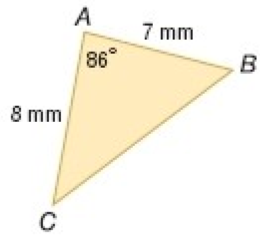
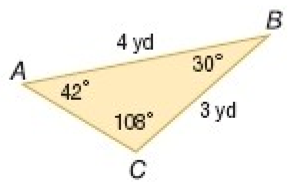
Find the area of ΔABC to the nearest tenth, if necessary.

1. 

SOLUTION:
Substitute c = 7, b = 8 and A = 86º in the area formula.

$$\begin{aligned}\text{Area} &= \frac{1}{2}bc\sin A\\ &= \frac{1}{2}(8)(7)\sin 86^\circ\\ &\approx 27.9\text{ mm}^2\end{aligned}$$

2. 

SOLUTION:
Substitute c = 4, a = 3 and B = 30º in the area formula.

$$\begin{aligned}\text{Area} &= \frac{1}{2}ac\sin B\\ &= \frac{1}{2}(3)(4)\sin 30^\circ\\ &= 3\text{ yd}^2\end{aligned}$$

3. A = 40°, b = 11 cm, c = 6 cm

SOLUTION:
Substitute c = 6, b = 11 and A = 40º in the area formula.

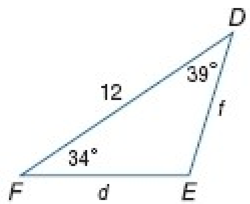
$$\begin{aligned}\text{Area} &= \frac{1}{2}bc\sin A\\ &= \frac{1}{2}(11)(6)\sin 40^\circ\\ &\approx 21.2\text{ cm}^2\end{aligned}$$

4. B = 103°, a = 20 in., c =18 in.

SOLUTION:
Substitute c = 18, a = 20 and B = 103º in the area formula.

$$\begin{aligned}\text{Area} &= \frac{1}{2}ac\sin B\\ &= \frac{1}{2}(20)(18)\sin 103^\circ\\ &= 175.4\text{ in}^2\end{aligned}$$

Solve each triangle. Round side lengths to the nearest tenth and angle measure to the nearest degree.

5. 

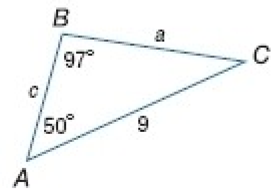
SOLUTION:

$$m\angle E = 180° - (34° + 39°)$$
$$= 180° - 73°$$
$$= 107°$$

Use the Law of Sines to find side lengths d and f.

$$\frac{\sin D}{d} = \frac{\sin E}{e}$$
$$\frac{\sin 39°}{d} = \frac{\sin 107°}{12}$$
$$d = \frac{12 \sin 39°}{\sin 107°}$$
$$d \approx 7.9$$

$$\frac{\sin F}{f} = \frac{\sin E}{e}$$
$$\frac{\sin 34°}{f} = \frac{\sin 107°}{12}$$
$$f = \frac{12 \sin 34°}{\sin 107°}$$
$$f \approx 7.0$$

6. 

SOLUTION:

$$m\angle C = 180° - (97° + 50°)$$
$$= 180° - 147°$$
$$= 33°$$

Use the Law of Sines to find side lengths a and c.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 50°}{a} = \frac{\sin 97°}{9}$$
$$a = \frac{9 \sin 50°}{\sin 97°}$$
$$a \approx 6.9$$

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$
$$\frac{\sin 33°}{c} = \frac{\sin 97°}{9}$$
$$c = \frac{9 \sin 33°}{\sin 97°}$$
$$c \approx 4.9$$

7. Solve ΔFGH if $G = 80°$, $H = 40°$, and $g = 14$.

SOLUTION:

$$m\angle F = 180° - (80° + 40°)$$
$$= 180° - 120°$$
$$= 60°$$

Use the Law of Sines to find side lengths f and h.

$$\frac{\sin F}{f} = \frac{\sin G}{g}$$
$$\frac{\sin 60°}{f} = \frac{\sin 80°}{14}$$
$$f = \frac{14 \sin 60°}{\sin 80°}$$
$$f \approx 12.3$$

$$\frac{\sin H}{h} = \frac{\sin G}{g}$$
$$\frac{\sin 40°}{h} = \frac{\sin 80°}{14}$$
$$h = \frac{14 \sin 40°}{\sin 80°}$$
$$h \approx 9.1$$

CCSS PERSEVERANCE Determine whether each $\triangle ABC$ has *no* solution, *one* solution, or *two* solutions. Then solve the triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

8. $A = 95°$, $a = 19$, $b = 12$

SOLUTION:

Because $\angle A$ is obtuse and $a > b$, one solution exists.

Use the Law of Sines to find $m\angle B$.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 95°}{19} = \frac{\sin B}{12}$$
$$\sin B = \frac{12 \sin 95°}{19}$$
$$B \approx 39°$$

$$m\angle C \approx 180° - (95° + 39°) \text{ or } 46°$$

Use the Law of Sines to find c.

$$\frac{\sin 95°}{19} \approx \frac{\sin 46°}{c}$$
$$c \approx \frac{19 \sin 46°}{\sin 95°}$$
$$c \approx 13.7$$

9. $A = 60°$, $a = 15$, $b = 24$

SOLUTION:

Since $\angle A$ is acute and $a < b$, find h and compare it to a.

$$h = b \sin A$$
$$= 24 \sin 60°$$
$$\approx 20.8$$

Since $15 < 20.8$ or $a < h$, there is no solution.

