ &= \frac{4\pi}{3} - \pi \\ &= \frac{\pi}{3}\end{aligned}$$

The sine function is negative in Quadrant III.

$$\begin{aligned}\sin \frac{4\pi}{3} &= -\sin \frac{\pi}{3} \\ &= -\sin 60^\circ \\ &= -\frac{\sqrt{3}}{2}\end{aligned}$$

29. $\cos\frac{5\pi}{3}$

SOLUTION:

The terminal side $\frac{5\pi}{3}$ lies in Quadrant IV.

$$\begin{aligned}\theta' &= 2\pi - \theta\\ &= 2\pi - \frac{5\pi}{3}\\ &= \frac{\pi}{3}\end{aligned}$$

The cosine function is positive in Quadrant IV.

$$\begin{aligned}\cos\frac{5\pi}{3} &= \cos\frac{\pi}{3}\\ &= \cos 60^\circ\\ &= \frac{1}{2}\end{aligned}$$

30. $\cot\frac{5\pi}{4}$

SOLUTION:

The terminal side $\frac{5\pi}{4}$ lies in Quadrant III.

$$\begin{aligned}\theta' &= \theta - \pi\\ &= \frac{5\pi}{4} - \pi\\ &= \frac{\pi}{4}\end{aligned}$$

The cotangent function is positive in Quadrant III.

$$\begin{aligned}\cot\frac{5\pi}{4} &= \cot\frac{\pi}{4}\\ &= \cot 45^\circ\\ &= 1\end{aligned}$$

31. $\sec\frac{11\pi}{6}$

SOLUTION:

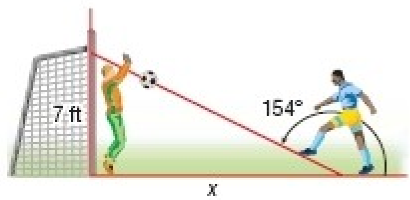
The terminal side $\frac{11\pi}{6}$ lies in Quadrant IV.

$$\begin{aligned}\theta' &= 2\pi - \theta\\ &= 2\pi - \frac{11\pi}{6}\\ &= \frac{\pi}{6}\end{aligned}$$

The secant function is positive in Quadrant IV.

$$\begin{aligned}\sec\frac{11\pi}{6} &= \sec\frac{\pi}{6}\\ &= \sec 30^\circ\\ &= \frac{2\sqrt{3}}{3}\end{aligned}$$

32. **CCSS REASONING** A soccer player x feet from the goalie kicks the ball toward the goal, as shown in the figure. The goalie jumps up and catches the ball 7 feet in the air.



a. Find the reference angle. Then write a trigonometric function that can be used to find how far from the goalie the soccer player was when he kicked the ball.

b. About how far away from the goalie was the soccer player?

SOLUTION:

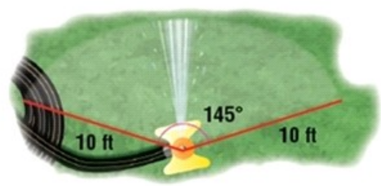
a. Reference angle: $180^\circ - 154^\circ = 26^\circ$

The trigonometric function $\tan 26^\circ = \frac{7}{x}$ can be used to find the distance from the goalie was the soccer player.

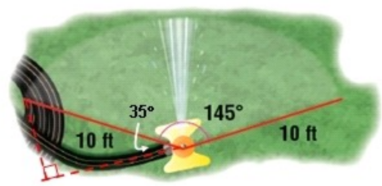
b.

$$\tan 26^\circ = \frac{7}{x}$$
$$x = \frac{7}{\tan 26^\circ}$$
$$x \approx 14.4 \text{ ft}$$

33. **SPRINKLER** A sprinkler rotating back and forth shoots water out a distance of 10 feet. From the horizontal position, it rotates 145° before reversing its direction. At a 145° angle, about how far to the left of the sprinkler does the water reach?



SOLUTION:



$$\cos 35^\circ = \frac{d}{10}$$
$$10 \cdot \cos 35^\circ = d$$
$$8.2 \approx d$$

The water reaches about 8.2 feet to the left of the sprinkler.

