2-1 Relations and Functions
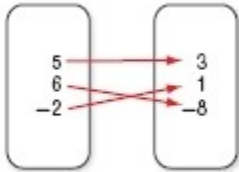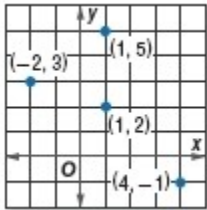
CCSS STRUCTURE **State the domain and range of each relation. Then determine whether each relation is a** *function* **. If it is a function, determine if it is** *one-to-one*, *onto*, *both*, **or** *neither*.

1.

ANSWER:
D = {−2, 5, 6}, R = {−8 1, 3}; function; both

2.

ANSWER:
D = {−2, 1, 4}, R = {−1, 2, 3, 5}; not a function

3.

ANSWER:
D = {−2, 1, 4, 8}, R = {−4, −2, 6}; function; onto

4. **BASKETBALL** The table shows the average points per game for Dwayne Wade of the Miami Heat for four years.

Season	Dwayne Wade's Age	Average Points Per Game
2005–2006	24	27.2
2006–2007	25	27.4
2007–2008	26	24.6
2008–2009	27	30.6

Source: Basketball-Reference

a. Assume that the ages are the domain. Identify the domain and range.
b. Write a relation of ordered pairs for the data.
c. State whether the relation is *discrete* or *continuous*.
d. Graph the relation. Is this relation a function?

ANSWER:
a. D = {24, 25, 26, 27}, R = {24.6, 27.2, 27.4, 30.2}
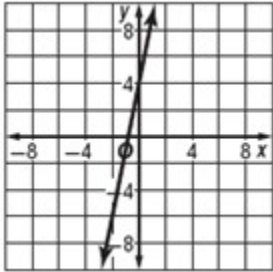**b.** {(24, 27.2), (25, 27.4), (26, 24.6), (27, 30.2)}
c. discrete
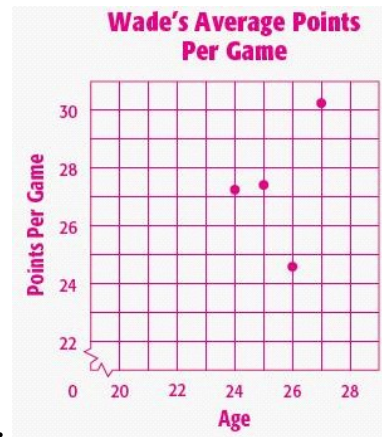
d.
yes

Graph each equation, and determine the domain and range. Determine whether the equation is a *function*, is *one-to-one, onto, both,* or *neither*. Then state whether it is *discrete* or *continuous*.
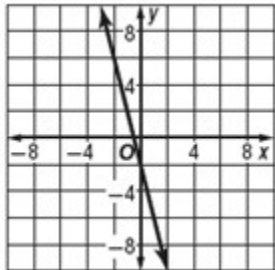
5. $y = 5x + 4$

ANSWER:

D = {all real numbers};
R = {all real numbers};
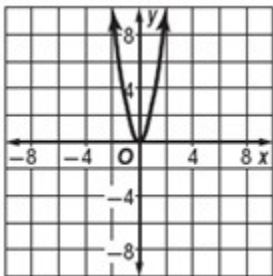function; both; continuous

6. $y = -4x - 2$

ANSWER:

D = {all real numbers};
R = {all real numbers};
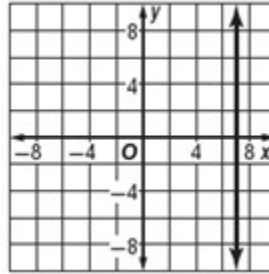function; both; continuous

7. $y = 3x^2$

ANSWER:

D = {all real numbers};
$R = \{y \mid y \geq 0\}$;
function; neither; continuous

8. $x = 7$

ANSWER:

D = {7}; R = {all real numbers};
not a function; not continuous

Evaluate each function.

9. $f(-3)$ if $f(x) = -4x - 8$

ANSWER:
4

10. $g(5)$ if $g(x) = -2x^2 - 4x + 1$

ANSWER:
−69

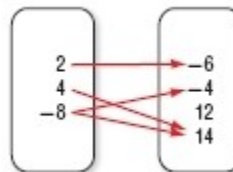**State the domain and range of each relation. Then determine whether each relation is a *function*. If it is a function, determine if it is *one-to-one, onto, both,* or *neither*.**

x	y
−0.3	−6
0.4	−3
1.2	−1
1.2	4

11.

