12-9 Inverse Trigonometric Functions

Find each value. Write angle measures in degrees and radians.

1. $\text{Sin}^{-1}\frac{1}{2}$

SOLUTION:
Use a calculator.

Keystrokes: 2nd [SIN^{-1}] 1 ÷ 2) ENTER 30

Therefore, $\sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$ or $\frac{\pi}{6}$.

2. $\text{Arctan}\left(-\sqrt{3}\right)$

SOLUTION:
Use a calculator.

Keystrokes: 2nd [TAN^{-1}] (–) 2nd [$\sqrt{\ }$] 3)) ENTER –60

Therefore, $\arctan\left(-\sqrt{3}\right) = -60^\circ$ or $-\frac{\pi}{3}$.

3. Arccos(−1)

SOLUTION:
Use a calculator.

Keystrokes: 2nd [COS^{-1}] (–) 1) ENTER 180

Therefore, $\arccos(-1) = 180^\circ$ or π.

Find each value. Round to the nearest hundredth if necessary.

4. $\cos\left(\text{Arcsin}\frac{4}{5}\right)$

SOLUTION:
Use a calculator.

Keystrokes: COS 2nd [SIN^{-1}] 4 ÷ 5)) ENTER 0.6

Therefore, $\cos\left(\arcsin\frac{4}{5}\right) = 0.6$.

5. $\tan\left(\text{Cos}^{-1}1\right)$

SOLUTION:
Use a calculator.

Keystrokes: TAN 2nd [COS^{-1}] 1)) ENTER 0

Therefore, $\tan\left(\text{Cos}^{-1}1\right) = 0$.

6. $\sin\left(\text{Sin}^{-1}\frac{\sqrt{3}}{2}\right)$

SOLUTION:
Use a calculator.

Keystrokes: SIN 2nd [SIN^{-1}] 2nd [$\sqrt{\ }$] 3) ÷ 2)) ENTER 0.8660254038

Therefore, $\sin\left(\sin^{-1}\frac{\sqrt{3}}{2}\right) \approx 0.87$.

7. **MULTIPLE CHOICE** If Sin θ = 0.422, find θ.

A 25°

B 42°

C 48°

D 65°

SOLUTION:

$$\sin\theta = 0.422$$
$$\theta = \arcsin(0.422)$$

Use a calculator.

Keystrokes: 2nd [SIN^{-1}] 0 . 4 2 2) ENTER 24.96092039

Therefore, $\theta \approx 25^\circ$.

The option A is the correct option.

Solve each equation. Round to the nearest tenth if necessary.

8. Cos θ = 0.9

SOLUTION:

$$\text{Cos}\,\theta = 0.9$$
$$\theta = \text{Arccos}(0.9)$$

Use a calculator.

Keystrokes: 2nd [COS^{-1}] 0 . 9) ENTER 25.84193276

Therefore, $\theta \approx 25.8^\circ$.

9. Sin θ = −0.46

SOLUTION:

$$\text{Sin}\,\theta = -0.46$$
$$\theta = \text{Arcsin}(-0.46)$$

Use a calculator.

Keystrokes: 2nd [SIN^{-1}] (–) 0 . 4 6) ENTER –27.3871075

Therefore, $\theta \approx -27.4^\circ$.

10. Tan θ = 2.1

SOLUTION:

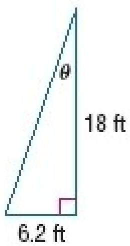
$$\text{Tan}\,\theta = 2.1$$
$$\theta = \text{Arctan}(2.1)$$

Use a calculator.

Keystrokes: 2nd [TAN^{-1}] 2 . 1) ENTER 64.53665494

Therefore, $\theta \approx 64.5^\circ$.

11. **SNOWBOARDING** A cross section of a superpipe for snowboarders is shown at the right. Write an inverse trigonometric function that can be used to find θ, the angle that describes the steepness of the superpipe. Then find the angle to the nearest degree.



SOLUTION:
Since the measures of the opposite side and the adjacent side are known, use the tangent function.

$$\text{Tan}\,\theta = \frac{6.2}{18}$$
$$\theta = \text{Arctan}\left(\frac{6.2}{18}\right)$$

Use a calculator.

Keystrokes: 2nd [TAN^{-1}] 6 . 2 ÷ 18) ENTER 19.0059842

So, the angle of the steepness of superpipe is 19°.

