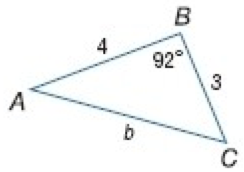
Solve each triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

1. 

SOLUTION:

Use the Law of Cosines to find the missing side length.

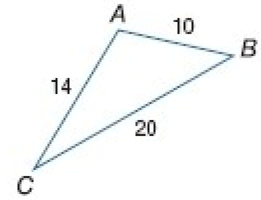
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$b^2 = 3^2 + 4^2 - 2(3)(4)\cos 92^\circ$$
$$b^2 \approx 25.8$$
$$b \approx 5.1$$

Use the Law of Sines to find a missing angle measure.

$$\frac{\sin A}{3} \approx \frac{\sin 92^\circ}{5.1}$$
$$\sin A \approx \frac{3\sin 92^\circ}{5.1}$$
$$A \approx 36^\circ$$

Find the measure of $\angle C$.

$$m\angle C \approx 180^\circ - (36^\circ + 92^\circ)$$
$$\approx 52^\circ$$

2. 

SOLUTION:

Use the Law of Cosines to find the measure of the largest angle, $\angle A$.

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$20^2 = 14^2 + 10^2 - 2(14)(10)\cos A$$
$$\frac{20^2 - 14^2 - 10^2}{-2(14)(10)} = \cos A$$
$$112^\circ \approx A$$

Use the Law of Sines to find the measure of angle, $\angle B$.

$$\frac{\sin B}{14} \approx \frac{\sin 112^\circ}{20}$$
$$\sin B \approx \frac{14\sin 112^\circ}{20}$$
$$B \approx 40^\circ$$

Find the measure of $\angle C$.

$$m\angle C \approx 180^\circ - (112^\circ + 40^\circ)$$
$$\approx 28^\circ$$

3. a = 5, b = 8, c = 12

SOLUTION:

Use the Law of Cosines to find the measure of the largest angle, $\angle C$.

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$12^2 = 5^2 + 8^2 - 2(5)(8)\cos C$$
$$\frac{12^2 - 5^2 - 8^2}{-2(5)(8)} = \cos C$$
$$133^\circ \approx C$$

Use the Law of Sines to find the measure of angle, $\angle B$.

$$\frac{\sin B}{8} \approx \frac{\sin 133^\circ}{12}$$
$$\sin B \approx \frac{8\sin 133^\circ}{12}$$
$$B \approx 29^\circ$$

Find the measure of $\angle A$.

$$m\angle A \approx 180^\circ - (133^\circ + 29^\circ)$$
$$\approx 18^\circ$$

4. B = 110°, a = 6, c = 3

SOLUTION:

Use the Law of Cosines to find the missing side length.

$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$b^2 = 6^2 + 3^2 - 2(6)(3)\cos 110^\circ$$
$$b^2 \approx 57.3$$
$$b \approx 7.6$$

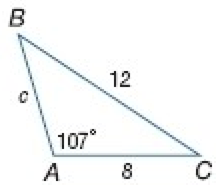
Use the Law of Sines to find a missing angle measure.

$$\frac{\sin A}{6} \approx \frac{\sin 110^\circ}{7.6}$$
$$\sin A \approx \frac{6\sin 110^\circ}{7.6}$$
$$A \approx 48^\circ$$

Find the measure of $\angle C$.

$$m\angle C \approx 180^\circ - (48^\circ + 110^\circ)$$
$$\approx 22^\circ$$

CCSS PRECISION Determine whether each triangle should be solved by beginning with the Law of *Sines* or the Law of *Cosines*. Then solve the triangle.

5. 

SOLUTION:

Since two sides and an angle opposite one of them of a triangle are given, the triangle should be solved by beginning with the Law of Sines.

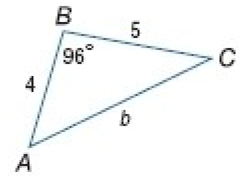
$$\frac{\sin 107^\circ}{12} = \frac{\sin B}{8}$$
$$\sin B = \frac{8\sin 107^\circ}{12}$$
$$\sin B \approx 0.6375$$
$$B \approx 40^\circ$$

Find the measure of $\angle C$.

$$m\angle C \approx 180^\circ - (107^\circ + 40^\circ)$$
$$\approx 33^\circ$$

Use Law of Sines to find *c*.

$$\frac{\sin 107^\circ}{12} \approx \frac{\sin 33^\circ}{c}$$
$$c \approx \frac{12\sin 33^\circ}{\sin 107^\circ}$$
$$c \approx 6.8$$

6. 

SOLUTION:

Since two sides and their included angle of a triangle are given, the triangle should be solved by beginning with the Law of Cosines.

$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$b^2 = 5^2 + 4^2 - 2(5)(4)\cos 96^\circ$$
$$b \approx 6.7$$

Use the Law of Sines to find a missing angle measure.

$$\frac{\sin A}{5} \approx \frac{\sin 96^\circ}{6.7}$$
$$\sin A \approx \frac{5\sin 96^\circ}{6.7}$$
$$A \approx 48^\circ$$

Find the measure of $\angle C$.

$$m\angle C \approx 180^\circ - (48^\circ + 96^\circ)$$
$$\approx 36^\circ$$

7. In ΔRST, $R = 35°$, $s = 16$, and $t = 9$.

SOLUTION:

Since two sides and their included angle of a triangle are given, the triangle should be solved by beginning with the Law of Cosines.

$$r^2 = s^2 + t^2 - 2st\cos R$$
$$r^2 = 16^2 + 9^2 - 2(16)(9)\cos 35°$$
$$r \approx 10.1$$

Use the Law of Sines to find a missing angle measure.

$$\frac{\sin T}{t} = \frac{\sin R}{r}$$
$$\frac{\sin T}{9} \approx \frac{\sin 35°}{10.1}$$
$$\sin T \approx \frac{9\sin 35°}{10.1}$$
$$T \approx 31°$$

Find the measure of $\angle S$.

$$m\angle S \approx 180° - (35° + 31°) \text{ or } 114°$$

8. **FOOTBALL** In a football game, the quarterback is 20 yards from Receiver A. He turns 40° to see Receiver B, who is 16 yards away. How far apart are the two receivers?