10. $A = 34°$, $a = 8$, $b = 13$

SOLUTION:

Since $\angle A$ is acute and $a < b$, find h and compare it to a.

$$\begin{aligned} h &= b \sin A \\ &= 13 \sin 34° \\ &\approx 7.3 \end{aligned}$$

Since 7.3 < 8 < 13 or $h < a < b$, there is two solutions. So, there are two triangles to be solved.

Case 1 $\angle B$ is acute	Case 2 $\angle B$ is obtuse
$\frac{\sin B}{b} = \frac{\sin A}{a}$	
$\frac{\sin B}{13} = \frac{\sin 34°}{8}$	The sine function also has a positive value in Quadrant II. So, find an obtuse angle B for which $\sin B \approx 0.9087$.
$\sin B = \frac{13 \sin 34°}{8}$	
$\sin B \approx 0.9087$	$m\angle B = 180° - 65°$ or $115°$
$B \approx 65°$	
$m\angle C \approx 180° - (34° + 65°)$ or $81°$	$m\angle C \approx 180° - (34° + 115°)$ or $31°$
$\frac{\sin 81°}{c} \approx \frac{\sin 34°}{8}$	$\frac{\sin 31°}{c} \approx \frac{\sin 34°}{8}$
$c \approx \frac{8 \sin 81°}{\sin 34°}$	$c \approx \frac{8 \sin 31°}{\sin 34°}$
$c \approx 14.1$	$c \approx 7.4$

11. $A = 30°$, $a = 3$, $b = 6$

SOLUTION:

Since $\angle A$ is acute and $a < b$, find h and compare it to a.

$$\begin{aligned} h &= b \sin A \\ &= 6 \sin 30° \\ &= 3 \end{aligned}$$

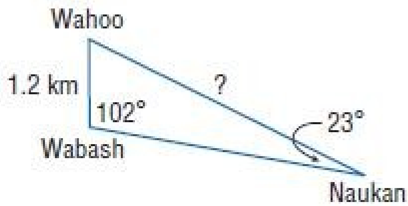
Since $a = h$, there is one solution and $m\angle B = 90°$

$$m\angle C = 180° - (30° + 90°) \text{ or } 60°$$

Use the Law of Sines to find c.

$$\begin{aligned} \frac{\sin A}{a} &= \frac{\sin C}{c} \\ \frac{\sin 30°}{3} &= \frac{\sin 60°}{c} \\ c &= \frac{3 \sin 60°}{\sin 30°} \\ c &\approx 5.2 \end{aligned}$$

12. **SPACE** Refer to the beginning of the lesson. Find the distance between the Wahoo Crater and the Naukan Crater on Mars.



SOLUTION:

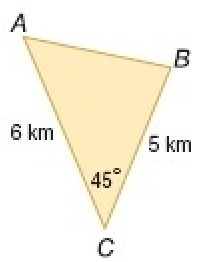
Use the Law of Sines to find *x*.

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 23}{1.2} = \frac{\sin 102}{x}$$
$$x = \frac{1.2 \sin 102}{\sin 23}$$
$$x = 3$$

The distance between the Wahoo Crater and the Naukan Crater is 3 kilometers.

Find the area of $\triangle ABC$ to the nearest tenth.

13.

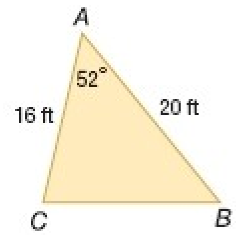


SOLUTION:

Substitute *a* = 5, *b* = 6 and *C* = 45º in the area formula.

$$\text{Area} = \frac{1}{2} ab \sin C$$
$$= \frac{1}{2}(5)(6)\sin 45$$
$$\approx 10.6 \text{ km}^2$$

14.

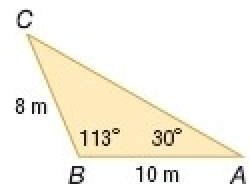


SOLUTION:

Substitute *b* = 16, *c* = 20 and *A* = 52º in the area formula.

$$\text{Area} = \frac{1}{2} bc \sin A$$
$$= \frac{1}{2}(16)(20)\sin 52$$
$$\approx 126.1 \text{ ft}^2$$

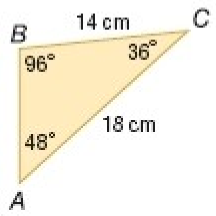
15.



SOLUTION:

Substitute *a* = 8, *c* = 10 and *B* = 113º in the area formula.

$$\text{Area} = \frac{1}{2} ac \sin B$$
$$= \frac{1}{2}(8)(10)\sin 113$$
$$\approx 36.8 \text{ m}^2$$

16. 

SOLUTION:
Substitute a = 14, b = 18 and C = 36º in the area formula.

$$\begin{aligned}\text{Area} &= \frac{1}{2}ab\sin C \\ &= \frac{1}{2}(14)(18)\sin 36^\circ \\ &\approx 74.1 \text{ cm}^2\end{aligned}$$

17. C = 25°, a = 4 ft, b = 7 ft

SOLUTION:
Substitute a = 4, b = 7 and C = 25º in the area formula.

$$\begin{aligned}\text{Area} &= \frac{1}{2}ab\sin C \\ &= \frac{1}{2}(4)(7)\sin 25^\circ \\ &\approx 5.9 \text{ ft}^2\end{aligned}$$

18. A = 138°, b = 10 in., c = 20 in.

SOLUTION:
Substitute b = 10, c = 20 and A = 138º in the area formula.

$$\begin{aligned}\text{Area} &= \frac{1}{2}bc\sin A \\ &= \frac{1}{2}(10)(20)\sin 138^\circ \\ &\approx 66.9 \text{ in}^2\end{aligned}$$

19. B = 92°, a = 14.5 m, c = 9 m

SOLUTION:
Substitute a = 14.5, c = 9 and B = 92º in the area formula.