34. **BASKETBALL** The formula $R = \frac{v_0^2 \sin 2\theta}{32}$ gives the distance of a basketball shot with an initial velocity of v_0 feet per second at an angle θ with the ground.

a. If the basketball was shot with an initial velocity of 24 feet per second at an angle of 75°, how far will the basketball travel?

b. If the basketball was shot at an angle of 65° and traveled 10 feet, what was its initial velocity?

c. If the basketball was shot with an initial velocity of 30 feet per second and traveled 12 feet, at what angle was it shot?

SOLUTION:

a. Substitute 24 for v_0 and 75º for θ in the given formula and simplify.

$$R = \frac{v_0^2 \sin 2\theta}{32}$$
$$= \frac{24^2 \sin 2(75°)}{32}$$
$$= \frac{576 \sin 150°}{32}$$
$$= 9 \text{ ft}$$

So, the basketball will travel 9 feet.

b. Substitute 65º for θ and 10 for R in the given formula and solve for v_0.

$$R = \frac{v_0^2 \sin 2\theta}{32}$$
$$10 = \frac{v_0^2 \sin 2(65°)}{32}$$
$$v_0^2 = \frac{320}{\sin 130°}$$
$$v_0 = \sqrt{\frac{320}{\sin 130°}}$$
$$\approx 20.4 \text{ft / sec}$$

Thus, the initial velocity of the basketball is about 20.4 feet per second.

c. Substitute 30 for v_0 and 12 for R in the given formula and solve for θ.

$$R = \frac{v_0^2 \sin 2\theta}{32}$$
$$12 = \frac{30^2 \sin 2\theta}{32}$$
$$\frac{384}{900} = \sin 2\theta$$
$$2\theta = \sin^{-1}\left(\frac{384}{900}\right)$$
$$2\theta \approx 25.3$$
$$\theta = 12.6°$$

So, the basketball was shot at an angle of about 12.6º.

35. **PHYSICS** A rock is shot off the edge of a ravine with a slingshot at an angle of 65° and with an initial velocity of 6 meters per second. The equation that represents the horizontal distance of the rock x is $x = v_0 (\cos \theta)t$, where v_0 is the initial velocity, θ is the angle at which it is shot, and t is the time in seconds. About how far does the rock travel after 4 seconds?

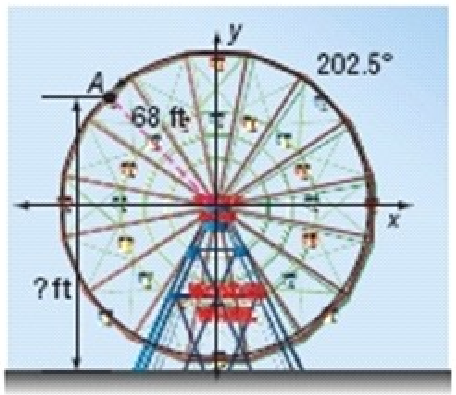
SOLUTION:

Substitute $V_0 = 6$, $\theta = 65º$ and $t = 4$ in the given equation and solve for x.

$$x = V_0(\cos\theta)t$$
$$= 6(\cos 65)4$$
$$\approx 10.1 \text{meters}$$

The rock travels about 10.1 m after 4 seconds.

36. **FERRIS WHEELS** The Wonder Wheel Ferris wheel at Coney Island has a radius of about 68 feet and is 15 feet off the ground. After a person gets on the bottom car, the Ferris wheel rotates 202.5° counterclockwise before stopping. How high above the ground is this car when it has stopped?



SOLUTION:

Since the angle measured from the negative y-axis, the terminal angle is 202.5º – 90º or 112.5º.

Therefore, the reference angle is(θ') 180º – 112.5º or 67.5º.

Substitute r = 68 and θ = 67.5º in the sine ratio.

$$\sin\theta = \frac{y}{r}$$
$$\sin 67.5^\circ = \frac{y}{68}$$
$$y = 68\sin 67.5^\circ$$
$$y \approx 62.8$$

The height above the ground to the car is 62.8 + 68 + 15 or 145.8 feet.

Suppose θ is an angle in standard position whose terminal side is in the given quadrant. For each function, find the exact values of the remaining five trigonometric functions of θ.