ANSWER:
D = {−0.3, 0.4, 1.2}, R = {−6, −3, −1, 4}; not a function

12.

ANSWER:
D = {−8, 2, 4}; R = {−6, −4, 14}; not a function

2-1 Relations and Functions

13. {(−3, −4), (−1, 0), (3, 0), (5, 3)}

    ANSWER:
    D = {−3, −1, 3, 5}; R = {−4, 0, 3}; function; onto

14. **POLITICS** The table below shows the population of several states and the number of U.S. representatives from those states.

    a. Make a graph of the data with population on the horizontal axis and representatives on the vertical axis.

    b. Identify the domain and range.

    c. Is the relation *discrete* or *continuous*?

    d. Does the graph represent a function? Explain your reasoning.

State	Population (millions)	Number of Representatives
California	33.93	53
Florida	16.03	25
Illinois	12.44	19
New York	19.00	29
North Carolina	8.07	13
Texas	20.90	32

Source: U.S. Bureau of the Census
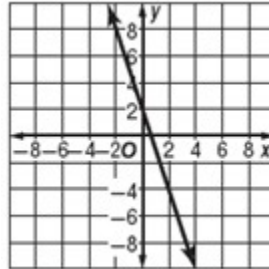
ANSWER:

a.
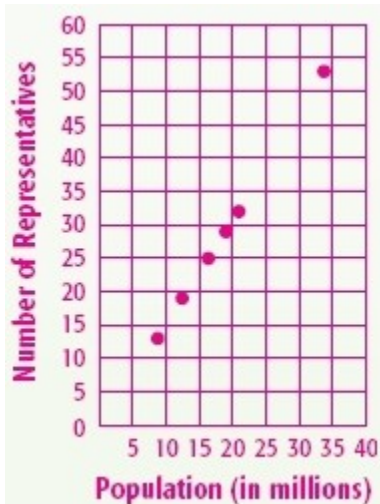
b. D = {8.07, 12.44, 16.03, 19.00, 20.90, 33.93}; R= {13, 19, 25, 29, 32, 53}

c. discrete

d. yes; each domain value is paired with only one range value so the relation is a function.

CCSS STRUCTURE Graph each equation, and determine the domain and range. Determine whether the equation is a *function,* is *one-to-one, onto, both,* or *neither*. Then state whether it is *discrete* or *continuous*.
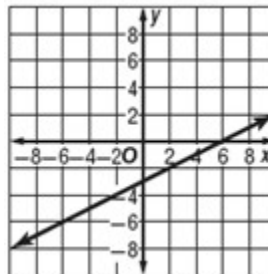
15. $y = -3x + 2$

    ANSWER:

    D = {all real numbers};
    R = {all real numbers};
    function; both; continuous

16. $y = 0.5x - 3$
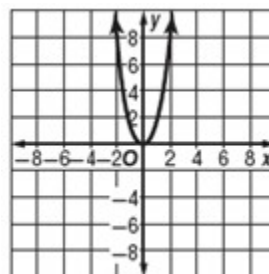
    ANSWER:

    D = {all real numbers};
    R = {all real numbers};
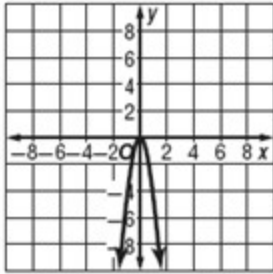   function; both; continuous

17. $y = 2x^2$

    ANSWER:

    D = {all real numbers};
    $R = \{y \mid y \geq 0\}$;
    function; neither; continuous

18. $y = -5x^2$

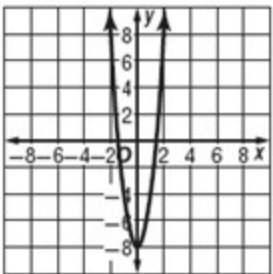   *ANSWER:*

    D = {all real numbers};
    $R = \{y \mid y \leq 0\}$;
    function; neither; continuous

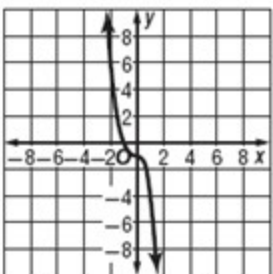19. $y = 4x^2 - 8$

    ANSWER:

    D = {all real numbers};
    $R = \{y \mid y \geq -8\}$;
    function; neither; continuous

20. $y = -3x^3 - 1$

    ANSWER:

    D = {all real numbers};
    R = {all real numbers};
    function; both; continuous

Evaluate each function.