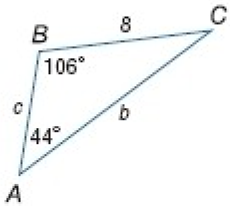
$$\begin{aligned}\text{Area} &= \frac{1}{2}ac\sin B \\ &= \frac{1}{2}(14.5)(9)\sin 92^\circ \\ &\approx 65.2 \text{ m}^2\end{aligned}$$

20. C = 116°, a = 2.7 cm, b = 4.6 cm

SOLUTION:
Substitute a = 2.7, b = 4.6 and C = 116º in the area formula.

$$\begin{aligned}\text{Area} &= \frac{1}{2}ab\sin C \\ &= \frac{1}{2}(2.7)(4.6)\sin 116^\circ \\ &\approx 5.6 \text{ cm}^2\end{aligned}$$

CCSS REASONING Solve each triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

21. 

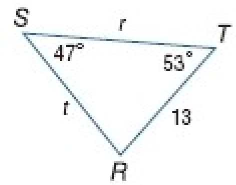
SOLUTION:

$m\angle C = 180^\circ - (106^\circ + 44^\circ)$ or 30°

Use the Law of Sines to find side lengths c and b.

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$
$$\frac{\sin 30^\circ}{c} = \frac{\sin 44^\circ}{8}$$
$$c = \frac{8\sin 30^\circ}{\sin 44^\circ}$$
$$c \approx 5.8$$

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$
$$\frac{\sin 106^\circ}{b} = \frac{\sin 44^\circ}{8}$$
$$b = \frac{8\sin 106^\circ}{\sin 44^\circ}$$
$$b \approx 11.1$$

22. 

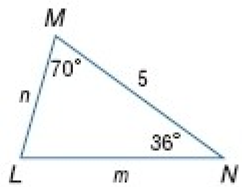
SOLUTION:

$m\angle R = 180^\circ - (47^\circ + 53^\circ)$ or 80°

Use the Law of Sines to find side lengths r and t.

$$\frac{\sin R}{r} = \frac{\sin S}{s}$$
$$\frac{\sin 80^\circ}{r} = \frac{\sin 47^\circ}{13}$$
$$r = \frac{13\sin 80^\circ}{\sin 47^\circ}$$
$$r \approx 17.5$$

$$\frac{\sin T}{t} = \frac{\sin S}{s}$$
$$\frac{\sin 53^\circ}{t} = \frac{\sin 47^\circ}{13}$$
$$t = \frac{13\sin 53^\circ}{\sin 47^\circ}$$
$$t \approx 14.2$$

23. 

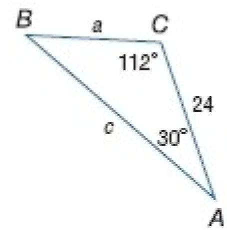
SOLUTION:

$m\angle L = 180° - (70° + 36°) \text{ or } 74°$

Use the Law of Sines to find side lengths *n* and *m*.

$$\frac{\sin N}{n} = \frac{\sin L}{l}$$
$$\frac{\sin 36°}{n} = \frac{\sin 74°}{5}$$
$$n = \frac{5 \sin 36°}{\sin 74°}$$
$$n \approx 3.1$$

$$\frac{\sin M}{m} = \frac{\sin L}{l}$$
$$\frac{\sin 70°}{m} = \frac{\sin 74°}{5}$$
$$m = \frac{5 \sin 70°}{\sin 74°}$$
$$m \approx 4.9$$

24. 

SOLUTION:

$m\angle B = 180° - (112° + 30°) \text{ or } 38°$

Use the Law of Sines to find side lengths *a* and *c*.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 30°}{a} = \frac{\sin 38°}{24}$$
$$a = \frac{24 \sin 30°}{\sin 38°}$$
$$a \approx 19.5$$

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$
$$\frac{\sin 112°}{c} = \frac{\sin 38°}{24}$$
$$c = \frac{24 \sin 112°}{\sin 38°}$$
$$c \approx 36.1$$

12-4 Law of Sines

25. Solve ΔHJK if $H = 53°$, $J = 20°$, and $h = 31$.

SOLUTION:

$m\angle K = 180° - (53° + 20°)$ or $107°$

Use the Law of Sines to find side lengths j and k.

$$\frac{\sin J}{j} = \frac{\sin H}{h}$$
$$\frac{\sin 20°}{j} = \frac{\sin 53°}{31}$$
$$j = \frac{31 \sin 20°}{\sin 53°}$$
$$j \approx 13.3$$

$$\frac{\sin K}{k} = \frac{\sin H}{h}$$
$$\frac{\sin 107°}{k} = \frac{\sin 53°}{31}$$
$$k = \frac{31 \sin 107°}{\sin 53°}$$
$$k \approx 37.1$$

26. Solve ΔNPQ if $P = 109°$, $Q = 57°$, and $n = 22$.

SOLUTION:

$$m\angle N = 180° - (109° + 57°)$$
$$= 180° - 166°$$
$$= 14°$$

Use the Law of Sines to find side lengths p and q.

$$\frac{\sin P}{p} = \frac{\sin N}{n}$$
$$\frac{\sin 109°}{p} = \frac{\sin 14°}{22}$$
$$p = \frac{22 \sin 109°}{\sin 14°}$$
$$p \approx 86.0$$

$$\frac{\sin Q}{q} = \frac{\sin N}{n}$$
$$\frac{\sin 57°}{q} = \frac{\sin 14°}{22}$$
$$q = \frac{22 \sin 57°}{\sin 14°}$$
$$q \approx 76.3$$

27. Solve ΔABC if $A = 50°$, $a = 2.5$, and $C = 67°$.

SOLUTION:

$m\angle B = 180° - (50° + 67°)$ or $63°$

Use the Law of Sines to find side lengths b and c.

