

12-6 Circular and Periodic Functions

CCSS STRUCTURE The terminal side of angle θ in standard position intersects the unit circle at each point P . Find $\cos \theta$ and $\sin \theta$.

1. $P\left(\frac{15}{17}, \frac{8}{17}\right)$

SOLUTION:

$$P\left(\frac{15}{17}, \frac{8}{17}\right) = P(\cos \theta, \sin \theta)$$

$$\cos \theta = \frac{15}{17} \quad \sin \theta = \frac{8}{17}$$

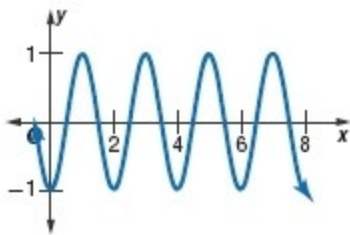
2. $P\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

SOLUTION:

$$P\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) = P(\cos \theta, \sin \theta)$$

$$\cos \theta = -\frac{\sqrt{2}}{2} \quad \sin \theta = \frac{\sqrt{2}}{2}$$

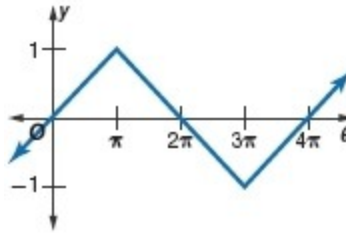
Determine the period of each function.



3.

SOLUTION:

The pattern repeats after 2, 4, 6, 8, ... units. So, the period of the function is 2.



4.

SOLUTION:

The pattern repeats after $4\pi, 8\pi, \dots$ units. So, the period of the function is 4π .

5. **SWINGS** The height of a swing varies periodically as the function of time. The swing goes forward and reaches its high point of 6 feet. It then goes backward and reaches 6 feet again. Its lowest point is 2 feet. The time it takes to swing from its high point to its low point is 1 second.

a. How long does it take for the swing to go forward and back one time?

b. Graph the height of the swing h as a function of time t .

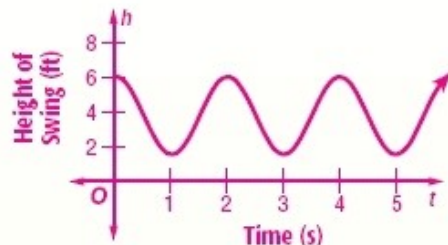
SOLUTION:

a. It takes 4 seconds for the swing to go forward and back one time.

b. Sample answer: Let the x -axis represents the time and the y -axis represents the height in feet.

The maximum point is 6 ft, and the minimum point is 2 ft.

It takes 1 second to reach from the maximum to the minimum. So, period is 2 seconds.



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Find the exact value of each function.

6. $\sin \frac{13\pi}{6}$

SOLUTION:

$$\begin{aligned}\sin \frac{13\pi}{6} &= \sin \left(\frac{\pi}{6} + \frac{12\pi}{6} \right) \\ &= \sin \frac{\pi}{6} \\ &= \frac{1}{2}\end{aligned}$$

7. $\sin (-60^\circ)$

SOLUTION:

$$\begin{aligned}\sin (-60^\circ) &= -\sin 60^\circ \\ &= -\frac{\sqrt{3}}{2}\end{aligned}$$

8. $\cos 540^\circ$

SOLUTION:

$$\begin{aligned}\cos 540^\circ &= \cos (360^\circ + 180^\circ) \\ &= \cos 180^\circ \\ &= -1\end{aligned}$$

The terminal side of angle θ in standard position intersects the unit circle at each point P . Find $\cos \theta$ and $\sin \theta$.

9. $P\left(\frac{6}{10}, -\frac{8}{10}\right)$

SOLUTION:

$$\begin{aligned}P\left(\frac{6}{10}, -\frac{8}{10}\right) &= P(\cos \theta, \sin \theta) \\ \cos \theta &= \frac{6}{10} \text{ or } \frac{3}{5} \\ \sin \theta &= -\frac{8}{10} \text{ or } -\frac{4}{5}\end{aligned}$$

10. $P\left(-\frac{10}{26}, -\frac{24}{26}\right)$

SOLUTION:

$$\begin{aligned}P\left(-\frac{10}{26}, -\frac{24}{26}\right) &= P(\cos \theta, \sin \theta) \\ \cos \theta &= -\frac{10}{26} \text{ or } -\frac{5}{13} \\ \sin \theta &= -\frac{24}{26} \text{ or } -\frac{12}{13}\end{aligned}$$