37. $\sin\theta = \frac{4}{5}$, Quadrant II

SOLUTION:

Opposite side = 4
Hypotenuse = 5

Use the Pythagorean Theorem to find the adjacent angle.

$$\text{Adjacent side} = \sqrt{5^2 - 4^2} = \sqrt{9} = 3$$

Only sine function is positive in Quadrant II
.

$$\cos\theta = -\frac{3}{5}$$
$$\tan\theta = -\frac{4}{3}$$
$$\csc\theta = \frac{5}{4}$$
$$\sec\theta = -\frac{5}{3}$$
$$\cot\theta = -\frac{3}{4}$$

38. $\tan\theta = -\frac{2}{3}$, Quadrant IV

SOLUTION:

Opposite side = 2
Adjacent side = 3

Use the Pythagorean Theorem to find the hypotenuse.

$$\begin{aligned}\text{Hypotenuse} &= \sqrt{2^2 + 3^2}\\ &= \sqrt{13}\end{aligned}$$

Only cosine function is positive in Quadrant IV.

$$\begin{aligned}\sin\theta &= -\frac{2}{\sqrt{13}} \text{ or } -\frac{2\sqrt{13}}{13}\\ \cos\theta &= \frac{3}{\sqrt{13}} \text{ or } \frac{3\sqrt{13}}{13}\\ \csc\theta &= -\frac{\sqrt{13}}{2}\\ \sec\theta &= \frac{\sqrt{13}}{3}\\ \cot\theta &= -\frac{3}{2}\end{aligned}$$

39. $\cos\theta = -\frac{8}{17}$, Quadrant III

SOLUTION:

Adjacent side = 8
Hypotenuse = 17

Use the Pythagorean Theorem to find the opposite side.

$$\begin{aligned}\text{Opposite side} &= \sqrt{17^2 - 8^2}\\ &= \sqrt{225}\\ &= 15\end{aligned}$$

Only tangent function is positive in Quadrant III.

$$\begin{aligned}\sin\theta &= -\frac{15}{17}\\ \tan\theta &= \frac{15}{8}\\ \csc\theta &= -\frac{17}{15}\\ \sec\theta &= -\frac{17}{8}\\ \cot\theta &= \frac{8}{15}\end{aligned}$$

40. $\cot\theta = -\frac{12}{5}$, Quadrant IV

SOLUTION:
Opposite side = 5
Adjacent side = 12

Use the Pythagorean Theorem to find the hypotenuse.

$$\begin{aligned}\text{Hypotenuse} &= \sqrt{5^2 + 12^2} \\ &= \sqrt{169} \\ &= 13\end{aligned}$$

Only cosine function is positive in Quadrant IV.

$$\begin{aligned}\sin\theta &= -\frac{5}{13} \\ \cos\theta &= \frac{12}{13} \\ \tan\theta &= -\frac{5}{12} \\ \csc\theta &= -\frac{13}{5} \\ \sec\theta &= \frac{13}{12}\end{aligned}$$

Find the exact value of each trigonometric function.

41. cot 270°

SOLUTION:
Since the angle 270º is a quadrant angle, the coordinates of the point on its terminal side is (0, –*y*).

$$\begin{aligned}\cot 270^\circ &= \frac{x}{y} \\ &= \frac{0}{-y} \\ &= 0\end{aligned}$$

42. csc 180°

SOLUTION:
Since the angle 180º is a quadrant angle, the coordinates of the point on its terminal side is (–*x*, 0).

Find the value of *r*.

$$\begin{aligned}r &= \sqrt{a^2 + b^2} \\ &= \sqrt{(-x)^2 + 0^2} \\ r &= x\end{aligned}$$

$$\begin{aligned}\csc 180^\circ &= \frac{r}{y} \\ &= \frac{y}{0} \text{ undefined}\end{aligned}$$

43. sin 570°

SOLUTION:
The coterminal angle of 570º is 570º – 360º or 210º.

The terminal side of 210° lies in Quadrant III.

$$\begin{aligned}\theta' &= \theta - 180^\circ \\ &= 210^\circ - 180^\circ \\ &= 30^\circ\end{aligned}$$

The sine function is negative in Quadrant III.

$$\begin{aligned}\sin 570^\circ &= -\sin 30^\circ \\ &= -\frac{1}{2}\end{aligned}$$

44. $\tan\left(-\frac{7\pi}{6}\right)$

SOLUTION:

The coterminal angle of $\frac{5\pi}{6}$ is $-\frac{7\pi}{6}+2\pi=\frac{5\pi}{6}$.