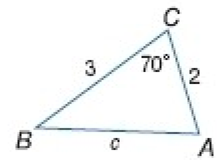
SOLUTION:

Use the Law of Cosines to find the missing side length.

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$c^2 = 16^2 + 20^2 - 2(16)(20)\cos 40°$$
$$c^2 \approx 165.7$$
$$c \approx 12.9$$

The two receivers are about 12.9 yards apart.

Solve each triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

9. 

SOLUTION:

Use the Law of Cosines to find the missing side length.

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$c^2 = 3^2 + 2^2 - 2(3)(2)\cos 70°$$
$$c^2 \approx 8.8958$$
$$c \approx 3.0$$

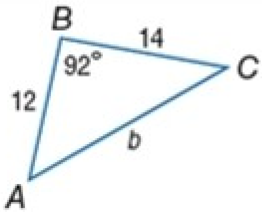
Use the Law of Sines to find a missing angle measure.

$$\frac{\sin A}{3} \approx \frac{\sin 70°}{3}$$
$$\sin A \approx \frac{3\sin 70°}{3}$$
$$A \approx 70°$$

Find the measure of $\angle B$.

$$m\angle B \approx 180° - (70° + 70°)$$
$$\approx 40°$$

12-5 Law of Cosines

10. 

SOLUTION:

Use the Law of Cosines to find the missing side length.

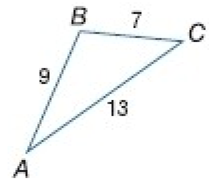
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$b^2 = 14^2 + 12^2 - 2(14)(12)\cos 92^\circ$$
$$b^2 \approx 351.7262$$
$$b \approx 18.75$$

Use the Law of Sines to find a missing angle measure.

$$\frac{\sin A}{14} \approx \frac{\sin 92^\circ}{18.8}$$
$$\sin A \approx \frac{5\sin 96^\circ}{6.7}$$
$$A \approx 48^\circ$$

Find the measure of $\angle C$.

$$m\angle C \approx 180^\circ - (48^\circ + 92^\circ)$$
$$\approx 40^\circ$$

11. 

SOLUTION:

Use the Law of Cosines to find the measure of the largest angle, $\angle B$.

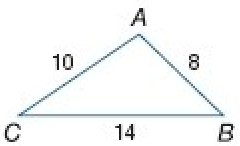
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$13^2 = 7^2 + 9^2 - 2(7)(9)\cos B$$
$$\frac{13^2 - 7^2 - 9^2}{-2(7)(9)} = \cos B$$
$$-0.3095 \approx \cos B$$
$$108^\circ \approx B$$

Use the Law of Sines to find the measure of angle, $\angle A$.

$$\frac{\sin 108^\circ}{13} \approx \frac{\sin A}{7}$$
$$\sin A \approx \frac{7\sin 108^\circ}{13}$$
$$A \approx 31^\circ$$

Find the measure of $\angle C$.

$$m\angle C \approx 180^\circ - (108^\circ + 31^\circ)$$
$$\approx 41^\circ$$

12. 

SOLUTION:

Use the Law of Cosines to find the measure of the largest angle, $\angle A$.

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$14^2 = 10^2 + 8^2 - 2(10)(8)\cos A$$
$$\frac{14^2 - 10^2 - 8^2}{-2(10)(8)} = \cos A$$
$$102^\circ \approx A$$

Use the Law of Sines to find the measure of angle, $\angle B$.

$$\frac{\sin 102^\circ}{14} \approx \frac{\sin B}{10}$$
$$\sin B \approx \frac{10\sin 102^\circ}{14}$$
$$B \approx 44^\circ$$

Find the measure of $\angle C$.

$$m\angle C \approx 180^\circ - (102^\circ + 44^\circ)$$
$$\approx 34^\circ$$

13. $A = 116°$, $b = 5$, $c = 3$

SOLUTION:

Use the Law of Cosines to find the missing side length.

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$a^2 = 5^2 + 3^2 - 2(5)(3)\cos 116^\circ$$
$$a^2 \approx 47.2$$
$$a \approx 6.9$$

Use the Law of Sines to find a missing angle measure.

$$\frac{\sin B}{5} \approx \frac{\sin 116^\circ}{6.9}$$
$$\sin B \approx \frac{5\sin 116^\circ}{6.9}$$
$$B \approx 41^\circ$$

Find the measure of $\angle C$.

$$m\angle C \approx 180^\circ - (116^\circ + 41^\circ)$$
$$\approx 23^\circ$$

14. $C = 80°$, $a = 9$, $b = 2$

SOLUTION:

Use the Law of Cosines to find the missing side length.