11. $P\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

SOLUTION:

$$\begin{aligned}P\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right) &= P(\cos \theta, \sin \theta) \\ \cos \theta &= \frac{\sqrt{3}}{2} \\ \sin \theta &= \frac{1}{2}\end{aligned}$$

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12. $P\left(\frac{\sqrt{6}}{5}, \frac{\sqrt{19}}{5}\right)$

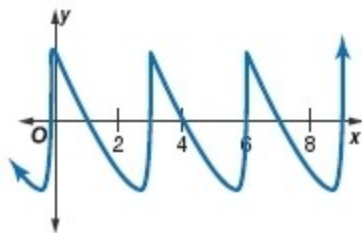
SOLUTION:

$$P\left(\frac{\sqrt{6}}{5}, \frac{\sqrt{19}}{5}\right) = P(\cos \theta, \sin \theta)$$

$$\cos \theta = \frac{\sqrt{6}}{5}$$

$$\sin \theta = \frac{\sqrt{19}}{5}$$

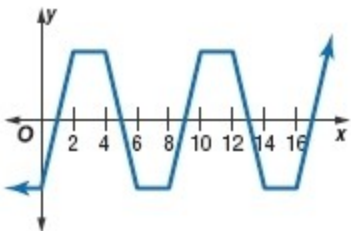
Determine the period of each function.



13.

SOLUTION:

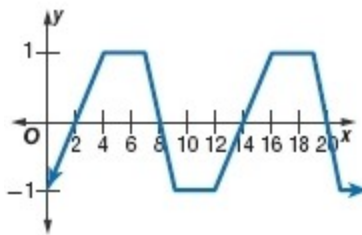
The pattern repeats every 3 units. So, the period of the given function is 3.



14.

SOLUTION:

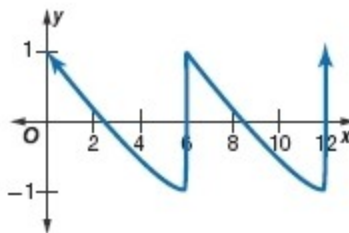
The pattern repeats every 8 units. So, the period of the given function is 8.



15.

SOLUTION:

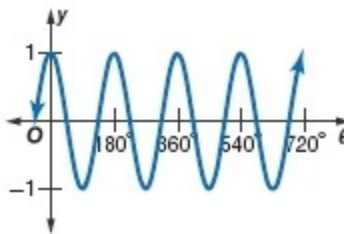
The pattern repeats every 12 units. So, the period of the given function is 12.



16.

SOLUTION:

The pattern repeats every 6 units. So, the period of the given function is 6.

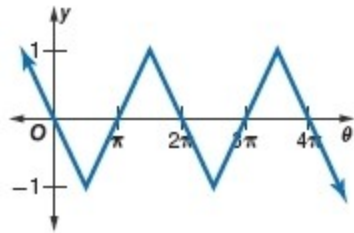


17.

SOLUTION:

The pattern repeats every 180° . So, the period of the given function is 180° .

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18.

SOLUTION:

The pattern repeats every 2π units. So the period of the given function is 2π .

19. **WEATHER** In a city, the average high temperature for each month is shown in the table.

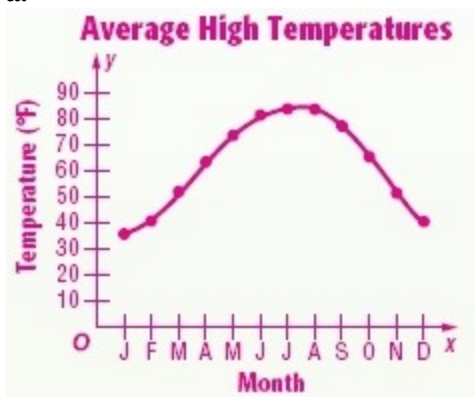
Average High Temperatures			
Month	Temperature (°F)	Month	Temperature (°F)
Jan.	36	July	85
Feb.	41	Aug.	84
Mar.	52	Sept.	78
Apr.	64	Oct.	66
May	74	Nov.	52
Jun.	82	Dec.	41

Source: The Weather Channel

- Sketch a graph of the function representing this situation.
- Describe the period of the function.

SOLUTION:

a.



- The period of the function is 12 months or 1 year.