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$
$$\frac{\sin 63°}{b} = \frac{\sin 50°}{2.5}$$
$$b = \frac{2.5 \sin 63°}{\sin 50°}$$
$$b \approx 2.9$$

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$
$$\frac{\sin 67°}{c} = \frac{\sin 50°}{2.5}$$
$$c = \frac{2.5 \sin 67°}{\sin 50°}$$
$$c \approx 3.0$$

28. Solve ΔABC if $B = 18°$, $C = 142°$, and $b = 20$.

SOLUTION:

$m\angle A = 180° - (18° + 142°)$ or $20°$

Use the Law of Sines to find side lengths a and c.

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$
$$\frac{\sin 18°}{20} = \frac{\sin 20°}{a}$$
$$a = \frac{20 \sin 20°}{\sin 18°}$$
$$a \approx 22.1$$

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$
$$\frac{\sin 18°}{20} = \frac{\sin 142°}{c}$$
$$c = \frac{20 \sin 142°}{\sin 18°}$$
$$c \approx 39.8$$

Determine whether each triangle $\triangle ABC$ has *no* solution, *one* solution, or *two* solutions. Then solve the triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

29. $A = 100°$, $a = 7$, $b = 3$

SOLUTION:

Because $\angle A$ is obtuse and $a > b$, one solution exists.

Use the Law of Sines to find $m\angle B$.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 100°}{7} = \frac{\sin B}{3}$$
$$\sin B = \frac{3\sin 100°}{7}$$
$$B = \sin^{-1}\left(\frac{3\sin 100°}{7}\right)$$
$$B \approx 25°$$

$$m\angle C \approx 180° - (100° + 25°) \text{ or } 55°$$

Use the Law of Sines to find c.

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 100°}{7} = \frac{\sin 55°}{c}$$
$$c = \frac{7\sin 55°}{\sin 100°}$$
$$c \approx 5.8$$

30. $A = 75°$, $a = 14$, $b = 11$

SOLUTION:

Because $\angle A$ is acute and $a > b$, one solution exists.

Use the Law of Sines to find $m\angle B$.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 75°}{14} = \frac{\sin B}{11}$$
$$\sin B = \frac{11\sin 75°}{14}$$
$$B = \sin^{-1}\left(\frac{11\sin 75°}{14}\right)$$
$$B \approx 49°$$

$$m\angle C \approx 180° - (75° + 49°) \text{ or } 56°$$

Use the Law of Sines to find c.

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 75°}{14} = \frac{\sin 56°}{c}$$
$$c = \frac{14\sin 56°}{\sin 75°}$$
$$c \approx 12.0$$

31. $A = 38°$, $a = 21$, $b = 18$

SOLUTION:

Because $\angle A$ is acute and $a > b$, one solution exists.

Use the Law of Sines to find $m\angle B$.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 38^\circ}{21} = \frac{\sin B}{18}$$
$$\sin B = \frac{18 \sin 38^\circ}{21}$$
$$B = \sin^{-1}\left(\frac{18 \sin 38^\circ}{21}\right)$$
$$B \approx 32^\circ$$

$$m\angle C \approx 180^\circ - (38^\circ + 32^\circ) \text{ or } 110^\circ$$

Use the Law of Sines to find c.

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 38^\circ}{21} = \frac{\sin 110^\circ}{c}$$
$$c = \frac{21 \sin 110^\circ}{\sin 38^\circ}$$
$$c \approx 32.1$$

32. $A = 52°$, $a = 9$, $b = 20$

SOLUTION:

Since $\angle A$ is acute and $a < b$, find h and compare it to a.

$$h = b \sin A$$
$$= 20 \sin 52^\circ$$
$$\approx 15.76$$

Since $9 < 15.76$ or $a < h$, there is no solution.

33. $A = 42°$, $a = 5$, $b = 6$

SOLUTION:

Since $\angle A$ is acute and $a < b$, find h and compare it to a.

$$h = b \sin A$$
$$= 6 \sin 42^\circ$$
$$\approx 4.0$$

Since $4 < 5 < 6$ or $h < a < b$, there are two solutions. So, there are two triangles to be solved.

Case 1 $\angle B$ is acute	Case 2 $\angle B$ is obtuse
$\frac{\sin B}{b} = \frac{\sin A}{a}$	
$\frac{\sin B}{6} = \frac{\sin 42^\circ}{5}$	The sine function also has a positive value in Quadrant II. So, find an obtuse angle B for which $\sin B \approx 0.8030$.
$\sin B = \frac{6 \sin 42^\circ}{5}$	$m\angle B \approx 180^\circ - 53^\circ$ or 127°
$B = \sin^{-1}\left(\frac{6 \sin 42^\circ}{5}\right)$	
$B \approx 53^\circ$	
$m\angle C \approx 180^\circ - (42^\circ + 53^\circ)$ or 85°	$m\angle C \approx 180^\circ - (42^\circ + 127^\circ)$ or 11°
$\frac{\sin C}{c} = \frac{\sin A}{a}$	$\frac{\sin C}{c} = \frac{\sin A}{a}$
$\frac{\sin 85^\circ}{c} = \frac{\sin 42^\circ}{5}$	$\frac{\sin 11^\circ}{c} = \frac{\sin 42^\circ}{5}$
$c = \frac{5 \sin 85^\circ}{\sin 42^\circ}$	$c = \frac{5 \sin 11^\circ}{\sin 42^\circ}$
$c \approx 7.4$	$c \approx 1.4$

34. $A = 44°$, $a = 14$, $b = 19$

SOLUTION:

Since $\angle A$ is acute and $a < b$, find h and compare it to a.

$$\begin{aligned} h &= b\sin A \\ &= 19\sin 44° \\ &\approx 13.2 \end{aligned}$$

Since 13.2 < 14 < 19 or $h < a < b$, there are two solutions. So, there are two triangles to be solved.