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$c^2 = 9^2 + 2^2 - 2(9)(2)\cos 80^\circ$$
$$c^2 \approx 78.7$$
$$c \approx 8.9$$

Use the Law of Sines to find a missing angle measure.

$$\frac{\sin B}{2} \approx \frac{\sin 80^\circ}{8.9}$$
$$\sin B \approx \frac{2\sin 80^\circ}{8.9}$$
$$B \approx 13^\circ$$
$$m\angle A \approx 180^\circ - (80^\circ + 13^\circ)$$
$$\approx 87^\circ$$

Find the measure of $\angle A$.

$$m\angle A \approx 180^\circ - (80^\circ + 13^\circ)$$
$$\approx 87^\circ$$

15. $f = 10$, $g = 11$, $h = 4$

SOLUTION:

Use the Law of Cosines to find the measure of the largest angle, $\angle G$.

$$g^2 = f^2 + h^2 - 2fh\cos G$$
$$11^2 = 10^2 + 4^2 - 2(10)(4)\cos G$$
$$\frac{11^2 - 10^2 - 4^2}{-2(10)(4)} = \cos G$$
$$94^\circ \approx G$$

Use the Law of Sines to find the measure of angle, $\angle F$.

$$\frac{\sin 94^\circ}{11} \approx \frac{\sin F}{10}$$
$$\sin F \approx \frac{10\sin 94^\circ}{11}$$
$$F \approx 65^\circ$$

Find the measure of $\angle H$.

$$m\angle H \approx 180^\circ - (94^\circ + 65^\circ)$$
$$\approx 21^\circ$$

16. $w = 20$, $x = 13$, $y = 12$

SOLUTION:

Use the Law of Cosines to find the measure of the largest angle, $\angle W$.

$$\begin{aligned} w^2 &= x^2 + y^2 - 2xy\cos W \\ 20^2 &= 13^2 + 12^2 - 2(13)(12)\cos W \\ \frac{20^2 - 13^2 - 12^2}{-2(13)(12)} &= \cos W \\ 106^\circ &\approx W \end{aligned}$$

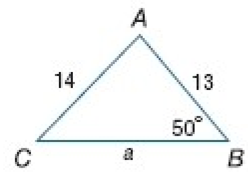
Use the Law of Sines to find the measure of angle, $\angle X$.

$$\begin{aligned} \frac{\sin 106^\circ}{20} &\approx \frac{\sin X}{13} \\ \sin X &\approx \frac{13\sin 106^\circ}{20} \\ X &\approx 39^\circ \end{aligned}$$

Find the measure of $\angle Y$.

$$\begin{aligned} m\angle Y &\approx 180^\circ - (39^\circ + 106^\circ) \\ &\approx 35^\circ \end{aligned}$$

Determine whether each triangle should be solved by beginning with the Law of *Sines* or the Law of *Cosines*. Then solve the triangle.

17. 

SOLUTION:

Since two sides and an angle opposite on of them of a triangle are given, the triangle should be solved by beginning with the Law of Sines.

$$\begin{aligned} \frac{\sin 50^\circ}{14} &= \frac{\sin C}{13} \\ \sin C &= \frac{13\sin 50^\circ}{14} \\ C &\approx 45^\circ \end{aligned}$$

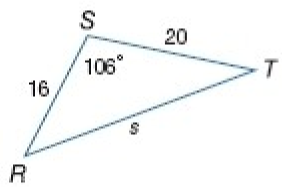
Find the measure of $\angle A$.

$$\begin{aligned} m\angle A &\approx 180^\circ - (50^\circ + 45^\circ) \\ &\approx 85^\circ \end{aligned}$$

Use Law of Sines to find a.

$$\begin{aligned} \frac{\sin 50^\circ}{14} &\approx \frac{\sin 85^\circ}{a} \\ a &\approx \frac{14\sin 85^\circ}{\sin 50^\circ} \\ a &\approx 18.2 \end{aligned}$$

12-5 Law of Cosines

18. 

SOLUTION:

Since two sides and their included angle of a triangle are given, the triangle should be solved by beginning with the Law of Cosines.

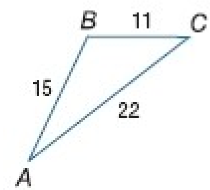
$$s^2 = r^2 + t^2 - 2rt\cos S$$
$$s^2 = 20^2 + 16^2 - 2(20)(16)\cos 106^\circ$$
$$s^2 \approx 832.4$$
$$s \approx 28.9$$

Use the Law of Sines to find a missing angle measure.

$$\frac{\sin R}{20} \approx \frac{\sin 106^\circ}{28.9}$$
$$\sin R \approx \frac{20\sin 106^\circ}{28.9}$$
$$R \approx 42^\circ$$

Find the measure of $\angle T$.

$$m\angle T \approx 180^\circ - (106^\circ + 42^\circ)$$
$$\approx 32^\circ$$

19. 

SOLUTION:

Since three sides of a triangle are given, the triangle should be solved by beginning with the Law of Cosines.

Find the measure of the largest angle, $\angle A$.

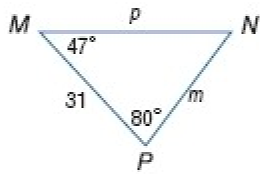
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$22^2 = 11^2 + 15^2 - 2(11)(15)\cos B$$
$$\frac{22^2 - 11^2 - 15^2}{-2(11)(15)} = \cos B$$
$$115^\circ \approx B$$

Use the Law of Sines to find the measure of angle, $\angle A$.

$$\frac{\sin 115^\circ}{22} \approx \frac{\sin A}{11}$$
$$\sin A \approx \frac{11\sin 115^\circ}{22}$$
$$\sin A \approx 0.4532$$
$$A \approx 27^\circ$$

Find the measure of $\angle C$.

$$m\angle C \approx 180^\circ - (27^\circ + 115^\circ)$$
$$\approx 38^\circ$$

20. 

SOLUTION:

Since two angles and any side of a triangle are given, the triangle should be solved by beginning with the Law of Sines.