Find the exact value of each function.

20. $\sin \frac{7\pi}{3}$

SOLUTION:

$$\begin{aligned} \sin \frac{7\pi}{3} &= \sin \left(\frac{\pi}{3} + \frac{6\pi}{3} \right) \\ &= \sin \frac{\pi}{3} \\ &= \frac{\sqrt{3}}{2} \end{aligned}$$

21. $\cos(-60^\circ)$

SOLUTION:

$$\begin{aligned} \cos(-60^\circ) &= \cos 60^\circ \\ &= \frac{1}{2} \end{aligned}$$

22. $\cos 450^\circ$

SOLUTION:

$$\begin{aligned} \cos 450^\circ &= \cos(90^\circ + 360^\circ) \\ &= \cos 90^\circ \\ &= 0 \end{aligned}$$

23. $\sin \frac{11\pi}{4}$

SOLUTION:

$$\begin{aligned} \sin \frac{11\pi}{4} &= \sin \left(\frac{3\pi}{4} + \frac{8\pi}{4} \right) \\ &= \sin \frac{3\pi}{4} \\ &= \frac{\sqrt{2}}{2} \end{aligned}$$

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24. $\sin(-45^\circ)$

SOLUTION:

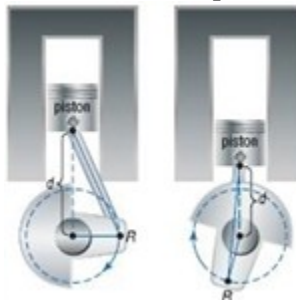
$$\begin{aligned}\sin(-45^\circ) &= -\sin 45^\circ \\ &= -\frac{\sqrt{2}}{2}\end{aligned}$$

25. $\cos 570^\circ$

SOLUTION:

$$\begin{aligned}\cos 570^\circ &= \cos(210^\circ + 360^\circ) \\ &= \cos 210^\circ \\ &= -\frac{\sqrt{3}}{2}\end{aligned}$$

26. **CCSS SENSE-MAKING** In the engine at the right, the distance d from the piston to the center of the circle, called the *crankshaft*, is a function of the speed of the piston rod. Point R on the piston rod rotates 150 times per second.

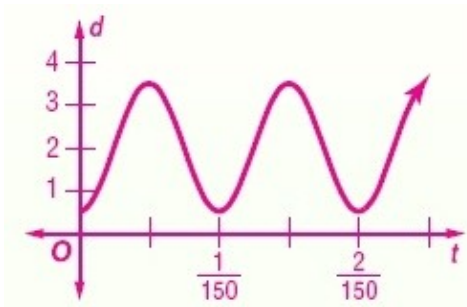


- a. Identify the period of the function as a fraction of a second.
- b. The shortest distance d is 0.5 inch, and the longest distance is 3.5 inches. Sketch a graph of the function. Let the horizontal axis represent the time t . Let the vertical axis represent the distance d .

SOLUTION:

- a. The period is the time it takes to complete one rotation. So, the period of the function as a fraction of a second is $\frac{1}{150}$.

- b. Sample answer:
The maximum distance d 3.5 inches, and the minimum distance d is 0.5 inch.



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27. **TORNADOES** A tornado siren makes 2.5 rotations per minute and the beam of sound has a radius of 1 mile. Ms. Miller's house is 1 mile from the siren. The distance of the sound beam from her house varies periodically as a function of time.

- Identify the period of the function in seconds.
- Sketch a graph of the function. Let the horizontal axis represent the time t from 0 seconds to 60 seconds. Let the vertical axis represent the distance d the sound beam is from Ms. Miller's house at time t .

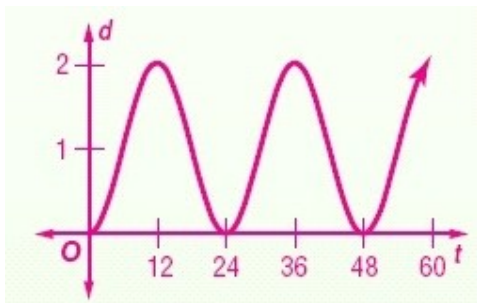
SOLUTION:

a. The period is the time it takes to complete one rotation.