Find each value. Write angle measures in degrees and radians.

12. $\text{Arcsin}\left(\frac{\sqrt{3}}{2}\right)$

SOLUTION:
Use a calculator.

Keystrokes: 2nd [SIN^{-1}] 2nd [$\sqrt{\ }$] 3) ÷ 2) ENTER 60

Therefore, $\arcsin\left(\frac{\sqrt{3}}{2}\right) = 60^\circ$ or $\frac{\pi}{3}$.

13. $\text{Arccos}\left(\frac{\sqrt{3}}{2}\right)$

SOLUTION:
Use a calculator.

Keystrokes: 2nd [COS^{-1}] 2nd [$\sqrt{\ }$] 3) ÷ 2) ENTER 30

Therefore, $\arccos\left(\frac{\sqrt{3}}{2}\right) = 30^\circ$ or $\frac{\pi}{6}$.

14. $\text{Sin}^{-1}(-1)$

SOLUTION:
Use a calculator.

Keystrokes: 2nd [SIN^{-1}] (–) 1) ENTER –90

Therefore, $\sin^{-1}(-1) = -90^\circ$ or $-\frac{\pi}{2}$.

15. $\text{Tan}^{-1}\sqrt{3}$

SOLUTION:
Use a calculator.

Keystrokes: 2nd [TAN^{-1}] 2nd [$\sqrt{\ }$] 3)) ENTER 60

Therefore, $\tan^{-1}\sqrt{3} = 60^\circ$ or $\frac{\pi}{3}$.

16. $\text{Cos}^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

SOLUTION:
Use a calculator.

Keystrokes: 2nd [COS^{-1}] (–) 2nd [$\sqrt{\ }$] 3) ÷ 2) ENTER 150

Therefore, $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = 150^\circ$ or $\frac{5\pi}{6}$.

17. $\text{Arctan}\left(-\frac{\sqrt{3}}{3}\right)$

SOLUTION:
Use a calculator.

Keystrokes: 2nd [TAN^{-1}] (–) 2nd [$\sqrt{\ }$] 3) ÷ 3) ENTER –30

Therefore, $\arctan\left(-\frac{\sqrt{3}}{3}\right) = -30^\circ$ or $-\frac{\pi}{6}$.

Find each value. Round to the nearest hundredth if necessary.

18. tan (Cos^{-1} 1)

SOLUTION:
Use a calculator.

Keystrokes: TAN 2nd [COS^{-1}] 1)) ENTER 0

Therefore, $\tan\left(\text{Cos}^{-1}1\right) = 0$.

19. $\tan\left[\text{Arcsin}\left(-\frac{1}{2}\right)\right]$

SOLUTION:
Use a calculator.

Keystrokes: TAN 2nd [SIN^{-1}] (–) 1 ÷ 2)) ENTER –0.5773502692

Therefore, $\tan\left[\text{Arcsin}\left(-\frac{1}{2}\right)\right] \approx -0.58$.

20. $\cos\left(\text{Tan}^{-1}\frac{3}{5}\right)$

SOLUTION:
Use a calculator.

Keystrokes: COS 2nd [TAN^{-1}] 3 ÷ 5)) ENTER 0.8574929257

Therefore, $\cos\left(\text{Tan}^{-1}\frac{3}{5}\right) \approx 0.86$.

21. $\sin\left(\text{Arctan}\sqrt{3}\right)$

SOLUTION:
Use a calculator.

Keystrokes: SIN 2nd [TAN^{-1}] 2nd [√] 3))) ENTER 0.8660254038

Therefore, $\sin\left(\text{Arctan}\sqrt{3}\right) \approx 0.87$.

22. $\cos\left(\text{Sin}^{-1}\frac{4}{9}\right)$

SOLUTION:
Use a calculator.

Keystrokes: COS 2nd [SIN^{-1}] 4 ÷ 9)) ENTER 0.8958064165

Therefore, $\cos\left(\text{Sin}^{-1}\frac{4}{9}\right) \approx 0.90$.

23. $\sin\left[\text{Cos}^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right]$

SOLUTION:
Use a calculator.

Keystrokes: SIN 2nd [COS^{-1}] (–) 2nd [√] 2) ÷ 2)) ENTER 0.7071067812

Therefore, $\sin\left[\text{Cos}^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right] \approx 0.71$.