Find the measure of $\angle N$.

$$\begin{aligned} m\angle N &= 180^\circ - (47^\circ + 80^\circ) \\ &= 53^\circ \end{aligned}$$

Find the length of m and p.

$$\begin{aligned} \frac{\sin 53^\circ}{31} &= \frac{\sin 47^\circ}{m} \\ m &= \frac{31\sin 47^\circ}{\sin 53^\circ} \\ m &\approx 28.4 \\ \frac{\sin 53^\circ}{31} &= \frac{\sin 80^\circ}{p} \\ p &= \frac{31\sin 80^\circ}{\sin 53^\circ} \\ p &\approx 38.2 \end{aligned}$$

21. In ΔABC, $C = 84°$, $c = 7$, and $a = 2$.

SOLUTION:

Since two sides and an angle opposite on of them of a triangle are given, the triangle should be solved by beginning with the Law of Sines.

Find the measure of angle, $\angle A$.

$$\begin{aligned} \frac{\sin 84^\circ}{7} &= \frac{\sin A}{2} \\ \sin A &= \frac{2\sin 84^\circ}{7} \\ A &\approx 17^\circ \end{aligned}$$

Find the measure of angle, $\angle B$.

$$\begin{aligned} m\angle B &\approx 180^\circ - (17^\circ + 84^\circ) \\ &\approx 79^\circ \end{aligned}$$

Find the length of b.

$$\begin{aligned} \frac{\sin 84^\circ}{7} &= \frac{\sin 79^\circ}{b} \\ b &= \frac{7\sin 79^\circ}{\sin 84^\circ} \\ b &\approx 6.9 \end{aligned}$$

22. In ΔHJK, h = 18, j = 10, and k = 23.

SOLUTION:
Since three sides of a triangle are given, the triangle should be solved by beginning with the Law of Cosines.

Find the measure of the largest angle, $\angle K$.

$$\begin{aligned} k^2 &= h^2 + j^2 - 2hj\cos K \\ 23^2 &= 18^2 + 10^2 - 2(18)(10)\cos K \\ \frac{23^2 - 18^2 - 10^2}{-2(18)(10)} &= \cos K \\ 107^\circ &\approx K \end{aligned}$$

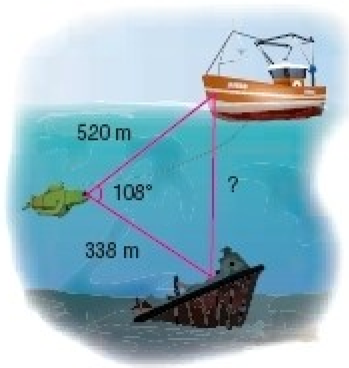
Use the Law of Sines to find the measure of angle $\angle H$.

$$\begin{aligned} \frac{\sin 107^\circ}{23} &\approx \frac{\sin H}{18} \\ \sin H &\approx \frac{18\sin 107^\circ}{23} \\ H &\approx 48^\circ \end{aligned}$$

Find the measure of $\angle J$.

$$\begin{aligned} m\angle J &\approx 180^\circ - (107^\circ + 48^\circ) \\ &\approx 25^\circ \end{aligned}$$

23. **EXPLORATION** Refer to the beginning of the lesson. Find the distance between the ship and the shipwreck. Round to the nearest tenth.



SOLUTION:
Use the Law of Cosines to find the missing side length.

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos A \\ a^2 &= 520^2 + 338^2 - 2(520)(338)\cos 70^\circ \\ a^2 &\approx 264417.0792 \\ a &\approx 514.215 \end{aligned}$$

The distance between the ship and the shipwreck is 514.2 meters.

24. **GEOMETRY** A parallelogram has side lengths 8 centimeters and 12 centimeters. One angle between them measures 42°. To the nearest tenth, what is the length of the shorter diagonal?

SOLUTION:
Use the Law of Cosines to find the shorter diagonal.

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos A \\ a^2 &= 8^2 + 12^2 - 2(8)(12)\cos 42^\circ \\ a^2 &\approx 65.3162 \\ a &\approx 8.1 \end{aligned}$$

The length of the shorter diagonal is 8.1 cm.

25. **RACING** A triangular cross-country course has side lengths 1.8 kilometers, 2 kilometers, and 1.2 kilometers. What are the angles formed between each pair of sides?

SOLUTION:

Let a = 2, b = 1.8 and c = 1.2.

Use the Law of Cosines to find the measure of the largest angle.

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos A \\ 2^2 &= 1.8^2 + 1.2^2 - 2(1.8)(1.2)\cos A \\ \frac{2^2 - 1.8^2 - 1.2^2}{-2(1.8)(1.2)} &= \cos A \\ 81^\circ &\approx A \end{aligned}$$

Use the Law of Sines to find the measure of $\angle B$.

$$\begin{aligned} \frac{\sin 81^\circ}{2} &\approx \frac{\sin B}{1.8} \\ \sin B &\approx \frac{1.8\sin 81^\circ}{2} \\ B &\approx 63^\circ \end{aligned}$$

Find the measure of $\angle C$.

$$\begin{aligned} m\angle C &\approx 180^\circ - (81^\circ + 63^\circ) \\ &\approx 36^\circ \end{aligned}$$

26. **CCSS MODELING** A triangular plot of farm land measures 0.9 by 0.5 by 1.25 miles.

a. If the plot of land is fenced on the border, what will be the angles at which the fences of the three sides meet? Round to the nearest degree.

b. What is the area of the plot of land?

SOLUTION:

a. Let a = 1.25, b = 0.9 and c = 0.5.

Use the Law of Cosines to find the measure of the largest angle.