So, the period of the function in seconds is

$$\frac{60}{2.5} \text{ or } 24 \text{ seconds.}$$

b. Sample answer:

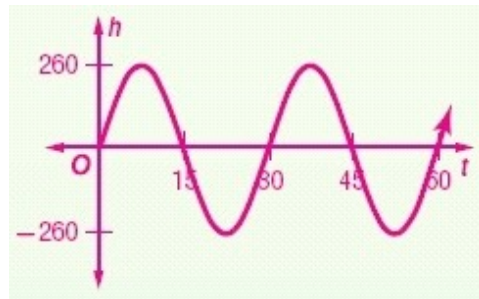


28. **FERRIS WHEEL** A Ferris wheel in China has a diameter of approximately 520 feet. The height of a compartment h is a function of time t . It takes about 30 seconds to make one complete revolution. Let the height at the center of the wheel represent the height at time 0. Sketch a graph of the function.

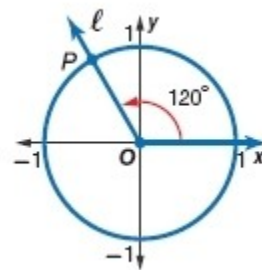
SOLUTION:

Sample answer:

Period of the function is 30 seconds.



29. **MULTIPLE REPRESENTATIONS** The terminal side of an angle in standard position intersects the unit circle at P , as shown in the figure.



a. **GEOMETRIC** Copy the figure. Draw lines representing 30° , 60° , 150° , 210° , and 315° .

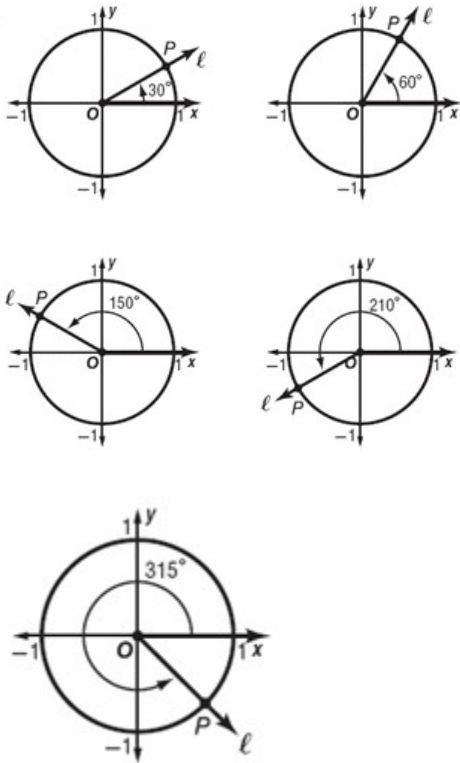
b. **TABULAR** Use a table of values to show the slope of each line to the nearest tenth.

c. **ANALYTICAL** What conclusions can you make about the relationship between the terminal side of the angle and the slope? Explain your reasoning.

SOLUTION:

a.

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b.

Angle	Slope
30	0.6
60	1.7
120	-1.7
150	-0.6
210	0.6
315	-1

c. Sample answer: The slope corresponds to the tangent of the angle. For $\theta = 120^\circ$, the x -coordinate of P is $-\frac{1}{2}$ and the y -coordinate is $\frac{\sqrt{3}}{2}$; slope = $\frac{\text{change in } y}{\text{change in } x}$. Since change in $x = -\frac{1}{2}$ and change in $y = \frac{\sqrt{3}}{2}$, slope = $\frac{\sqrt{3}}{2} \div \left(-\frac{1}{2}\right) = -\sqrt{3}$ or about -1.7 .

30. **POGO STICK** A person is jumping up and down on a pogo stick at a constant rate. The difference between his highest and lowest points is 2 feet. He jumps 50 times per minute.

a. Describe the independent variable and dependent variable of the periodic function that represents this situation. Then state the period of the function in seconds.

b. Sketch a graph of the jumper's change in height in relation to his starting point. Assume that his starting point is halfway between his highest and lowest points. Let the horizontal axis represent the time t in seconds. Let the vertical axis represent the height h .

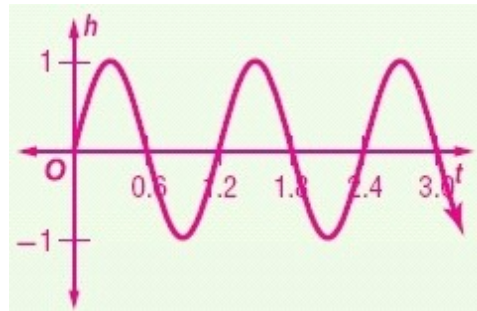
SOLUTION:

a. Sample answer: Independent variable: time t in seconds.