Solve each equation. Round to the nearest tenth if necessary.

24. Tan θ = 3.8

SOLUTION:

$$\text{Tan}\,\theta = 3.8$$
$$\theta = \text{Arctan}(3.8)$$

Use a calculator.

Keystrokes: 2nd [TAN^{-1}] 3 . 8) ENTER 75.25643716

Therefore, $\theta \approx 75.3^\circ$.

25. Sin θ = 0.9

SOLUTION:

$$\text{Sin}\,\theta = 0.9$$
$$\theta = \text{Arcsin}(0.9)$$

Use a calculator.

Keystrokes: 2nd [SIN^{-1}] 0 . 9) ENTER 64.15806724

Therefore, $\theta \approx 64.2^\circ$.

26. Sin θ = −2.5

SOLUTION:
Since the range of $\sin\theta$ is $\{y \mid -1 \le y \le 1\}$, $\sin\theta = -2.5$ has no solution.

27. Cos θ = −0.25

SOLUTION:

$$\cos\theta = -0.25$$
$$\theta = \text{Arccos}(-0.25)$$

Use a calculator.

Keystrokes: 2nd [COS^{-1}] (–) 0 . 2 5) ENTER 104.4775122

Therefore, $\theta \approx 104.5^\circ$.

28. Cos θ = 0.56

SOLUTION:

$$\cos\theta = 0.56$$
$$\theta = \text{Arccos}(0.56)$$

Use a calculator.

Keystrokes: 2nd [COS^{-1}] 0 . 5 6) ENTER 55.94420226

Therefore, $\theta \approx 55.9^\circ$.

29. Tan θ = −0.2

SOLUTION:

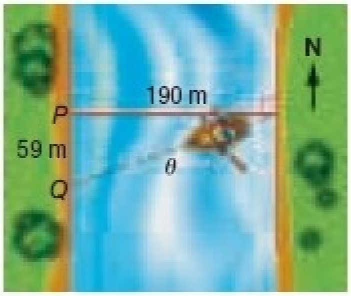
$$\tan\theta = -0.2$$
$$\theta = \text{Arctan}(-0.2)$$

Use a calculator.

Keystrokes: 2nd [TAN^{-1}] (–) 0 . 2) ENTER –11.30993247

Therefore, $\theta \approx -11.3^\circ$.

30. **CCSS SENSE-MAKING** A boat is traveling west to cross a river that is 190 meters wide. Because of the current, the boat lands at point *Q*, which is 59 meters from its original destination point *P*. Write an inverse trigonometric function that can be used to find θ, the angle at which the boat veered south of the horizontal line. Then find the measure of the angle to the nearest tenth.



SOLUTION:

Since the measures of the opposite side and the adjacent side are known, use the tangent function.

$$\tan\theta = \frac{59}{190}$$
$$\theta = \text{Arctan}\left(\frac{59}{190}\right)$$

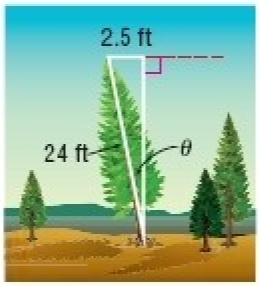
Use a calculator.

Keystrokes: 2nd [TAN^{-1}] 5 9 ÷ 1 9 0) ENTER 17.25094388

Therefore, $\theta \approx 17.3^\circ$.

So, the angle at which the boat veered south of the horizontal line is 17.3°.

31. **TREES** A 24-foot tree is leaning 2.5 feet left of vertical, as shown in the figure. Write an inverse trigonometric function that can be used to find θ, the angle at which the tree is leaning. Then find the measure of the angle to the nearest degree.



SOLUTION:
Since the measures of the opposite side and the hypotenuse are known, use the sine function.

$$\text{Sin}\,\theta = \frac{2.5}{24}$$

$$\theta = \text{Arcsin}\left(\frac{2.5}{24}\right)$$

Use a calculator.

Keystrokes: 2nd [SIN^{-1}] 2 . 5 ÷ 2 4) ENTER 5.979156796

Therefore, $\theta \approx 6^\circ$.

So, the angle at which the tree is leaning is about 6°.