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos A \\ 1.25^2 &= 0.9^2 + 0.5^2 - 2(0.9)(0.5)\cos A \\ \frac{1.25^2 - 0.9^2 - 0.5^2}{-2(0.9)(0.5)} &= \cos A \\ 124^\circ &\approx A \end{aligned}$$

Use the Law of Sines to find the measure of $\angle B$.

$$\begin{aligned} \frac{\sin 124^\circ}{1.25} &\approx \frac{\sin B}{0.9} \\ \sin B &\approx \frac{0.9\sin 124^\circ}{1.25} \\ B &\approx 37^\circ \end{aligned}$$

Find the measure of $\angle C$.

$$\begin{aligned} m\angle C &\approx 180^\circ - (124^\circ + 37^\circ) \\ &\approx 19^\circ \end{aligned}$$

b. Substitute b = 0.9, c = 0.5 and A = 124º in the area formula.

$$\begin{aligned} \text{Area} &= \frac{1}{2}bc\sin A \\ &= \frac{1}{2}(0.9)(0.5)\sin 124^\circ \\ &\approx 0.19 \text{ mi}^2 \end{aligned}$$

27. **LAND** Some land is in the shape of a triangle. The distances between each vertex of the triangle are 140 yd, 210 yd and 300 yd, respectively. Use the Law of Cosines to find the area of the land to the nearest square yard.

SOLUTION:

Use the Law of Cosines to find the measure of the largest angle.

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$300^2 = 210^2 + 140^2 - 2(210)(140)\cos A$$
$$\frac{300^2 - 210^2 - 140^2}{-2(210)(140)} = \cos A$$
$$-0.4473 \approx \cos A$$
$$117^\circ \approx A$$

Substitute b = 210, c = 140 and A = 117º in the area formula.

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} \text{ where } = (\frac{1}{2}(a+b+c)$$
$$= \sqrt{325(325-140)(325-210)(325-300)}$$
$$\approx 13{,}148 \text{ yd}^2$$

The area of the land is 13,148 yd^2.

28. **RIDES** Two bumper cars at an amusement park ride collide as shown

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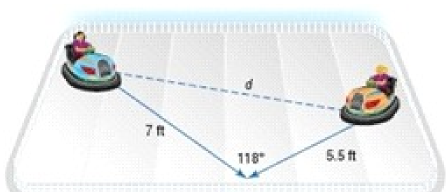


a. How far apart d were the two cars before they collided?

b. Before the collision, a third car was 10 feet from car 1 and 13 feet from car 2. Describe the angles formed by cars 1, 2, and 3 before the collision.

SOLUTION:

a. Let a = 5.5, b = 7, c = d and C = 118

Use the Law of Cosines to find the missing side length.

$$d^2 = 5.5^2 + 7^2 - 2(5.5)(7)\cos 118^\circ$$
$$d^2 \approx 115.399$$
$$d \approx 10.7$$

The distance between the two cars is about 10.7 ft.

b. Let a = 13, b = 10 and c = 10.7

Use the Law of Cosines to find the measure of the largest angle.

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$13^2 \approx 10^2 + 10.7^2 - 2(10)(10.7)\cos A$$
$$\frac{13^2 - 10^2 - 10.7^2}{-2(10)(10.7)} = \cos A$$
$$78^\circ \approx A$$

Use the Law of Sines to find the measure of $\angle B$.

$$\frac{\sin 78^\circ}{13} \approx \frac{\sin B}{10}$$
$$\sin B \approx \frac{10\sin 78^\circ}{13}$$
$$B \approx 49^\circ$$

Find the measure of $\angle C$.

$$m\angle C \approx 180^\circ - (78^\circ + 49^\circ)$$
$$\approx 53^\circ$$

29. **PICNICS** A triangular picnic area is 11 yards by 14 yards by 10 yards.

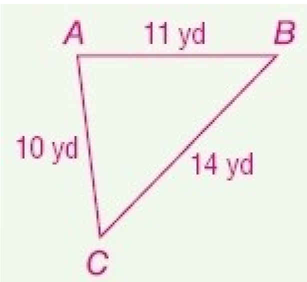
a. Sketch and label a drawing to represent the picnic area.

b. Describe how you could find the area of the picnic area.

c. What is the area? Round to the nearest tenth.

SOLUTION:

a.



b. Sample answer: Use the Law of Cosines to find the measure of $\angle A$. Then use the formula Area = $\frac{1}{2}bc\sin A$.

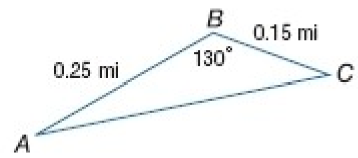
c. Use the Law of Cosines to find the measure of the largest angle.

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos A \\ 14^2 &\approx 10^2 + 11^2 - 2(10)(11)\cos A \\ \frac{14^2 - 10^2 - 11^2}{-2(10)(11)} &= \cos A \\ 0.1136 &\approx \cos A \\ 83 &\approx A \end{aligned}$$

Substitute b = 10, c = 11 and A = 83º in the area formula.

$$\begin{aligned} \text{Area} &= \frac{1}{2}bc\sin A \\ &= \frac{1}{2}(10)(11)\sin 83 \\ &\approx 54.6 \text{ yd}^2 \end{aligned}$$

30. **WATERSPORTS** A person on a personal watercraft makes a trip from point A to point B to point C traveling 28 miles per hour. She then returns from point C back to her starting point traveling 35 miles per hour. How many minutes did the entire trip take? Round to the nearest tenth.



SOLUTION:

Use the Law of Cosines to find the missing side length.

$$\begin{aligned} b^2 &= 0.15^2 + 0.25^2 - 2(0.15)(0.25)\cos 130^\circ \\ b^2 &\approx 0.1332 \\ b &\approx 0.36 \text{ mi} \end{aligned}$$

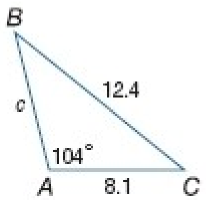
The time taken for the trip from point A to point B is $\frac{0.25}{28} \approx 0.0089$ per hour.