Case 1 $\angle B$ is acute	Case 2 $\angle B$ is obtuse
$\frac{\sin B}{b} = \frac{\sin A}{a}$	
$\frac{\sin B}{19} = \frac{\sin 44°}{14}$	The sine function also has a positive value in Quadrant II. So, find an obtuse angle B for which $\sin B \approx 0.9428$.
$\sin B = \frac{19\sin 44°}{14}$	
$\sin B \approx 0.9428$	
$B \approx 71°$	$m\angle B \approx 180° - 71°$ or $109°$
$m\angle C \approx 180° - (44° + 71°)$ or $65°$	$m\angle C \approx 180° - (44° + 109°)$ or $27°$
$\frac{\sin C}{c} \approx \frac{\sin A}{a}$	$\frac{\sin C}{c} \approx \frac{\sin A}{a}$
$\frac{\sin 65°}{c} \approx \frac{\sin 44°}{14}$	$\frac{\sin 27°}{c} \approx \frac{\sin 44°}{14}$
$c \approx \frac{14\sin 65°}{\sin 44°}$	$c \approx \frac{14\sin 27°}{\sin 44°}$
$c \approx 18.3$	$c \approx 9.1$

35. $A = 131°$, $a = 15$, $b = 32$

SOLUTION:

Since $\angle A$ is obtuse and $a < b$, so there is no solution.

36. $A = 30°$, $a = 17$, $b = 34$

SOLUTION:

Since $\angle A$ is acute and $a < b$, find h and compare it to a.

$$\begin{aligned} h &= b\sin A \\ &= 34\sin 30° \\ &= 17 \end{aligned}$$

Since $a = h$, one solution exists.

Use the Law of Sines to find $m\angle B$.

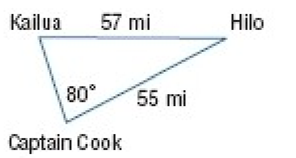
$$\begin{aligned} \frac{\sin A}{a} &= \frac{\sin B}{b} \\ \frac{\sin 30°}{17} &= \frac{\sin B}{34} \\ \sin B &= \frac{34\sin 30°}{17} \\ \sin B &= 1 \\ B &= 90° \end{aligned}$$

$$\begin{aligned} m\angle C &= 180° - (30° + 90°) \\ &= 180° - 120° \\ &= 60° \end{aligned}$$

Use the Law of Sines to find c.

$$\begin{aligned} \frac{\sin A}{a} &= \frac{\sin C}{c} \\ \frac{\sin 30°}{17} &= \frac{\sin 60°}{c} \\ c &= \frac{17\sin 60°}{\sin 30°} \\ c &\approx 29.4 \end{aligned}$$

GEOGRAPHY In Hawaii, the distance from Hilo to Kailua is 57 miles, and the distance from Hilo to Captain Cook is 55 miles.

37. 

What is the measure of the angle formed at Hilo?

SOLUTION:
Use the Law of Sines to find the measure of the angle formed at Kailua.

$$\frac{\sin 80^\circ}{57} = \frac{\sin K}{55}$$
$$\sin K = \frac{55 \sin 80^\circ}{57}$$
$$\sin K \approx 0.9503$$
$$K \approx 72^\circ$$

The measure of the angle formed at Hilo is about 28º (180º – 80° – 72º).

38. What is the distance between Kailua and Captain Cook?

SOLUTION:
Substitute 80, 57 and 28 for *C*, *c* and *H* and then solve for *h*.

$$\frac{\sin 80^\circ}{57} = \frac{\sin 28^\circ}{h}$$
$$h = \frac{57 \sin 28^\circ}{\sin 80^\circ}$$
$$h \approx 27.2$$

The distance between Kailua and Captain Cook is about 27.2 mi.

39. **TORNADOES** Tornado sirens *A*, *B*, and *C* form a triangular region in one area of a city. Sirens *A* and *B* are 8 miles apart. The angle formed at siren *A* is 112°, and the angle formed at siren *B* is 40°. How far apart are sirens *B* and *C*?

SOLUTION:
Measure of the angle formed at siren *C* = 180º – (112º + 40º) = 28º.

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$
$$\frac{\sin 28^\circ}{8} = \frac{\sin 112^\circ}{a}$$
$$a = \frac{8 \sin 112^\circ}{\sin 28^\circ}$$
$$a \approx 15.8$$

Sirens *B* and *C* are about 15.8 miles apart.

40. **MYSTERIES** The Bermuda Triangle is a region of the Atlantic Ocean between Bermuda, Miami, Florida, and San Juan, Puerto Rico. It is an area where ships and airplanes have been rumored to mysteriously disappear.

a. What is the distance between Miami and Bermuda?

b. What is the approximate area of the Bermuda Triangle?

SOLUTION:
a.
Use the Law of Sines to find the measure of angle at Bermuda..

$$\frac{\sin 53^\circ}{965} = \frac{\sin B}{1038}$$
$$\sin B = \frac{1038 \sin 53^\circ}{965}$$
$$B = \sin^{-1}\left(\frac{1038 \sin 53^\circ}{965}\right)$$
$$B \approx 59^\circ$$

The measure of angle at San Juan ≈ 180º – (53º + 59º) or 68º

Use the Law of Sines to find the distance between Miami and Bermuda.

$$\frac{\sin 53^\circ}{965} \approx \frac{\sin 68^\circ}{b}$$
$$b \approx \frac{965 \sin 68^\circ}{\sin 53^\circ}$$
$$b \approx 1120.3 \text{ mi}$$

b.

$$\text{Area} \approx \frac{1}{2} ab \sin C$$
$$\approx \frac{1}{2}(1038)(965)\sin 68^\circ$$
$$\approx 464366.1 \text{ mi}^2$$

41. **BICYCLING** One side of a triangular cycling path is 4 miles long. The angle opposite this side is 64°. Another angle formed by the triangular path measures 66°.