21. $f(-8)$ if $f(x) = 5x^3 + 1$

    ANSWER:
    -2559

22. $f(2.5)$ if $f(x) = 16x^2$

    ANSWER:
    100

23. **DIVING** The table below shows the pressure on a diver at various depths.

Depth (ft)	0	20	40	60	80	100
Pressure (atm)	1	1.6	2.2	2.8	3.4	4

    a. Write a relation to represent the data.
    b. Graph the relation.
    c. Identify the domain and range. Is the relation *discrete* or *continuous*?
    d. Is the relation a function? Explain your reasoning.

    ANSWER:
    a. {(0,1), (20,1.6), (40,2.2), (60, 2.8), (80, 3.4), (100, 4)}
    b.
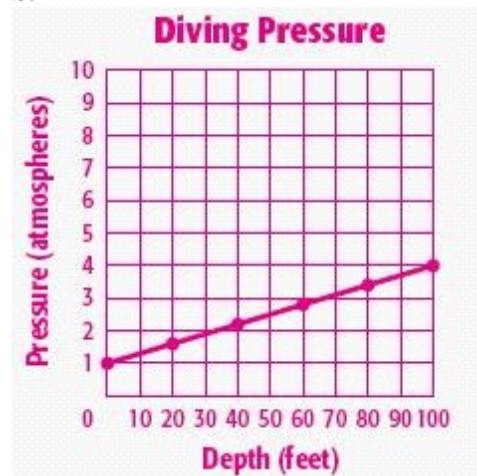
    c. D = $\{x \mid x \geq 0\}$; R= $\{y \mid y \geq 1\}$; continuous
    d. yes; each domain value is paired with only one range value so the relation is a function.

Find each value if
$$f(x) = 3x + 2, \ g(x) = -2x^2, \text{ and}$$
$$h(x) = -4x^2 - 2x + 5.$$

24. $f(-5)$

    ANSWER:
    –13

25. $f(9)$

    ANSWER:
    29

26. $g(-3)$

    ANSWER:
    –18

27. $g(-6)$

    ANSWER:
    –72

28. $h(3)$

    ANSWER:
    –37

29. $h(8)$

    ANSWER:
    –267

30. $f\left(\dfrac{2}{3}\right)$

    ANSWER:
    4

31. $g\left(\dfrac{3}{2}\right)$

    ANSWER:
    –4.5

32. $h\left(\dfrac{1}{5}\right)$

    ANSWER:
    $\dfrac{111}{25}$

33. **PODCASTS** Chaz has a collection of 15 podcasts downloaded on his digital audio player. He decides to download 3 more podcasts each month. The function $P(t) = 15 + 3t$ counts the number of podcasts $P(t)$ he has after t months. How many podcasts will he have after 8 months?

    ANSWER:
    39

34. **MULTIPLE REPRESENTATIONS** In this problem you will investigate one-to-one and onto functions.

    a. GRAPHICAL Graph each function on a separate graphing calculator screen.
    $$f(x) = x^2$$
    $$g(x) = 2^x$$
    $$h(x) = x^3 - 3x^2 - 5x + 6$$
    $$j(x) = x^3$$

    b. TABULAR Use the graphs to create a table showing the number of times a horizontal line could intersect the graph of each function. List all possibilities.

    c. ANALYTICAL For a function to be one-to-one, a horizontal line on the graph of the function can intersect the function at most once. Which functions meet this condition? Which do not? Explain your reasoning.
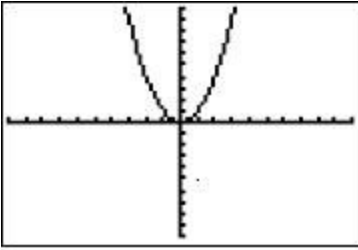
    d. ANALYTICAL For a function to be onto, every possible horizontal line on the graph of the function must intersect the function at least once. Which functions meet this condition? Which do not? Explain your reasoning.