The time taken for the trip from point B to point C is $\frac{0.15}{28} \approx 0.0054$ per hour.

The time taken for the trip from point C to point A is $\frac{0.36}{35} \approx 0.0103$ per hour.

Time taken for the entire trip is 0.0246 hrs or 1.5 minutes.

Solve each triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

31. 

SOLUTION:

Use the Law of Sines to find a missing angle measure.

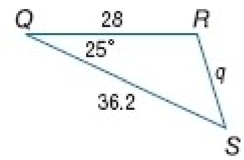
$$\frac{\sin 104^\circ}{12.4} = \frac{\sin B}{8.1}$$
$$\sin B = \frac{8.1 \sin 104^\circ}{12.4}$$
$$B \approx 39^\circ$$

Find the measure of $\angle C$.

$$m\angle C \approx 180^\circ - (104^\circ + 39^\circ)$$
$$\approx 37^\circ$$

Use the Law of Sines to find *c*.

$$\frac{\sin 104^\circ}{12.4} \approx \frac{\sin 37^\circ}{c}$$
$$c \approx \frac{12.4 \sin 37^\circ}{\sin 104^\circ}$$
$$c \approx 7.7$$

32. 

SOLUTION:

Use the Law of Cosines to find the missing side length.

$$q^2 = r^2 + s^2 - 2rs \cos Q$$
$$q^2 = 36.2^2 + 28^2 - 2(36.2)(28)\cos 25^\circ$$
$$q \approx 16.0$$

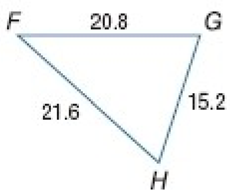
Use the Law of Sines to find the measure of angle $\angle R$.

$$\frac{\sin R}{36.2} \approx \frac{\sin 25^\circ}{16}$$
$$\sin R \approx \frac{36.2 \sin 25^\circ}{16}$$
$$R \approx 73^\circ$$

Since R is obtuse $R = 180 - 73^\circ = 107°$

Find the measure of $\angle S$.

$$m\angle S \approx 180^\circ - (107^\circ + 25^\circ) \text{ or } 48^\circ$$



33.

SOLUTION:

Use the Law of Cosines to find the measure of the largest angle, $\angle A$.

$$g^2 = h^2 + f^2 - 2hf\cos G$$
$$21.6^2 = 20.8^2 + 15.2^2 - 2(20.8)(15.2)\cos G$$
$$\frac{21.6^2 - 20.8^2 - 15.2^2}{-2(20.8)(15.2)} = \cos G$$
$$72^\circ \approx G$$

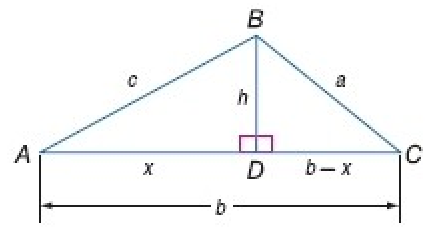
Use the Law of Sines to find the measure of angle $\angle H$.

$$\frac{\sin H}{20.8} \approx \frac{\sin 72^\circ}{21.6}$$
$$\sin H \approx \frac{20.8\sin 72^\circ}{21.6}$$
$$H \approx 66^\circ$$

Find the measure of $\angle F$.

$$m\angle F \approx 180^\circ - (72^\circ + 66^\circ)$$
$$\approx 42^\circ$$

34. **CHALLENGE** Use the figure and the Pythagorean Theorem to derive the Law of Cosines. Use the hints below.



- First, use the Pythagorean Theorem for ΔDBC.
- In ΔADB, $c^2 = x^2 + h^2$.
- $\cos A = \frac{x}{c}$

SOLUTION:

$a^2 = (b - x)^2 + h^2$ Use the Pythagorean Theorem for ΔDBC.

$= b^2 - 2bx + x^2 + h^2$ Expand $(b - x)^2$.

$= b^2 - 2bx + c^2$ In ΔADB, $c^2 = x^2 + h^2$.

$= b^2 - 2b(c \cos A) + c^2$ $\cos A = \frac{x}{c}$, so $x = c \cos A$.

$= b^2 + c^2 - 2bc \cos A$ Commutative Property

35. **CCSS ARGUMENTS** Three sides of a triangle measure 10.6 centimeters, 8 centimeters, and 14.5 centimeters. Explain how to find the measure of the largest angle. Then find the measure of the angle to the nearest degree.

SOLUTION:

The longest side is 14.5 centimeters. Use the Law of Cosines to find the measure of the angle opposite the longest side.

36. **OPEN ENDED** Create an application problem involving right triangles and the Law of Cosines. Then solve your problem, drawing diagrams if necessary.

SOLUTION:

See students' work.

37. **WRITING IN MATH** How do you know which method to use when solving a triangle?

SOLUTION:

Sample answer: To solve a right triangle, you can use the Pythagorean Theorem to find side lengths and trigonometric ratios to find angle measures and side lengths. To solve a nonright triangle, you can use the Law of Sines or the Law of Cosines, depending on what information is given. When two angles and a side are given or when two sides and an angle opposite one of the sides are given, you can use the Law of Sines. When two sides and an included angle or three sides are given, you can use the Law of Cosines.

38. **SAT/ACT** If *c* and *d* are different positive integers and $4c + d = 26$, what is the sum of all possible values of *c*?

A 6

B 10

C 15

D 21

E 28

SOLUTION:

$$4c + d = 26$$
$$d = 26 - 4c$$

Since the values of *c* and *d* are different positive integers, the values of *c* are 1, 2, 3, 4, 5 and 6. Therefore, the sum of all possible values of *c* is 1 + 2 + 3 + 4 + 5 + 6 or 21.