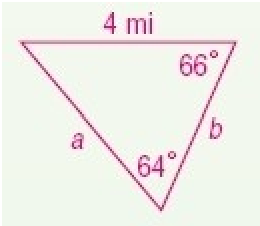
a. Sketch a drawing of the situation. Label the missing sides *a* and *b*.

b. Write equations that could be used to find the lengths of the missing sides.

c. What is the perimeter of the path?

SOLUTION:

a.



b.

$$\frac{\sin 66^\circ}{a} = \frac{\sin 64^\circ}{4};$$
$$\frac{\sin 50^\circ}{b} = \frac{\sin 64^\circ}{4}$$

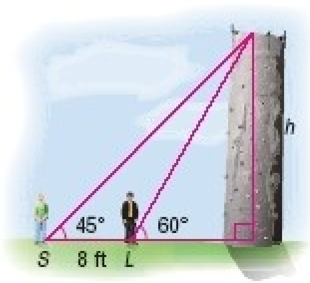
c.

$$\frac{\sin 66^\circ}{a} = \frac{\sin 64^\circ}{4}$$
$$a = \frac{4 \sin 66^\circ}{\sin 64^\circ}$$
$$a \approx 4.1$$

$$\frac{\sin 50^\circ}{b} = \frac{\sin 64^\circ}{4}$$
$$b = \frac{4 \sin 50^\circ}{\sin 64^\circ}$$
$$b \approx 3.4$$

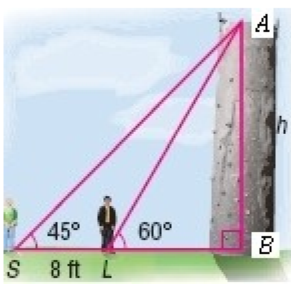
Perimeter of the triangle = 4 + 4.1 + 3.3 = 11.5 miles

42. **ROCK CLIMBING** Savannah *S* and Leon *L* are standing 8 feet apart in front of a rock climbing wall, as shown at the right. What is the height of the wall? Round to the nearest tenth.



SOLUTION:

Label the triangles.



Consider the ΔABL, $m\angle LAB$ is $30°$, so $m\angle LAS$ is $15°$.

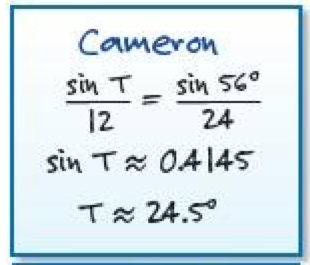
Use the Law of Sines to find the length $\overline{LA}$.

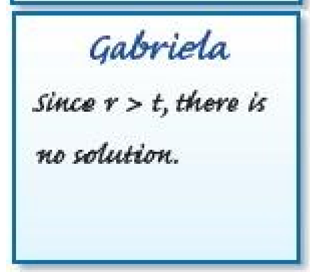
$$\frac{\sin A}{a} = \frac{\sin S}{s}$$
$$\frac{\sin 15°}{8} = \frac{\sin 45°}{s}$$
$$s = \frac{8 \sin 45°}{\sin 15°}$$

In a 30º-60º-90º triangle the sides are in the ratio $1 : \sqrt{3} : 2$.

In the ΔABL side opposite to the $m\angle B$ is about $\frac{8 \sin 45°}{\sin 15°}$ ft, so *h* is about $\frac{1}{2} \cdot \frac{8 \sin 45°}{\sin 15°} \cdot \sqrt{3}$ ft or 18.9 ft.

43. **CCSS CRITIQUE** In ΔRST, $R = 56°$, $r = 24$, and $t = 12$. Cameron and Gabriela are using the Law of Sines to find *T*. Is either of them correct? Explain your reasoning.





SOLUTION:

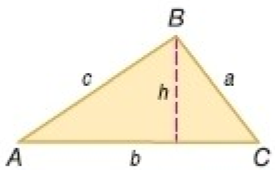
Cameron is correct; *R* is acute and $r > t$, so there is one solution.

44. **OPEN ENDED** Create an application problem involving right triangles and the Law of Sines. Then solve your problem, drawing diagrams if necessary.

SOLUTION:

See students' work.

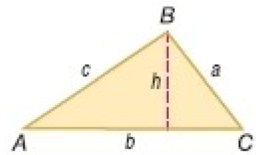
45. **CHALLENGE** Using the figure, derive the formula Area = $\frac{1}{2}bc\sin A$.



SOLUTION:

$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$	Definition of sine
$\sin A = \frac{h}{c}$	h = opposite side, c = hypotenuse
$c\sin A = h$	Multiply both sides by c.
$\text{Area} = \frac{1}{2}\cdot\text{base}\cdot\text{height}$	Area of a triangle
$\text{Area} = \frac{1}{2}bh$	b = base, h = height
$\text{Area} = \frac{1}{2}bc\sin A$	Substitution

46. **REASONING** Find the side lengths of two different triangles ABC that can be formed if $A = 55°$ and $C = 20°$.