32. **DRIVING** An expressway off-ramp curve has a radius of 52 meters and is designed for vehicles to safely travel at speeds up to 45 kilometers per hour (or 12.5 meters per second). The equation below represents the angle θ of the curve. What is the measure of the angle to the nearest degree?

$$\tan\theta = \frac{(12.5\text{m}/\text{s})^2}{(52\text{m})(9.8\text{m}/\text{s}^2)}$$

SOLUTION:
Use a calculator.

Keystrokes: 2nd [TAN^{-1}] 1 2 . 5 x^2 ÷ (5 2 × 9 . 8)) ENTER 17.04622194

Therefore, $\theta \approx 17^\circ$.

33. **TRACK AND FIELD** A shot-putter throws the shot with an initial speed of 15 meters per second. The expression $\frac{15\text{m/s}(\sin x)}{9.8\text{m/s}^2}$ represents the time in seconds at which the shot reached its maximum height. In the expression, *x* is the angle at which the shot was thrown. If the maximum height of the shot was reached in 1.0 second, at what angle was it thrown? Round to the nearest tenth.

SOLUTION:

The expression to find time, *t* at which the shot reached its maximum height is $t = \frac{15\text{m/s}(\sin x)}{9.8\text{m/s}^2}$.

Substitute 1.0 for *t* to find the angle, *x*.

$$1.0 = \frac{15(\sin x)}{9.8}$$
$$x = \sin^{-1}\left(\frac{9.8}{15}\right)$$

Use a calculator.

Keystrokes: 2nd [SIN^{-1}] 9 . 8 ÷ 1 5) ENTER 40.79339495

So, the angle at which the maximum height of the shot was thrown is 40.8°.

Solve each equation for $0 \le \theta \le 2\pi$.

34. csc θ = 1

SOLUTION:

$$\csc\theta = 1$$
$$\sin\theta = 1$$
$$\theta = \sin^{-1}(1)$$

Use a calculator.

Keystrokes: 2nd [SIN^{-1}] 1) ENTER 90

Therefore, $\theta = \frac{\pi}{2}$.

35. sec θ = −1

SOLUTION:

$$\sec\theta = -1$$
$$\cos\theta = -1$$
$$\theta = \cos^{-1}(-1)$$

Use a calculator.

Keystrokes: 2nd [COS^{-1}] (–) 1) ENTER 180

Therefore, $\theta = \pi$.

36. sec θ = 1

SOLUTION:

$$\sec\theta = 1$$
$$\cos\theta = 1$$
$$\theta = \cos^{-1}(1)$$

Use a calculator.

Keystrokes: 2nd [COS^{-1}] 1) ENTER 0

Therefore, $\theta = 0, 2\pi$.

37. $\csc\theta = \frac{1}{2}$

SOLUTION:

$$\csc\theta = \frac{1}{2}$$
$$\sin\theta = 2$$

Since the range of $\sin\theta$ is $\{y \mid -1 \le y \le 1\}$, $\csc\theta = \frac{1}{2}$ has no solution.

38. cot θ = 1

SOLUTION:

$$\cot\theta = 1$$
$$\tan\theta = 1$$
$$\theta = \tan^{-1}(1)$$

Use a calculator.

Keystrokes: 2nd [TAN^{-1}] 1) ENTER 45

Therefore, $\theta = \frac{\pi}{4}, \frac{5\pi}{4}$.

39. sec θ = 2

SOLUTION:

$$\sec\theta = 2$$
$$\cos\theta = \frac{1}{2}$$
$$\theta = \cos^{-1}\left(\frac{1}{2}\right)$$

Use a calculator.

Keystrokes: 2nd [COS^{-1}] 1 ÷ 2) ENTER 60

Therefore, $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$.

40. **MULTIPLE REPRESENTATIONS** Consider $y = \text{Cos}^{-1}\, x$.

a. GRAPHICAL Sketch a graph of the function. Describe the domain and the range.

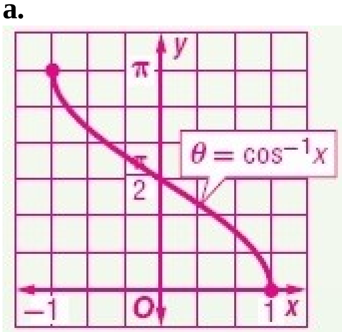
b. SYMBOLIC Write the function using different notation.

c. NUMERICAL Choose a value for x between −1 and 0. Then evaluate the inverse cosine function. Round to the nearest tenth.

d. ANALYTICAL Compare the graphs of $y = \cos x$ and $y = \text{Cos}^{-1}\, x$.