The terminal side of $\frac{5\pi}{6}$ lies in Quadrant II.

$$\begin{aligned}\theta' &= \pi-\theta\\ &= \pi-\frac{5\pi}{6}\\ &= \frac{\pi}{6}\end{aligned}$$

The tangent function is negative in Quadrant II.

$$\begin{aligned}\tan\left(-\frac{7\pi}{6}\right) &= -\tan\frac{\pi}{6}\\ &= -\tan 30^\circ\\ &= -\frac{\sqrt{3}}{3}\end{aligned}$$

45. $\cos\left(-\frac{11\pi}{6}\right)$

SOLUTION:

The coterminal angle of $-\frac{11\pi}{6}$ is $-\frac{11\pi}{6}+2\pi=\frac{\pi}{6}$.

The terminal side of $\frac{\pi}{6}$ lies in Quadrant I.

$$\begin{aligned}\theta' &= \theta\\ &= \frac{\pi}{6}\end{aligned}$$

$$\begin{aligned}\cos\left(-\frac{11\pi}{6}\right) &= \cos\frac{\pi}{6}\\ &= \cos 30^\circ\\ &= \frac{\sqrt{3}}{2}\end{aligned}$$

46. $\cot\frac{9\pi}{4}$

SOLUTION:

The coterminal angle of $\frac{9\pi}{4}$ is $\frac{9\pi}{4}-2\pi=\frac{\pi}{4}$.

The terminal side of $\frac{\pi}{4}$ lies in Quadrant I.

$$\begin{aligned}\theta' &= \theta\\ &= \frac{\pi}{4}\end{aligned}$$

$$\begin{aligned}\cot\frac{9\pi}{4} &= \cot\frac{\pi}{4}\\ &= \cot 45^\circ\\ &= 1\end{aligned}$$

47. **CHALLENGE** For an angle θ in standard position, $\sin\theta = \frac{\sqrt{2}}{2}$ and tan $\theta = -1$. Can the value of θ be 225°? Justify your reasoning.

SOLUTION:

No; for $\sin\theta = \frac{\sqrt{2}}{2}$ and tan $\theta = -1$, the reference angle is 45°. However, for sin θ to be positive and tan θ to be negative, the reference angle must be in the second quadrant. So, the value of θ must be 135° or an angle coterminal with 135°.

48. **CCSS ARGUMENTS** Determine whether 3 sin 60° = sin 180° is *true* or *false*. Explain your reasoning.

SOLUTION:

False;

$3\sin 60° = 3\cdot\frac{\sqrt{3}}{2}$ or $\frac{3\sqrt{3}}{2}$ and $\sin 180° = 0$

49. **REASONING** Use the sine and cosine functions to explain why cot 180° is undefined.

SOLUTION:

Sample answer: We know that $\cot\theta = \frac{x}{y}$, $\sin\theta = \frac{y}{r}$, and $\cos\theta = \frac{x}{r}$. Since $\sin 180° = 0$, it must be true that $y = 0$. Thus $\cot 180° = \frac{x}{0}$, which is undefined.

50. **OPEN ENDED** Give an example of a negative angle θ for which sin $\theta > 0$ and cos $\theta < 0$.

SOLUTION:

Sample answer:
$\theta = -200°$

51. **WRITING IN MATH** Describe the steps for evaluating a trigonometric function for an angle θ that is greater than 90°. Include a description of a reference angle.

SOLUTION:

First, sketch the angle and determine in which quadrant it is located. Then use the appropriate rule for finding its reference angle θ'. A reference angle is the acute angle formed by the terminal side of θ and the x-axis. Next, find the value of the trigonometric function for θ'. Finally, use the quadrant location to determine the sign of the trigonometric function value of θ.

52. **GRIDDED RESPONSE** If the sum of two numbers is 21 and their difference is 3, what is their product?

SOLUTION:

Let the two unknown numbers be x and y.
The system of equations that represent the situation are $x + y = 21$ and $x - y = 3$.

$$\begin{aligned} x + y &= 21 \\ \underline{x - y} &\underline{= 3} \\ 2x &= 24 \\ x &= 12 \end{aligned}$$

Substitute 12 for x in the first equation and solve for y.

$$\begin{aligned} x + y &= 21 \\ 12 + y &= 21 \\ y &= 21 - 12 \\ y &= 9 \end{aligned}$$

The product of 9 and 12 is 108.