SOLUTION:
Sample answer:

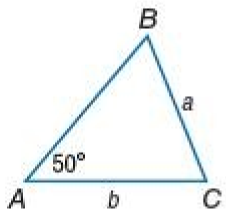
$$m\angle B = 180° - (55° + 20°)$$
$$= 180° - 75°$$
$$= 105°$$

$$\frac{\sin 55°}{a} = \frac{\sin 105°}{b}$$
$$b = \frac{a\sin 105°}{\sin 55°}$$
$$b \approx 1.18a$$
$$\frac{\sin 55°}{a} = \frac{\sin 20°}{c}$$
$$c = \frac{a\sin 20°}{\sin 55°}$$
$$c \approx 0.42a$$

The side lengths a, b and c of the triangle will be in the ratio 1 : 1.18 : 0.42.

If $a = 12$, then $b \approx 1.18 \times 12$ or 14.2 and $c \approx 0.42 \times 12$ or 5.0.
If $a = 6$, then $b \approx 1.18 \times 6$ or 7.1 and $c \approx 0.42 \times 62$ or 2.5.

47. **WRITING IN MATH** Use the Law of Sines to explain why a and b do not have unique values in the figure shown.



SOLUTION:

In the triangle, $B = 115°$. Using the Law of Sines, $\frac{\sin 50°}{a} = \frac{\sin 115°}{b}$. This equation cannot be solved because there are two unknown sides. To solve a triangle using the Law of Sines, two sides and an angle must be given or two angles and a side opposite one of the angles must be given.

48. **OPEN ENDED** Given that $E = 62°$ and $d = 38$, find a value for e such that no triangle DEF can exist. Explain your reasoning.

SOLUTION:

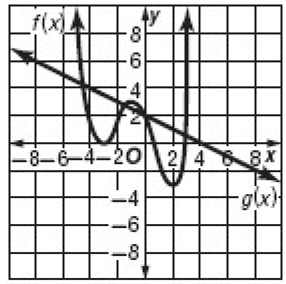
Sample answer:

$$\frac{\sin 62°}{e} = \frac{\sin D}{38}$$
$$e = \frac{38 \sin 62°}{\sin D}$$
$$e \approx \frac{33.6}{\sin D}$$

Since the value of sin D is between 0 to 1, the value e should be greater than or equal to 33.6.

For $e = 30$, the triangle will not exist.
For no triangle to exist, the length of the side opposite angle E must be less than 33.6 to satisfy the Law of Sines.

49. **SHORT RESPONSE** Given the graphs of $f(x)$ and $g(x)$, what is the value of $f(g(4))$?



SOLUTION:

At $x = 4$, the graph of $g(x)$ intersect the x-axis. Therefore, $g(4) = 0$

$$f(g(4)) = f(0)$$

At $x = 0$, the graph of $f(x)$ intersect the y-axis at 2. So, $f(g(4)) = 2$.

50. **STATISTICS** If the average of seven consecutive odd integers is n, what is the median of these seven integers?

A 0

B 7

C n

D $n - 2$

SOLUTION:
Let x be the first odd integer.

So, the other integers are $x + 2$, $x + 4$, $x + 6$, $x + 8$, $x + 10$, $x + 12$.

The median of the data is $x + 6$.

The equation that represents the situation is $\frac{x+(x+2)+(x+4)+(x+6)+(x+8)+(x+10)+(x+12)}{7}=n$.

$$\begin{aligned}\frac{x+(x+2)+(x+4)+(x+6)+(x+8)+(x+10)+(x+12)}{7}&=n\\ \frac{7x+42}{7}&=n\\ \frac{7(x+6)}{7}&=n\\ x+6&=n\end{aligned}$$

C is the correct option.

51. One zero of $f(x) = x^3 - 7x^2 - 6x + 72$ is 4. What is the factored form of the expression $x^3 - 7x^2 - 6x + 72$?

F $(x - 6)(x + 3)(x + 4)$

G $(x - 6)(x + 3)(x - 4)$

H $(x + 6)(x + 3)(x - 4)$

J $(x + 12)(x - 1)(x - 4)$

SOLUTION:
Divide the polynomial by $x - 4$.

$x^3 - 7x^2 - 6x + 72 = (x^2 - 3x - 18)(x - 4)$

To factor the quadratic, find a pair of numbers with the product −18 and the sum −3.

$= (x - 6)(x + 3)(x - 4)$

G is the correct option.

52. **SAT/ACT** Three people are splitting \$48,000 using the ratio 5:4:3. What is the amount of the greatest share?

A \$12,000

B \$16,000

C \$20,000

D \$24,000

E \$30,000

SOLUTION:
The total number of shares is 5 + 4 + 3 or 12.

$\frac{5}{12}$ of \$48,000 is \$20,000.

$\frac{4}{12}$ of \$48,000 is \$16,000.

$\frac{3}{12}$ of \$48,000 is \$12,000.

\$20,000 is the greatest share.

C is the correct option.

Find the exact value of each trigonometric function.

53. sin 210°

SOLUTION:
The terminal side of 210° lies in Quadrant III.

Find the reference angle.

$$\begin{aligned}\theta' &= \theta - 180^\circ \\ &= 210^\circ - 180^\circ \\ &= 30^\circ\end{aligned}$$

The sine function is negative in Quadrant III.

$$\begin{aligned}\sin 210^\circ &= -\sin 30^\circ \\ &= -\frac{1}{2}\end{aligned}$$

54. $\cos\frac{3}{4}\pi$

SOLUTION:
The terminal side of $\frac{3}{4}\pi$ lies in Quadrant II.

Find the reference angle.

$$\begin{aligned}\theta' &= \pi - \theta \\ &= \pi - \frac{3\pi}{4} \\ &= \frac{\pi}{4}\end{aligned}$$

The cosine function is negative in Quadrant II.

$$\begin{aligned}\cos\frac{3}{4}\pi &= -\cos\frac{\pi}{4} \\ &= -\cos 45^\circ \\ &= -\frac{\sqrt{2}}{2}\end{aligned}$$

55. cot 60°

SOLUTION:
The terminal side of 60° lies in Quadrant I.