SOLUTION:

a.



domain: $-1 \le x \le 1$; range: $0 \le y \le \pi$

b. $y = \text{Arccos}\, x$

c. Sample answer: Let $x = -0.2$. To find the value of $y = \text{Cos}^{-1}(-0.2)$, use a calculator.

Keystrokes: 2nd [COS^{-1}] (–) . 2) ENTER 101.536959

Therefore, $y = 101.5°$.

d. Sample answer: The graph of $y = \cos x$ has a domain of all real numbers and a range from –1 to 1. The graph of $y = \text{Cos}^{-1} x$ has a domain from –1 to 1 and a range of 0 to $180°$.

41. **CHALLENGE** Determine whether cos (Arccos x) = x *for all values of* x is *true* or *false*. If false, give a counterexample.

SOLUTION:
false; $x = 2\pi$

42. **CCSS CRITIQUE** Desiree and Oscar are solving $\cos \theta = 0.3$ where $90 < \theta < 180$. Is either of them correct? Explain your reasoning.

SOLUTION:
Sample answer: Neither; cosine is not positive in the second quadrant.

43. **REASONING** Explain how the domain of $y = \text{Sin}^{-1}\, x$ is related to the range of $y = \text{Sin}\, x$.

SOLUTION:
The domain of $y = \text{Sin}^{-1} x$ is $-1 \le x \le 1$. This is the same as the range of $y = \text{Sin}\, x$.

44. **OPEN ENDED** Write an equation with an Arcsine function and an equation with a Sine function that both involve the same angle measure.

SOLUTION:
Sample answer:

$$\text{Arcsin}\frac{1}{2} = 30^\circ$$
$$\frac{1}{2} = \text{Sin}30^\circ$$

45. **WRITING IN MATH** Compare and contrast the relations $y = \tan^{-1} x$ and $y = \text{Tan}^{-1} x$. Include information about the domains and ranges.

SOLUTION:
Sample answer: $y = \tan^{-1}x$ is a relation that has a domain of all real numbers and a range of all real numbers except odd multiples of $\frac{\pi}{2}$.

The relation is not a function. $y = \text{Tan}^{-1}x$ is a function that has a domain of all real numbers and a range of $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.

46. **REASONING** Explain how $\text{Sin}^{-1}\ 8$ and $\text{Cos}^{-1}\ 8$ are undefined while $\text{Tan}^{-1}\ 8$ is defined.

SOLUTION:
The range of $y = \sin x$ and $y = \cos x$ is $-1 \le x \le 1$.
The range of $y = \text{Tan}^{-1}x$ is all real numbers.

47. Simplify $\dfrac{\frac{2}{x}+2}{\frac{2}{x}-2}$

A $\dfrac{1+x}{1-x}$

B $\dfrac{2}{x}$

C $\dfrac{1-x}{1+x}$

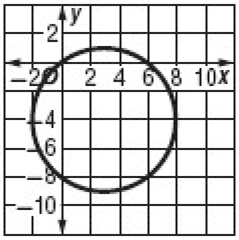
D $-x$

SOLUTION:

$$\frac{\frac{2}{x}+2}{\frac{2}{x}-2} = \frac{\frac{2+2x}{x}}{\frac{2-2x}{x}}$$
$$= \frac{2+2x}{\cancel{x}} \cdot \frac{\cancel{x}}{2-2x}$$
$$= \frac{\cancel{2}(1+x)}{\cancel{2}(1-x)}$$
$$= \frac{1+x}{1-x}$$

The option A is the correct option.

48. **SHORT RESPONSE** What is the equation of the graph?



SOLUTION:

The center of the circle is (3, –4) and the radius is 5 units.

The equation of the circle with center (h, k) and radius, r is $(x-h)^2+(y-k)^2=r^2$.

Substitute 3 for h, –4 for k and 5 for r.

$$(x-3)^2+(y-(-4))^2=5^2$$
$$(x-3)^2+(y+4)^2=25$$

49. If $f(x) = 2x^2 - 3x$ and $g(x) = 4 - 2x$, what is $g[f(x)]$?

F $g[f(x)] = 4 + 6x - 8x^2$

G $g[f(x)] = 4 + 6x - 4x^2$

H $g[f(x)] = 20 - 26x + 8x^2$

J $g[f(x)] = 44 - 38x + 8x^2$

SOLUTION:

$$\begin{aligned} g[f(x)] &= g(f(x)) \\ &= g(2x^2-3x) \\ &= 4-2(2x^2-3x) \\ &= 4-4x^2+6x \\ &= 4+6x-4x^2 \end{aligned}$$

The option G is the correct option.