Dependent variable: height h in feet;

The period of the function is $\frac{60}{50}$ or 1.2 second.

b.



Find the exact value of each function.

31. $\cos 45^\circ - \cos 30^\circ$

SOLUTION:

$$\begin{aligned} \cos 45^\circ - \cos 30^\circ &= \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \\ &= \frac{\sqrt{2} - \sqrt{3}}{2} \end{aligned}$$

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32. $6(\sin 30^\circ)(\sin 60^\circ)$

SOLUTION:

$$\begin{aligned}6(\sin 30^\circ)(\sin 60^\circ) &= 6\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) \\ &= \frac{3\sqrt{3}}{2}\end{aligned}$$

33. $2\sin\frac{4\pi}{3} - 3\cos\frac{11\pi}{6}$

SOLUTION:

$$\begin{aligned}2\sin\frac{4\pi}{3} - 3\cos\frac{11\pi}{6} &= 2\sin\left(\pi + \frac{\pi}{3}\right) - 3\cos\left(2\pi - \frac{\pi}{6}\right) \\ &= -2\sin\frac{\pi}{3} - 3\cos\frac{\pi}{6} \\ &= -2\left(\frac{\sqrt{3}}{2}\right) - 3\left(\frac{\sqrt{3}}{2}\right) \\ &= \frac{-2\sqrt{3} - 3\sqrt{3}}{2} \\ &= -\frac{5\sqrt{3}}{2}\end{aligned}$$

34. $\cos\left(-\frac{2\pi}{3}\right) + \frac{1}{3}\sin 3\pi$

SOLUTION:

$$\begin{aligned}\cos\left(-\frac{2\pi}{3}\right) + \frac{1}{3}\sin 3\pi &= \cos\frac{2\pi}{3} + \frac{1}{3}\sin(\pi + 2\pi) \\ &= -\frac{1}{2} + \frac{1}{3}\sin\pi \\ &= -\frac{1}{2} + \frac{1}{3}(0) \\ &= -\frac{1}{2}\end{aligned}$$

35. $(\sin 45^\circ)^2 + (\cos 45^\circ)^2$

SOLUTION:

$$\begin{aligned}(\sin 45^\circ)^2 + (\cos 45^\circ)^2 &= \left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 \\ &= \frac{1}{2} + \frac{1}{2} \\ &= 1\end{aligned}$$

36. $\frac{(\cos 30^\circ)(\cos 150^\circ)}{\sin 315^\circ}$

SOLUTION:

$$\begin{aligned}\frac{(\cos 30^\circ)(\cos 150^\circ)}{\sin 315^\circ} &= \frac{(\cos 30^\circ)(\cos(180^\circ - 30^\circ))}{\sin(360^\circ - 45^\circ)} \\ &= \frac{\cos 30^\circ(-\cos 30^\circ)}{-\sin 45^\circ} \\ &= \frac{\left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right)}{-\frac{\sqrt{2}}{2}} \\ &= -\frac{3}{4} \div -\frac{\sqrt{2}}{2} \\ &= -\frac{3}{4} \times -\frac{2}{\sqrt{2}} \\ &= \frac{3}{2\sqrt{2}} \\ &= \frac{3\sqrt{2}}{4}\end{aligned}$$

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37. **CCSS CRITIQUE** Francis and Benita are finding the exact value of $\cos \frac{-\pi}{3}$. Is either of them correct? Explain your reasoning.

$$\begin{array}{l} \text{Francis} \\ \cos \frac{-\pi}{3} = -\cos \frac{\pi}{3} \\ = -0.5 \end{array}$$

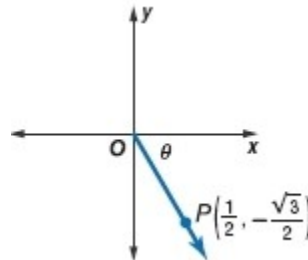
$$\begin{array}{l} \text{Benita} \\ \cos \frac{-\pi}{3} = \cos \left(-\frac{\pi}{3} + 2\pi \right) \\ = \cos \frac{5\pi}{3} \\ = 0.5 \end{array}$$

SOLUTION:

Benita is correct.

Francis incorrectly wrote $\cos \frac{-\pi}{3} = -\cos \frac{\pi}{3}$.