The cotangent function is positive in Quadrant I.

cot 60° = $\frac{\sqrt{3}}{3}$

Find an angle with a positive measure and an angle with a negative measure that are coterminal with each angle.

56. 125°

SOLUTION:
Positive angle: 125° + 360° = 485°

Negative angle: 125° – 360° = –235°

57. −32°

SOLUTION:
Positive angle: –32° + 360° = 328°

Negative angle: –32° – 360° = –392°

58. $\frac{2}{3}\pi$

SOLUTION:
Positive angle: $\frac{2}{3}\pi + 2\pi = \frac{8\pi}{3}$

Negative angle: $\frac{2}{3}\pi - 2\pi = -\frac{4\pi}{3}$

59. **CLOCKS** Jun's grandfather clock is broken. When she sets the pendulum in motion by holding it against the side of the clock and letting it go, it swings 24 centimeters to the other side, then 18 centimeters back, then 13.5 centimeters, and so on. What is the total distance that the pendulum swings before it stops?

SOLUTION:
Find the common ratio.

$r = \frac{18}{24} = \frac{13.5}{18}$ or 0.75

Substitute 24 for a_1 and 0.75 for r in the sum formula.

$$\begin{aligned} S_n &= \frac{a_1}{1-r} \\ &= \frac{24}{1-0.75} \\ &= 96 \end{aligned}$$

The total distance traveled by the pendulum is 96 cm.

Find the sum of each infinite series, if it exists.

60. 64 + 48 + 36 + …

SOLUTION:
Find the common ratio.

$$r = \frac{48}{64} = \frac{3}{4}$$

Since $\left|\frac{3}{4}\right|$ < 1, the series is convergent.

Use the formula to find the sum.

$$S = \frac{a_1}{1-r} = \frac{64}{1-\frac{3}{4}} = 64 \div \frac{1}{4} = 256$$

61. 27 + 36 + 48 + …

SOLUTION:
Find the common ratio.

$$r = \frac{36}{27} = \frac{4}{3} = 1\frac{1}{3}$$

Since $\left|1\frac{1}{3}\right|$ > 1, the series is divergent. So, the sum does not exist.

62. $\sum_{n=1}^{\infty} 0.5(1.1)^n$

SOLUTION:
Here r = 1.1.

Since $|1.1|$ > 1, the series is divergent. So, the sum does not exist.

63. **ASTRONOMY** At its closest point, Earth is 91.8 million miles from the center of the Sun. At its farthest point, Earth is 94.9 million miles from the center of the Sun. Write an equation for the orbit of Earth, assuming that the center of the orbit is the origin and the Sun lies on the *x*-axis.

SOLUTION:

The value of *a* is one half the length of the major axis.

$$a = \frac{1}{2}(91.8 + 94.9)$$
$$= 93.35 \text{ million miles}$$

$$a^2 = 8714.22 \text{ million miles}$$
$$= 8.714 \times 10^{15}$$

The value of *c* is the distance from the center of the ellipse to the focus. This distance is equal to *a* minus the perihelion.

$$c = 93.35 - 91.8$$
$$c = 1.55 \text{ million miles}$$

$$c^2 = a^2 - b^2$$
$$2.40 = 8714.22 - b^2$$
$$b^2 = 8711.82 \text{ million miles}$$
$$= 8.712 \times 10^{15}$$

The equation of the ellipse is

$$\frac{x^2}{8.714 \times 10^{15}} + \frac{y^2}{8.712 \times 10^{15}} = 1$$

Simplify.

64. $\sqrt{(x-4)^2}$

SOLUTION:

$$\sqrt{(x-4)^2} = |x-4|$$

Since the index 2 is even and the exponent 1 is odd, we must use absolute value.

65. $\sqrt{(y+2)^4}$

SOLUTION:

$$\sqrt{(y+2)^4} = \sqrt{\left((y+2)^2\right)^2}$$
$$= (y+2)^2$$

66. $\sqrt[3]{(a-b)^6}$

SOLUTION:

$$\sqrt[3]{(a-b)^6} = \sqrt[3]{\left((a-b)^2\right)^3}$$
$$= (a-b)^2$$

Evaluate each expression if $w = 6$, $x = -4$, $y = 1.5$, and $z = \frac{3}{4}$.

67. $w^2 + y^2 - 6xz$

SOLUTION:

Substitute $w = 6$, $x = -4$, $y = 1.5$ and $z = \frac{3}{4}$ in the given expression and evaluate.

$$\begin{aligned}&w^2 + y^2 - 6xz\\&= 6^2 + 1.5^2 - 6(-4)\left(\frac{3}{4}\right)\\&= 36 + 2.25 + 18\\&= 56.25\end{aligned}$$

68. $x^2 + z^2 + 5wy$

SOLUTION:

Substitute $w = 6$, $x = -4$, $y = 1.5$ and $z = \frac{3}{4}$ in the given expression and evaluate.

$$\begin{aligned}&x^2 + z^2 + 5wy\\&= (-4)^2 + \left(\frac{3}{4}\right)^2 + 5(6)(1.5)\\&= 16 + \frac{9}{16} + 45\\&= 61\frac{9}{16}\end{aligned}$$

69. $wy + xz + w^2 - x^2$

SOLUTION:

Substitute $w = 6$, $x = -4$, $y = 1.5$ and $z = \frac{3}{4}$ in the given expression and evaluate.

$$\begin{aligned}&wy + xz + w^2 - x^2\\&= (6)(1.5) + (-4)\left(\frac{3}{4}\right) + 6^2 - (-4)^2\\&= 9 - 3 + 36 - 16\\&= 26\end{aligned}$$