53. **GEOMETRY** *D* is the midpoint of $\overline{BC}$, and *A* and *E* are the midpoints of $\overline{BD}$ and $\overline{DC}$, respectively. If the length of $\overline{AE}$ is 12, what is the length of $\overline{BC}$?

A 6

B 12

C 24

D 48

SOLUTION:

D is the midpoint of $\overline{BC}$, so $BD = DC$.

A and *E* are the midpoints of $\overline{BD}$ and $\overline{DC}$ respectively, so $BA = AD = DE = EC$. They are at equal distance. Thus, the length of $\overline{BC}$ is 4×6 or 24.

C is the correct option.

54. The expression $(-6 + i)^2$ is equivalent to which of the following expressions?

F $-12i$

G $36 - i$

H $36 - 12i$

J $35 - 12i$

SOLUTION:

$$\begin{aligned}(-6+i)^2 &= (-6)^2 + (2 \cdot -6 \cdot i) + i^2 \\ &= 36 - 12i - 1 \\ &= 35 - 12i\end{aligned}$$

J is the correct option.

55. **SAT/ACT** Of the following, which is least?

A $1 + \frac{1}{4}$

B $1 - \frac{1}{4}$

C $1 \div \frac{1}{4}$

D $1 \times \frac{1}{4}$

E $\frac{1}{4} - 1$

SOLUTION:

$$1 + \frac{1}{4} = 1\frac{1}{4}$$
$$1 - \frac{1}{4} = \frac{3}{4}$$
$$\frac{1}{4} - 1 = -\frac{3}{4}$$
$$1 \times \frac{1}{4} = \frac{1}{4}$$

$-\frac{3}{4}$ is the least number.

E is the correct option.

Rewrite each radian measure in degrees.

56. $\frac{4}{3}\pi$

SOLUTION:

$$\begin{aligned}\frac{4\pi}{3} &= \frac{4\pi}{3}\text{radians} \cdot \frac{180^\circ}{\pi\,\text{radians}} \\ &= \frac{720^\circ}{3} \\ &= 240^\circ\end{aligned}$$

57. $\frac{11}{6}\pi$

SOLUTION:

$$\frac{11\pi}{6} = \frac{11\pi}{6}\text{radians} \cdot \frac{180^\circ}{\pi\,\text{radians}}$$
$$= \frac{1980^\circ}{6}$$
$$= 330^\circ$$

58. $-\frac{17}{4}\pi$

SOLUTION:

$$-\frac{17\pi}{4} = \frac{-17\pi}{4}\text{radians} \cdot \frac{180^\circ}{\pi\,\text{radians}}$$
$$= -\frac{3060^\circ}{4}$$
$$= -765^\circ$$

Solve each equation.

59. $\cos a = \frac{13}{17}$

SOLUTION:

$$\cos a = \frac{13}{17}$$
$$\cos^{-1}\frac{13}{17} = a$$
$$a \approx 40.1^\circ$$

60. $\sin 30^\circ = \frac{b}{6}$

SOLUTION:

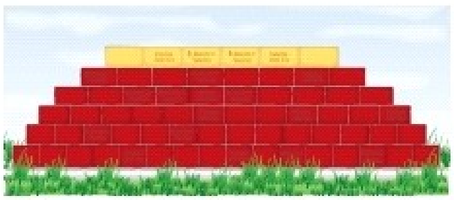
$$\sin 30^\circ = \frac{b}{6}$$
$$6\sin 30^\circ = b$$
$$b = 6 \times \frac{1}{2}$$
$$b = 3$$

61. $\tan c = \frac{9}{4}$

SOLUTION:

$$\tan c = \frac{9}{4}$$
$$\tan^{-1}\frac{9}{4} = c$$
$$c \approx 66.0^\circ$$

62. **ARCHITECTURE** A memorial being constructed in a city park will be a brick wall, with a top row of six gold-plated bricks engraved with the names of six local war veterans. Each row has two more bricks than the row above it. Prove that the number of bricks in the top n rows is $n^2 + 5n$.



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SOLUTION:

Sample answer:

Step 1: There are 6 bricks in the top row, and $1^2 + 5(1) = 6$, so the formula is true for $n = 1$.

Step 2: Assume that there are $k^2 + 5k$ bricks in the top k rows for some positive integer k.