38. **CHALLENGE** A ray has its endpoint at the origin of the coordinate plane, and point $P\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ lies on the ray. Find the angle θ formed by the positive x -axis and the ray.



SOLUTION:

$$P\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right) = P(\cos \theta, \sin \theta)$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = 60^\circ$$

$$\sin \theta = -\frac{\sqrt{3}}{2}$$

$$\theta = -60^\circ$$

In Quadrant IV, cosines will have positive values and sines will have negative values. So, the angle θ is -60° .

39. **REASONING** Is the period of a sine curve *sometimes*, *always*, or *never* a multiple of π ? Justify your reasoning.

SOLUTION:

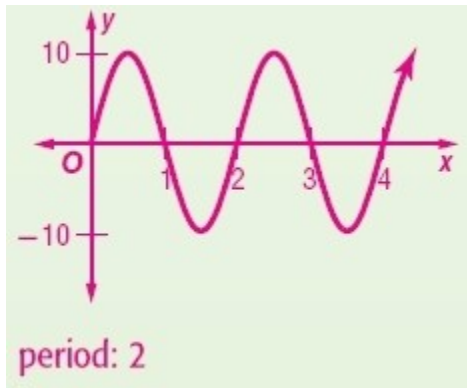
The period of a sine curve is sometimes a multiple of π . The period of a sine curve could be $\frac{\pi}{2}$, which is not a multiple of π .

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40. **OPEN ENDED** Draw the graph of a periodic function that has a maximum value of 10 and a minimum value of -10 . Describe the period of the function.

SOLUTION:

Sample answer:



41. **WRITING IN MATH** Explain how to determine the period of a periodic function from its graph. Include a description of a cycle.

SOLUTION:

The period of a periodic function is the horizontal distance of the part of the graph that is nonrepeating. Each nonrepeating part of the graph is one cycle.

42. **SHORT RESPONSE** Describe the translation of the graph of $f(x) = x^2$ to the graph of $g(x) = (x + 4)^2 - 3$.

SOLUTION:

Sample answer: Move the graph of $f(x)$ 4 units to the left and 3 units down to obtain the graph of $g(x)$.

43. The rate of population decline of Hampton Cove is modeled by $P(t) = 24,000e^{-0.0064t}$, where t is time in years from this year and 24,000 is the current population. In how many years will the population be 10,000?

A 14

B 104

C 137

D 375

SOLUTION:

Substitute 10,000 for $P(t)$ in the equation and solve for t .

$$\begin{aligned}P(t) &= 24000e^{-0.0064t} \\10000 &= 24000e^{-0.0064t} \\ \frac{10000}{24000} &= e^{-0.0064t} \\ \ln \frac{10000}{24000} &= \ln e^{-0.0064t} \\ &= -0.0064t \\ 137 &\approx t\end{aligned}$$

After about 137 years the population will be 10,000.

Option C is the correct answer.

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44. **SAT/ACT** If $d^2 + 8 = 21$, then $d^2 - 8 =$

F 0

G 5

H 13

J 31

K 161

SOLUTION:

$$d^2 + 8 = 21$$

$$d^2 = 13$$

Substitute 13 for d^2 .

$$\begin{aligned}d^2 - 8 &= 13 - 8 \\ &= 5\end{aligned}$$

So, G is the correct option.

45. **STATISTICS** If the average of three different positive integers is 65, what is the greatest possible value of one of the integers?

A 192

B 193

C 194

D 195

SOLUTION:

The sum of three different positive integers is 195. So, the greatest possible value of one of the integer is 192 ($195 - (1 + 2)$).

A is the correct option.

46. **GRIDDED RESPONSE** If $8xy + 3 = 3$, what is the value of xy ?

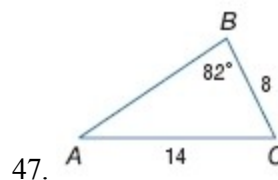
SOLUTION:

$$8xy + 3 = 3$$

$$8xy = 0$$

$$xy = 0$$

Solve each triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.