50. If g is a positive number, which of the following is equal to $12g$?

A $\sqrt{144g}$

B $\sqrt{12g^2}$

C $\sqrt{24g^2}$

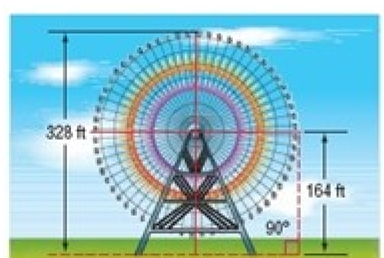
D $6\sqrt{4g^2}$

SOLUTION:

$$\begin{aligned} (12g)^2 &= 144g^2 \\ 12g &= \sqrt{144g^2} \\ &= \sqrt{36\times4\times g^2} \\ &= 6\sqrt{4g^2} \end{aligned}$$

The option D is the correct option.

51. **RIDES** The Cosmoclock 21 is a huge Ferris wheel in Japan. The diameter is 328 feet. Suppose a rider enters the ride at 0 feet, and then rotates in 90° increments counterclockwise. The table shows the angle measures of rotation and the height above the ground of the rider.



Angle	Height	Angle	Height
0°	0	450°	164
90°	164	540°	328
180°	328	630°	164
270°	164	720°	0
360°	0		

a. A function that models the data is $y=164\cdot\sin(x-90°)+164$. Identify the vertical shift, amplitude, period, and phase shift of the graph.

b. Write an equation using the sine that models the position of a rider on the Vienna Giant Ferris Wheel in Austria, with a diameter of 200 feet. Check your equation by plotting the points and the equation with a graphing calculator.

SOLUTION:

a. From the equation, we can find a = 164, b = 1, h = 90° and k = 164.

Vertical shift:
Since k = 164, the vertical shift is 164.

Amplitude:

$$|a| = |164| = 164$$

Period :

$$\frac{360^\circ}{|b|} = \frac{360^\circ}{|1|} = 360^\circ$$

Phase shift:

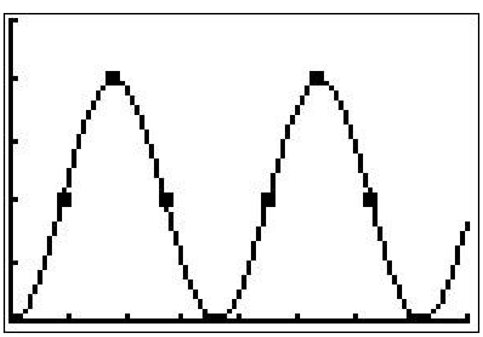
$$h = 90^\circ$$

b. Here, the diameter of the Giant Wheel is 200 feet. So the amplitude is 100 and the vertical shift is $k = 100$.

Substitute 100 for a, 1 for b, 90 for h, and 100 for k in $y = a \sin b(x-h)+k$.

$$y = 100\sin(x - 90^\circ) + 100$$

Check the equation by plotting the points and graphing the equation using graphing calc.



52. **TIDES** The world's record for the highest tide is held by the Minas Basin in Nova Scotia, Canada, with a tidal range of 54.6 feet. A tide is at equilibrium when it is at its normal level halfway between its highest and lowest points. Write an equation to represent the height h of the tide. Assume that the tide is at equilibrium at t = 0, that the high tide is beginning, and that the tide completes one cycle in 12 hours.

SOLUTION:

A tide is at equilibrium when it is at its normal level halfway between its highest and lowest points. As the midline lies halfway between the maximum and the minimum values, the equation of the midline is y = 0.

Therefore, there is no vertical shift.