Step 3: Since each row has 2 more bricks than the one above, the numbers of bricks in the rows form an arithmetic sequence.

The number of bricks in the $(k + 1)^{st}$ row is $6 + [(k + 1) - 1](2)$ or $2k + 6$.
Then the number of bricks in the top $k + 1$ rows is $k^2 + 5k + (2k + 6)$ or $k^2 + 7k + 6$.
$k^2 + 7k + 6 = (k + 1)^2 + 5(k + 1)$, which is the formula to be proved, where $n = k + 1$.

Therefore, the formula is true for $n = k + 1$.
Therefore, the number of bricks in the top n rows in $n^2 + 5n$ for all positive integers n.

63. **LEGENDS** There is a legend of a king who wanted to reward a boy for a good deed. The king gave the boy a choice. He could have $1,000,000 at once, or he could be rewarded daily for a 30-day month, with one penny on the first day, two pennies on the second day, and so on, receiving twice as many pennies each day as the previous day. How much would the second option be worth?

SOLUTION:

Substitute $a_1 = 1$ and $r = 2$ in the sum formula.

$$\begin{aligned} S_n &= \frac{a_1 - a_1 r^n}{1 - r} \\ &= \frac{1 - 2^{30}}{1 - 2} \\ &= \frac{1 - 1073741824}{-1} \\ &= 1073741823 \text{ pennies} \\ &= \$10{,}737{,}418.23 \end{aligned}$$

The worth of the second option is $10,737,418.23.

Write an equation for each circle given the endpoints of a diameter.

64. (2, −4), (10, 2)

SOLUTION:
Find the center.

$$(x_1, y_1) = (2, -4), (x_2, y_2) = (10, 2)$$
$$(h, k) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{2+10}{2}, \frac{-4+2}{2}\right) = (6, -1)$$

Find the radius.

$$(x_1, y_1) = (6, -1), (x_2, y_2) = (10, 2)$$
$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(10-6)^2 + (2+1)^2} = \sqrt{25} = 5$$

The equation of the circle is $(x-6)^2 + (y+1)^2 = 25$.

65. (−1, −10), (−7, 6)

SOLUTION:
Find the center.

$$(x_1, y_1) = (-1, -10), (x_2, y_2) = (-7, 6)$$

$$(h, k) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-1-7}{2}, \frac{-10+6}{2}\right) = (-4, -2)$$

Find the radius.

$$(x_1, y_1) = (-4, -2), (x_2, y_2) = (-7, 6)$$

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-7+4)^2 + (6+2)^2} = \sqrt{73}$$

The equation of the circle is $(x+4)^2 + (y+2)^2 = 73$.

66. (9, 0), (4, −7)

SOLUTION:

Find the center.

$$(x_1, y_1) = (9,0), (x_2, y_2) = (4,-7)$$

$$\begin{aligned}(h,k) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \\ &= \left(\frac{9+4}{2}, \frac{0-7}{2}\right) \\ &= (6.5, -3.5)\end{aligned}$$

Find the radius.

$$(x_1, y_1) = (6.5,-3.5), (x_2, y_2) = (4,-7)$$

$$\begin{aligned}r &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(4-6.5)^2 + (-7+3.5)^2} \\ &= \sqrt{18.5}\end{aligned}$$

The equation of the circle is $(x-6.5)^2 + (y+3.5)^2 = 18.5$.

Simplify each expression.

67. $\dfrac{5}{x^2+6x+8} + \dfrac{x}{x^2-3x-28}$

SOLUTION:

$$\begin{aligned}\frac{5}{x^2+6x+8} + \frac{x}{x^2-3x-28} &= \frac{5}{(x+2)(x+4)} + \frac{x}{(x-7)(x+4)} \\ &= \frac{5(x-7)+x(x+2)}{(x+2)(x+4)(x-7)} \\ &= \frac{x^2+7x-35}{(x+2)(x+4)(x-7)}\end{aligned}$$

68. $\dfrac{3x}{x^2+8x-20} - \dfrac{6}{x^2+7x-18}$

SOLUTION:

$$\begin{aligned}\frac{3x}{x^2+8x-20} - \frac{6}{x^2+7x-18} &= \frac{3x}{(x+10)(x-2)} - \frac{6}{(x+9)(x-2)} \\ &= \frac{3x(x+9)-6(x+10)}{(x+10)(x-2)(x+9)} \\ &= \frac{3x^2+21x-60}{(x+10)(x-2)(x+9)} \\ &= \frac{3(x^2+7x-20)}{(x+10)(x-2)(x+9)}\end{aligned}$$