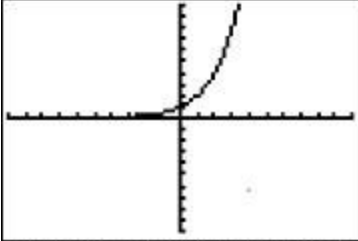   **e. GRAPHICAL** Create a table showing whether each function is one-to-one and/or onto.
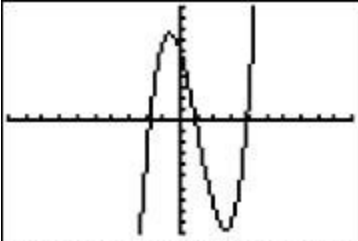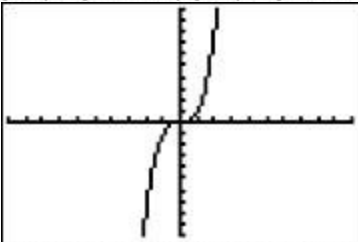
    ANSWER:

a.

[-10, 10] scl: 1 by [-10, 10] scl: 1

[-10, 10] scl: 1 by [-10, 10] scl: 1

[-10, 10] scl: 1 by [-10, 10] scl: 1

[-10, 10] scl: 1 by [-10, 10] scl: 1

b.

Function	Possible Intersection Points
$f(x) = x^2$	0, 1, 2
$g(x) = 2^x$	0, 1
$h(x) = x^3 - 3x^2 - 5x + 6$	1, 2, 3
$j(x) = x^3$	1

c. $g(x)$ and $j(x)$ are one-to-one, and $f(x)$ and $h(x)$ are not.

d. $h(x)$ and $j(x)$ are onto, and $f(x)$ and $g(x)$ are not.

e.

Function	One-to-one	Onto
$f(x) = x^2$	no	no
$g(x) = 2^x$	yes	no
$h(x) = x^3 - 3x^2 - 5x + 6$	no	yes
$j(x) = x^3$	yes	yes

35. **CCSS CRITIQUE** Omar and Madison are finding f (3d) for the function $f(x) = -4x^2 - 2x + 1$. Is either of them correct? Explain your reasoning.

Omar	Madison
$f(3d) = -4(3d)^2 - 2(3d) + 1$	$f(3d) = -4(3d)^2 - 2(3d) + 1$
$= -4(9d^2) - 6d + 1$	$= 12d^2 - 6d + 1$
$= -36d^2 - 6d + 1$	

ANSWER:
Sample answer: Omar; Madison did not square the 3 before multiplying by –4.

36. **CHALLENGE** Consider the functions $f(x)$ and $g(x) \cdot f(a) = 19$ and $g(a) = 33$, while $f(b) = 31$ and $g(b) = 51$. If $a = 5$ and $b = 8$, find two possible functions to represent $f(x)$ and $g(x)$.

ANSWER:
Sample answer: $f(x) = 4x - 1$; $g(x) = 6x + 3$

37. **REASONING** If the graph of a relation crosses the y-axis at more than one point, is the relation *sometimes, always,* or *never* a function? Explain your reasoning.

ANSWER:
Never; if the graph crosses the y-axis twice, then there will be two separate y-values that correspond to $x = 0$, which violates the vertical line test.
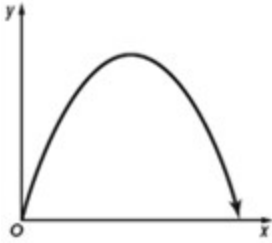
38. **OPEN ENDED** Graph a relation that can be used to represent each of the following.
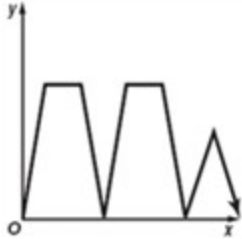   **a.** the height of a baseball that is hit into the outfield
    b. the speed of a car that travels to the store, stopping at two lights along the way
    c. the height of a person from age 5 to age 80
    d. the temperature on a typical day from 6 A.M. to 11 P.M.

    ANSWER:
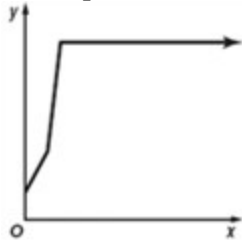   **a**. Sample answer:
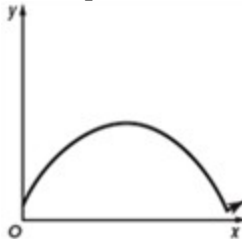

    b. Sample answer:

    c. Sample answer:

    d. Sample answer:

39. **REASONING** Determine whether the following statement is *true* or *false*. Explain your reasoning.
    If a function is onto, then it must be one-to-one as well.