Amplitude:

$$|a| = \left|\frac{54.6}{2} - 0\right| = 27.3$$

Period:
Since the tide completes one cycle in 12 hours, the period is 12.

$$12 = \frac{2\pi}{|b|}$$

$$|b| = \frac{2\pi}{12}$$

$$b = \pm\frac{\pi}{6}$$

Substitute 27.3 for a, $\frac{\pi}{6}$ for b, 0 for h, and 0 for k in $h = a \sin b(t-h)+k$.

$$h = 27.3 \sin \frac{\pi}{6}(t-0)+0$$

$$h = 27.3 \sin \frac{\pi}{6}t$$

12-9 Inverse Trigonometric Functions

Solve each equation.

53. $\log_3 5 + \log_3 x = \log_3 10$

SOLUTION:

$$\begin{aligned} \log_3 5 + \log_3 x &= \log_3 10 \\ \log_3(5x) &= \log_3 10 && \text{[Product property of logarithm]} \\ 5x &= 10 && \text{[Property of equality]} \\ x &= 2 \end{aligned}$$

54. $\log_4 a + \log_4 9 = \log_4 27$

SOLUTION:

$$\begin{aligned} \log_4 a + \log_4 9 &= \log_4 27 \\ \log_4(9a) &= \log_4 27 && \text{[Product property of logarithm]} \\ 9a &= 27 && \text{[Property of equality]} \\ a &= 3 \end{aligned}$$

55. $\log_{10} 16 - \log_{10} 2t = \log_{10} 2$

SOLUTION:

$$\begin{aligned} \log_{10} 16 - \log_{10} 2t &= \log_{10} 2 \\ \log_{10}\left(\frac{16}{2t}\right) &= \log_{10} 2 && \text{[Quotient property of logarithm]} \\ \frac{16}{2t} &= 2 && \text{[Property of equality]} \\ 4t &= 16 \\ t &= 4 \end{aligned}$$

56. $\log_7 24 - \log_7 (y + 5) = \log_3 8$

SOLUTION:

$$\begin{aligned} \log_7 24 - \log_7(y+5) &= \log_7 8 \\ \log_7\left(\frac{24}{y+5}\right) &= \log_7 8 && \text{[Quotient property of logarithm]} \\ \frac{24}{y+5} &= 8 && \text{[Property of equality]} \\ 24 &= 8(y+5) \\ 8y + 40 &= 24 \\ 8y &= -16 \\ y &= -2 \end{aligned}$$

Find the exact value of each trigonometric function.

57. $\cos 3\pi$

SOLUTION:

$$\begin{aligned} \cos 3\pi &= \cos(2\pi + \pi) \\ &= \cos \pi \end{aligned}$$

Since the angle π is a quadrant angle, the coordinates of the point on its terminal side is $(-x, 0)$.

Find the value of r.

$$\begin{aligned} r &= \sqrt{a^2 + b^2} \\ &= \sqrt{(-x)^2 + 0^2} \\ r &= x \end{aligned}$$

$$\begin{aligned} \cos 3\pi &= \cos \pi \\ &= \frac{x}{r} \\ &= \frac{-x}{x} \\ &= -1 \end{aligned}$$

58. $\tan 120°$

SOLUTION:

The terminal side of 120° lies in Quadrant II.

Find the measure of the reference angle.

$$\begin{aligned} \theta' &= 180° - \theta \\ &= 180° - 120° \\ &= 60° \end{aligned}$$

The tangent function is negative in quadrant II.

$$\begin{aligned} \tan 120° &= -\tan 60° \\ &= -\sqrt{3} \end{aligned}$$

59. sin 300°

SOLUTION:

The terminal side of 300° lies in Quadrant III.

Find the measure of the reference angle.

$$\begin{aligned}\theta' &= 360^\circ - \theta \\ &= 360^\circ - 300^\circ \\ &= 60^\circ\end{aligned}$$

The sine function is negative in quadrant III.

$$\begin{aligned}\sin 300^\circ &= -\sin 60^\circ \\ &= -\frac{\sqrt{3}}{2}\end{aligned}$$

60. $\sec \frac{7\pi}{6}$

SOLUTION:

The terminal side of $\frac{7\pi}{6}$ lies in Quadrant III.

Find the measure of the reference angle.

$$\begin{aligned}\theta' &= \frac{7\pi}{6} - \pi \\ &= \frac{\pi}{6}\end{aligned}$$

The secant function is negative in quadrant III.

$$\begin{aligned}\sec \frac{7\pi}{6} &= -\sec \frac{\pi}{6} \\ &= -\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \\ &= -\frac{2\sqrt{3}}{3}\end{aligned}$$