SOLUTION:

Use the Law of Sines to find the measure of angle $\angle A$.

$$\frac{\sin A}{8} = \frac{\sin 82^\circ}{14}$$

$$\sin A = \frac{8 \sin 82^\circ}{14}$$

$$A \approx 34^\circ$$

Find the measure of $\angle C$.

$$\begin{aligned}\angle C &\approx 180^\circ - (34^\circ + 82^\circ) \\ &\approx 64^\circ\end{aligned}$$

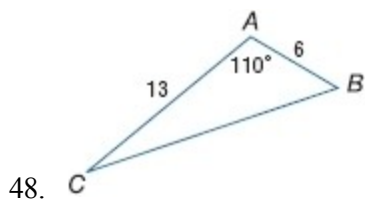
Use the Law of Sines to find c .

$$\frac{\sin 64^\circ}{c} \approx \frac{\sin 82^\circ}{14}$$

$$c \approx \frac{14 \sin 64^\circ}{\sin 82^\circ}$$

$$c \approx 12.7$$

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SOLUTION:

Use the Law of Cosines to find the missing side length.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = 13^2 + 6^2 - 2(13)(6)\cos 110^\circ$$

$$a \approx 16.1$$

Use the Law of Sines to find a missing angle measure.

$$\frac{\sin B}{13} \approx \frac{\sin 110^\circ}{16.1}$$

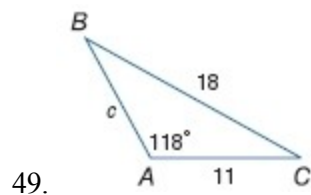
$$\sin B \approx \frac{13 \sin 110^\circ}{16.1}$$

$$B \approx 49^\circ$$

Find the measure of $\angle C$.

$$\angle C \approx 180^\circ - (49^\circ + 110^\circ)$$

$$\approx 21^\circ$$



SOLUTION:

Use the Law of Sines to find the measure of angle $\angle B$.

$$\frac{\sin B}{11} = \frac{\sin 118^\circ}{18}$$

$$\sin B = \frac{11 \sin 118^\circ}{18}$$

$$B \approx 33^\circ$$

Find the measure of $\angle C$.

$$\angle C \approx 180^\circ - (118^\circ + 33^\circ)$$

$$\approx 29^\circ$$

Use the Law of Sines to find c .

$$\frac{\sin 29^\circ}{c} \approx \frac{\sin 118^\circ}{18}$$

$$c \approx \frac{18 \sin 29^\circ}{\sin 118^\circ}$$

$$c \approx 9.9$$

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Determine whether each triangle has *no* solution, *one* solution, or *two* solutions. Then solve the triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

50. $A = 72^\circ$, $a = 6$, $b = 11$

SOLUTION:

Since $\angle A$ is acute and $a < b$, find h and compare it to a .

$$\begin{aligned}h &= b \sin A \\ &= 11 \sin 72^\circ \\ &\approx 10.5\end{aligned}$$

Since $6 < 10.5$ or $a < h$, there is no solution.

51. $A = 46^\circ$, $a = 10$, $b = 8$

SOLUTION:

Since $\angle A$ is acute and $a > b$, there is one solution.

Use the Law of Sines to find $m\angle B$.

$$\begin{aligned}\frac{\sin A}{a} &= \frac{\sin B}{b} \\ \frac{\sin 46^\circ}{10} &= \frac{\sin B}{8} \\ \sin B &= \frac{8 \sin 46^\circ}{10} \\ B &\approx 35^\circ\end{aligned}$$

Find the measure of $\angle C$.

$$\begin{aligned}\angle C &\approx 180^\circ - (46^\circ + 35^\circ) \\ &\approx 99^\circ\end{aligned}$$

Use the Law of Sines to find c .

$$\begin{aligned}\frac{\sin 46^\circ}{10} &\approx \frac{\sin 99^\circ}{c} \\ c &\approx \frac{10 \sin 99^\circ}{\sin 46^\circ} \\ c &\approx 13.7\end{aligned}$$

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52. $A = 110^\circ$, $a = 9$, $b = 5$

SOLUTION:

Because $\angle A$ is obtuse and $a > b$, one solution exists.

Use the Law of Sines to find $m\angle B$.

$$\begin{aligned}\frac{\sin A}{a} &= \frac{\sin B}{b} \\ \frac{\sin 110^\circ}{9} &= \frac{\sin B}{5} \\ \sin B &= \frac{5 \sin 110^\circ}{9} \\ B &\approx 31^\circ\end{aligned}$$

Find the measure of $\angle C$.

$$\begin{aligned}\angle C &\approx 180^\circ - (110^\circ + 31^\circ) \\ &\approx 39^\circ\end{aligned}$$

Use the Law of Sines to find c .

$$\begin{aligned}\frac{\sin A}{a} &= \frac{\sin C}{c} \\ \frac{\sin 110^\circ}{9} &= \frac{\sin 39^\circ}{c} \\ c &= \frac{9 \sin 39^\circ}{\sin 110^\circ} \\ c &\approx 6.0\end{aligned}$$

A binomial distribution has a 70% rate of success. There are 10 trials.