D is the correct option.

39. 4If $6^y = 21$, what is y?

F log 12 – log 6

G $\frac{\log 21}{\log 6}$

H $\frac{\log 6}{\log 21}$

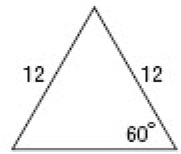
J $\log\left(\frac{6}{21}\right)$

SOLUTION:

$$\begin{aligned} 6^y &= 21 \\ \log 6^y &= \log 21 \\ y \log 6 &= \log 21 \\ y &= \frac{\log 21}{\log 6} \end{aligned}$$

G is the correct option.

40. **GEOMETRY** Find the perimeter of the figure.



A 24

B 30

C 36

D 48

SOLUTION:

Two sides of the triangle are equal, so the angle opposite to the side length 12 units is 60º.

Measure of the third angles is also 60º, so it is an equilateral triangle.

The side lengths of an equilateral triangle are equal.

Thus, the perimeter of the figure is 12 + 12 + 12 or 36 units.

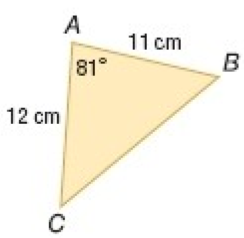
41. **SHORT RESPONSE** Solve the equation for x.

$$\frac{1}{x-1}+\frac{5}{8}=\frac{23}{6x}$$

SOLUTION:

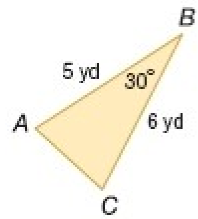
$$\begin{aligned}\frac{1}{x-1}+\frac{5}{8}&=\frac{23}{6x}\\ \frac{8+5(x-1)}{8(x-1)}&=\frac{23}{6x}\\ \frac{5x+3}{8x-8}&=\frac{23}{6x}\\ (5x+3)6x&=23(8x-8)\\ 30x^2+18x&=184x-184\\ 30x^2-166x+184&=0\\ x&=4 \text{ or } \frac{23}{15}\end{aligned}$$

Find the area of $\triangle ABC$ to the nearest tenth.

42. 

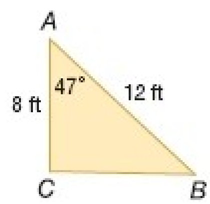
SOLUTION:
Substitute $c = 11$, $b = 12$ and $A = 81°$ in the area formula.

$$\begin{aligned}\text{Area}&=\frac{1}{2}bc\sin A\\ &=\frac{1}{2}(12)(11)\sin 81°\\ &\approx 65.2 \text{ cm}^2\end{aligned}$$

43. 

SOLUTION:
Substitute $c = 5$, $a = 6$ and $A = 30°$ in the area formula.

$$\begin{aligned}\text{Area}&=\frac{1}{2}ac\sin B\\ &=\frac{1}{2}(6)(5)\sin 30°\\ &\approx 7.5 \text{ yd}^2\end{aligned}$$

44. 

SOLUTION:
Substitute $b = 8$, $c = 12$ and $A = 47°$ in the area formula.

$$\begin{aligned}\text{Area}&=\frac{1}{2}bc\sin A\\ &=\frac{1}{2}(8)(12)\sin 47°\\ &\approx 35.1 \text{ ft}^2\end{aligned}$$

The terminal side of θ in standard position contains each point. Find the exact values of the six trigonometric functions of θ.

45. (8, 5)

SOLUTION:
Find the value of *r*.

$$r = \sqrt{x^2 + y^2}$$
$$= \sqrt{8^2 + 5^2}$$
$$= \sqrt{89}$$

Use $x = 8$, $y = 5$, and $r = \sqrt{89}$ to write the six trigonometric ratios.

$$\sin\theta = \frac{y}{r} = \frac{5}{\sqrt{89}} \text{ or } \frac{5\sqrt{89}}{89}$$
$$\cos\theta = \frac{x}{r} = \frac{8}{\sqrt{89}} \text{ or } \frac{8\sqrt{89}}{89}$$
$$\tan\theta = \frac{y}{x} = \frac{5}{8}$$
$$\csc\theta = \frac{r}{y} = \frac{\sqrt{89}}{5}$$
$$\sec\theta = \frac{r}{x} = \frac{\sqrt{89}}{8}$$
$$\cot\theta = \frac{x}{y} = \frac{8}{5}$$

46. (–4, –2)

SOLUTION:
Find the value of *r*.

$$r = \sqrt{x^2 + y^2}$$
$$= \sqrt{(-4)^2 + (-2)^2}$$
$$= 2\sqrt{5}$$

Use $x = -4$, $y = -2$, and $r = 2\sqrt{5}$ to write the six trigonometric ratios.

$$\sin\theta = \frac{y}{r} = \frac{-2}{2\sqrt{5}} \text{ or } -\frac{\sqrt{5}}{5}$$
$$\cos\theta = \frac{x}{r} = \frac{-4}{2\sqrt{5}} \text{ or } -\frac{2\sqrt{5}}{5}$$
$$\tan\theta = \frac{y}{x} = \frac{-2}{-4} \text{ or } 0.5$$
$$\csc\theta = \frac{r}{y} = \frac{2\sqrt{5}}{-2} \text{ or } -\sqrt{5}$$
$$\sec\theta = \frac{r}{x} = \frac{2\sqrt{5}}{-4} \text{ or } -\frac{\sqrt{5}}{2}$$
$$\cot\theta = \frac{x}{y} = \frac{-4}{-2} \text{ or } 2$$

47. (6, –9)

SOLUTION:

Find the value of r.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{6^2 + (-9)^2} \\ &= 3\sqrt{13} \end{aligned}$$

Use $x = 6$, $y = -9$, and $r = 3\sqrt{13}$ to write the six trigonometric ratios.

$$\sin\theta = \frac{y}{r} = \frac{-9}{3\sqrt{13}} \text{ or } -\frac{3\sqrt{13}}{13}$$

$$\cos\theta = \frac{x}{r} = \frac{6}{3\sqrt{13}} \text{ or } \frac{2\sqrt{13}}{13}$$

$$\tan\theta = \frac{y}{x} = \frac{-9}{6} \text{ or } -1.5$$

$$\csc\theta = \frac{r}{y} = -\frac{\sqrt{13}}{3}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{13}}{2}$$

$$\cot\theta = \frac{x}{y} = -\frac{6}{9} \text{ or } -\frac{2}{3}$$

48. **ATHLETIC SHOES** The prices for a random sample of athletic shoes are shown.

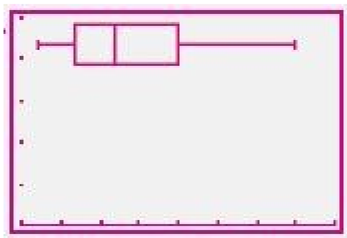
Price (dollars)				
70	300	400	250	250
150	120	250	100	70
150	160	200	170	300

a. Use a graphing calculator to create a box-and-whisker plot. Then describe the shape of the distribution.

b. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.

SOLUTION:

a.



[50, 450] scl: 50 by [0, 5] scl: 1

The distribution is positively skewed.

b. Sample answer: The distribution is positively skewed, so use the five-number summary. The range is \$70 to \$400. The median is \$190, and half of the data are between \$120 and \$250.

49. **BUSINESS** During the month of June, MediaWorld had revenue of \$2700 from sales of a certain DVD box set. During the July Blowout Sale, the set was on sale for \$10 off. Revenue from the set was \$3750 in July with 30 more sets sold than were sold in June. Find the price of the DVD set for June and the price for July.

SOLUTION:

Let p be the price in June and q the quantity sold in June. Represent the problem with a system on non-linear equations.

$$pq = 2700 \quad \text{June}$$
$$(p - 10)(q + 30) = 3750 \quad \text{July}$$

Solve the system.

$$q = \frac{2700}{p}$$

$$(p - 10)(q + 30) = 3750$$

$$30p + pq - 10q = 4050$$

$$30p + p\left(\frac{2700}{p}\right) - 10\left(\frac{2700}{p}\right) = 4050$$

$$30p + 2700 - \frac{27,000}{p} = 4050$$

$$30p - \frac{27,000}{p} = 1350$$

$$30p^2 - 27000 = 1350p$$

$$p^2 - 45p - 900 = 0$$

$$(p - 60)(p + 15) = 0$$

$p = 60$ or -15

The price in June was \$60. Therefore, the price in July was \$50.

Without writing the equation in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

50. $x^2 + y^2 - 8x - 6y + 5 = 0$

SOLUTION:

$A = 1$, $B = 0$, and $C = 1$

The discriminant is $(0)^2 - 4(1)(1)$ or -4.

Because the discriminant is less than 0 and $B = 0$ and $A = C$, the conic is a circle.

51. $3x^2 - 2y^2 + 32y - 134 = 0$

SOLUTION:

$A = 3$, $B = 0$, and $C = -2$

The discriminant is $(0)^2 - 4(3)(-2)$ or 24.

Because the discriminant is greater than 0, the conic is a hyperbola.

52. $y^2 + 18y - 2x = -84$

SOLUTION:

$A = 0$, $B = 0$, and $C = 1$

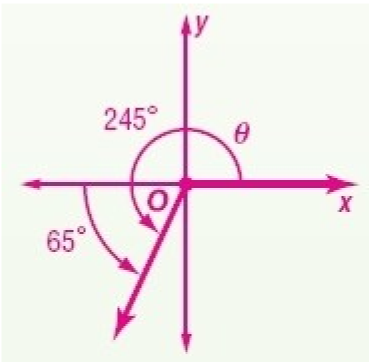
The discriminant is $(0)^2 - 4(0)(1)$ or 0.

Because the discriminant is equal to 0, the conic is a parabola.

Sketch each angle. Then find its reference angle.

53. 245°

SOLUTION:



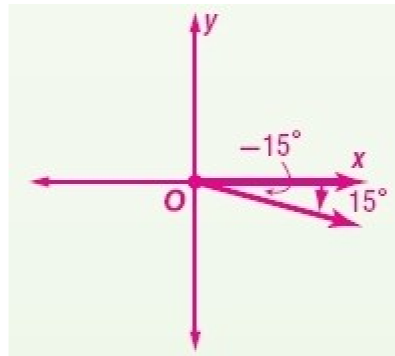
The terminal side of 245º lies in Quadrant III.

$$\begin{aligned}\theta' &= \theta - 180^\circ \\ &= 245^\circ - 180^\circ \\ &= 65^\circ\end{aligned}$$

54. –15°

SOLUTION:

coterminal angle: $-15^\circ + 360^\circ = 345^\circ$



The coterminal side of 345° lies in Quadrant IV.

$$\begin{aligned}\theta' &= 360^\circ - \theta \\ &= 360^\circ - 345^\circ \\ &= 15^\circ\end{aligned}$$

55. $\frac{5}{4}\pi$

SOLUTION:

The terminal side $\frac{5\pi}{4}$ lies in Quadrant III.

$$\begin{aligned}\theta' &= \theta - \pi \\ &= \frac{5\pi}{4} - \pi \\ &= \frac{\pi}{4}\end{aligned}$$