    ANSWER:
    Sample answer: False; a function is onto and not one-to-one if all of the elements of the domain correspond to an element of the range, but more than one element of the domain corresponds to the same element of the range.

40. **WRITING IN MATH** Explain why the vertical line test can determine if a relation is a function.

    ANSWER:
    Sample answer: A relation is a function if each x-value only pairs with one y-value. If the vertical line test fails then there is an x-value that pairs with more than one y-value, so the relation is not a function.

41. Patricia's swimming pool contains 19,500 gallons of water. She drains the pool at a rate of 6 gallons per minute. Which of these equations represents the number of gallons of water g, remaining in the pool after m minutes?

    A $g = 19,500 - 6m$

    B $g = 19,500 + 6m$

    C $g = \dfrac{19,500}{6m}$

    D $g = \dfrac{6m}{19,500}$

    ANSWER:
    A

42. **SHORT RESPONSE** Look at the pattern below.
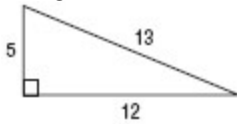 $$-\frac{5}{2}, -2, -\frac{3}{2}, -1, \ldots$$
    If the pattern continues, what will the next term be?

    ANSWER:
    $-\dfrac{1}{2}$

43. **GEOMETRY** Which set of dimensions represents a triangle similar to the triangle shown below?

    F 1 unit, 2 units, 3 units
    G 7 units, 11 units, 12 units
    H 10 units, 23 units, 24 units
    J 20 units, 48 units, 52 units

ANSWER:
J

44. **ACT/SAT** If $g(x) = x^2$, which expression is equal to $g(x + 1)$?
    A. 1
    B. $x^2 + 1$
    C. $x^2 + 2x + 1$
    D. $x^2 - x$
    E. $x^2 + x + 1$

ANSWER:
C

Solve each inequality.

45. $48 > 7y + 6 > 20$

ANSWER:
$6 > y > 2$

46. $z + 12 > 18 \text{ or } -2z + 16 > 12$

ANSWER:
$z > 6 \text{ or } z < 2$

47. $2|4x + 2| + 3 > 21$

ANSWER:
$x > \dfrac{7}{4} \text{ or } x < -\dfrac{11}{4}$

48. **CLUBS** Mr. Willis is starting a chess club at his high school. He sent the advertisement at the right to all of the homerooms. Write an absolute value inequality representing the situation.

ANSWER:
$|x - 14| \le 8$

49. **SALES** Ling can spend no more than \$120 at the summer sale of a department store. She wants to buy shirts on sale for \$15 each. Write and solve an inequality to determine the number of shirts she can buy.

ANSWER:
$15x \le 120$; She can buy up to 8 shirts.

Solve each equation. Check your solutions.

50. $18 = 2|2a + 6| - 2$

ANSWER:
2 or –8

51. $2 = -3|4c - 5| + 8$

ANSWER:
$\dfrac{3}{4} \text{ or } \dfrac{7}{4}$

52. $-5 = 2|3b + 4| - 9$

ANSWER:
$-2 \text{ or } -\dfrac{2}{3}$

Simplify each expression.

53. $6(3a - 2b) + 3(5a + 4b)$

ANSWER:
33a

54. $-4(5x-3y)+2(y+3x)$

    ANSWER:

    $-14x + 14y$

55. $-7(2c-4d)+8(3c+d)$

    ANSWER:

    $10c + 36d$

Solve each equation. Check your solutions.

56. $5x+2=32$

    ANSWER:

    6

57. $6a-3=21$

    ANSWER:

    4

58. $-2x+5=5x+19$

    ANSWER:

    -2

59. $6b+4=-2b-28$

    ANSWER:

    -4

60. $2(x+5)-3(x-4)=19$

    ANSWER:

    3

61. $4(2y-3)+5(3y+1)=-99$

    ANSWER:

    -4

62. $5c-8+2c=4c+10$

    ANSWER:

    6

63. $8d-4+3d=2d-100-7d$

    ANSWER:

    -6

64. $10y-5-3y=4(2y+3)-20$

    ANSWER:

    3