53. What is the probability that there will be 3 failures?

SOLUTION:

The probability of a success is 0.7.

The probability of a failure is $1 - 0.7$ or 0.3.

The probability of 3 failures is ${}_{10}C_3 (0.7)^3 (0.3)^7$ or about 0.267.

54. What is the probability that there will be at least 7 successes?

SOLUTION:

The probability that at least 7 successes is ${}_{10}C_7 (0.7)^7 (0.3)^3 + \dots + {}_{10}C_{10} (0.7)^{10} (0.3)^0$.

That is approximately 0.6496.

55. What is the expected number of successes?

SOLUTION:

Expected value of binomial distribution.

$$\begin{aligned}E(X) &= np \\ &= 10(0.7) \\ &= 7\end{aligned}$$

The expected number of success is 7.

12-6 Circular and Periodic Functions

Find the exact solution(s) of each system of equations.

$$58. \begin{aligned} y &= x + 2 \\ y &= x^2 \end{aligned}$$

SOLUTION:

Substitute x^2 for y in the equation $y = x + 2$ and solve for x .

$$\begin{aligned} x^2 &= x + 2 \\ x^2 - x - 2 &= 0 \\ (x - 2)(x + 1) &= 0 \\ x &= 2 \text{ or } -1 \end{aligned}$$

Substitute 2 and -1 for x in anyone of the equation and find the respective y values.

$$\begin{aligned} y &= x + 2 \\ y &= 2 + 2 \\ y &= 4 \end{aligned}$$

$$\begin{aligned} y &= x + 2 \\ y &= -1 + 2 \\ y &= 1 \end{aligned}$$

The solutions of the system are $(2, 4)$ and $(-1, 1)$.

$$59. \begin{aligned} 4x + y^2 &= 20 \\ 4x^2 + y^2 &= 100 \end{aligned}$$

SOLUTION:

Solve the system of equations. Subtract both the equations.

$$\begin{aligned} 4x^2 + y^2 - (4x + y^2) &= 100 - 20 \\ 4x^2 - 4x &= 80 \\ x^2 - x - 20 &= 0 \\ (x - 5)(x + 4) &= 0 \\ x &= 5 \text{ or } -4 \end{aligned}$$

Substitute 5 and -4 for x in anyone of the equation and find the respective y values.

$$\begin{aligned} 4x + y^2 &= 20 \\ 4(5) + y^2 &= 20 \\ 20 + y^2 &= 20 \\ y^2 &= 0 \\ y &= 0 \end{aligned}$$

$$\begin{aligned} 4x + y^2 &= 20 \\ 4(-4) + y^2 &= 20 \\ -16 + y^2 &= 20 \\ y^2 &= 36 \\ y &= \pm 6 \end{aligned}$$

The solutions are $(5, 0)$ and $(-4, \pm 6)$.

12-6 Circular and Periodic Functions

Simplify each expression.

$$60. \frac{240}{\left|1 - \frac{5}{4}\right|}$$

SOLUTION:

$$\begin{aligned}\frac{240}{\left|1 - \frac{5}{4}\right|} &= \frac{240}{\left|-\frac{1}{4}\right|} \\ &= \frac{240}{\frac{1}{4}} \\ &= 240 \times 4 \\ &= 960\end{aligned}$$

$$61. \frac{180}{\left|2 - \frac{1}{3}\right|}$$

SOLUTION:

$$\begin{aligned}\frac{180}{\left|2 - \frac{1}{3}\right|} &= \frac{180}{\left|\frac{5}{3}\right|} \\ &= \frac{180}{\frac{5}{3}} \\ &= 180 \times \frac{3}{5} \\ &= 108\end{aligned}$$

$$62. \frac{90}{\left|2 - \frac{11}{4}\right|}$$

SOLUTION:

$$\begin{aligned}\frac{90}{\left|2 - \frac{11}{4}\right|} &= \frac{90}{\left|-\frac{3}{4}\right|} \\ &= \frac{90}{\frac{3}{4}} \\ &= 90 \times \frac{4}{3} \\ &= 120\end{aligned}$$