69. $\dfrac{4}{3x^2+12x} + \dfrac{2x}{x^2-2x-24}$

SOLUTION:

$$\begin{aligned}\frac{4}{3x^2+12x} + \frac{2x}{x^2-2x-24} &= \frac{4}{3x(x+4)} + \frac{2x}{(x-6)(x+4)} \\ &= \frac{4(x-6)+2x(3x)}{3x(x+4)(x-6)} \\ &= \frac{6x^2+4x-24}{3x(x+4)(x-6)} \\ &= \frac{2(3x^2+2x-12)}{3x(x+4)(x-6)}\end{aligned}$$

Solve each equation or inequality. Round to the nearest ten-thousandth.

70. $8^x = 30$

SOLUTION:

Use the property of equality for logarithmic functions.

$$\begin{aligned}8^x &= 30 \\ \log 8^x &= \log 30 \\ x\log 8 &= \log 30 \\ x &= \frac{\log 30}{\log 8} \\ x &\approx 1.6356\end{aligned}$$

71. $5^x = 64$

SOLUTION:

Use the property of equality for logarithmic functions.

$$\begin{aligned} 5^x &= 64 \\ \log 5^x &= \log 64 \\ x\log 5 &= \log 64 \\ x &= \frac{\log 64}{\log 5} \\ x &\approx 2.5841 \end{aligned}$$

72. $3^{x+2} = 41$

SOLUTION:

Use the property of equality for logarithmic functions.

$$\begin{aligned} 3^{x+2} &= 41 \\ \log 3^{x+2} &= \log 41 \\ (x+2)\log 3 &= \log 41 \\ x+2 &= \frac{\log 41}{\log 3} \\ x &= \frac{\log 41}{\log 3} - 2 \\ x &\approx 1.3802 \end{aligned}$$

Evaluate each expression.

73. $16^{-\frac{1}{4}}$

SOLUTION:

$$\begin{aligned} 16^{-\frac{1}{4}} &= \frac{1}{16^{\frac{1}{4}}} \\ &= \frac{1}{\sqrt[4]{16}} \\ &= \frac{1}{\sqrt[4]{2^4}} \\ &= \frac{1}{2} \end{aligned}$$

74. $27^{\frac{4}{3}}$

SOLUTION:

$$\begin{aligned} 27^{\frac{4}{3}} &= \left(3^3\right)^{\frac{4}{3}} \\ &= 3^{3\cdot\frac{4}{3}} \\ &= 3^4 \\ &= 81 \end{aligned}$$

75. $25^{-\frac{5}{2}}$

SOLUTION:

$$\begin{aligned} 25^{-\frac{5}{2}} &= \frac{1}{25^{\frac{5}{2}}} \\ &= \frac{1}{\left(5^2\right)^{\frac{5}{2}}} \\ &= \frac{1}{5^{2\cdot\frac{5}{2}}} \\ &= \frac{1}{5^5} \\ &= \frac{1}{3125} \end{aligned}$$

Solve for *x*.

76. $\frac{x+2}{18} = \frac{x-2}{9}$

SOLUTION:

$$\begin{aligned} \frac{x+2}{18} &= \frac{x-2}{9} \\ 9(x+2) &= 18(x-2) \\ 9x+18 &= 18x-36 \\ 9x &= 54 \\ x &= \frac{54}{9} \\ x &= 6 \end{aligned}$$

77. $\frac{x+5}{x-1} = \frac{7}{4}$

SOLUTION:

$$\begin{aligned} \frac{x+5}{x-1} &= \frac{7}{4} \\ 4(x+5) &= 7(x-1) \\ 4x+20 &= 7x-7 \\ 3x &= 27 \\ x &= \frac{27}{3} \\ x &= 9 \end{aligned}$$

78. $\frac{5}{x+8} = \frac{15}{2x+20}$

SOLUTION:

$$\begin{aligned} \frac{5}{x+8} &= \frac{15}{2x+20} \\ 5(2x+20) &= 15(x+8) \\ 10x+100 &= 15x+120 \\ 5x &= -20 \\ x &= -\frac{20}{5} \\ x &= -4 \end{aligned}$$