2-3 Rate of Change and Slope

CCSS REGULARITY Find the rate of change for each set of data.

1.

Time (min)	2	4	6	8	10
Distance (ft)	12	24	36	48	60

ANSWER:

6 feet/min

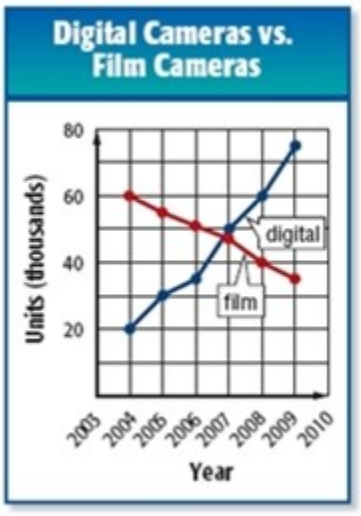
2.

Time (sec)	5	10	15	20	25
Volume (cm^3)	16	32	48	64	80

ANSWER:

3.2 cm^3/sec

3.



CAMERAS The graph shows the number of digital still cameras and film cameras sold by Yellow Camera Stores in recent years.

a. Find the average rate of change of the number of digital cameras sold from 2004 to 2009.

b. Find the average rate of change of the number of film cameras sold from 2004 to 2009.

c. What do the signs of each rate of change represent?

ANSWER:

a. about 11,000 per year
b. about –5000 per year
c. The positive rate in part a represents an increase in sales of digital cameras. The negative rate in part **b** represents a decrease in sales of film cameras.

Find the slope of the line that passes through each pair of points.

4. (3, 2), (8, 12)

ANSWER:

2

5. (–1, 4), (3, –8)

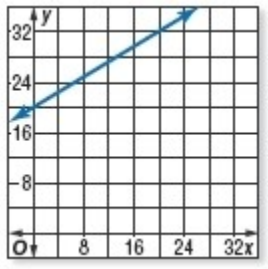
ANSWER:

–3

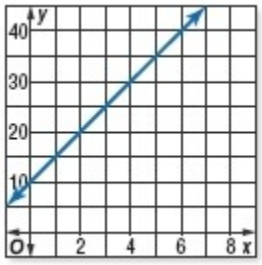
6. (–2, –5), (–7, 10)

ANSWER:
–3

Determine the rate of change of each graph.

7. 

ANSWER:
$\frac{3}{5}$

8. 

ANSWER:
5

Find the rate of change for each set of data.

9.

Time (day)	3	6	9	12	15
Height (mm)	20	40	60	80	100

ANSWER:
$\frac{20}{3}$ mm/day

10.

Weight (lb)	11	22	33	44	55
Cost ($)	8	16	24	32	40

ANSWER:
$\frac{8}{11}$ $/lb

11. **HEALTH** The table below shows Lisa's temperature during an illness over a 3-day period.

Day	Monday		Tuesday		Wednesday	
Time	8:00 A.M.	8:00 P.M.	8:00 A.M.	8:00 P.M.	8:00 A.M.	8:00 P.M.
Temp (°F)	100.5	102.3	103.1	100.7	99.9	98.6

a. What was the average rate of change in Lisa's temperature from 8:00 A.M. on Monday to 8:00 P.M. on Monday?

b. What was the average rate of change in Lisa's temperature from 8:00 A.M. on Tuesday to 8:00 P.M. on Wednesday? Is your answer reasonable? What does the sign of the rate mean?

c. During which 12-hour period was the average rate of change in Lisa's temperature the greatest?

ANSWER:
a. 0.15°/h
b. –0.125°/h; Yes; the number should be negative because her temperature is dropping.
c. Monday 8:00 A.M.-Monday 8:00 P.M.

Find the slope of the line that passes through each pair of points. Express as a fraction in simplest form.

12. (–2, 11), (5, 6)

ANSWER:
$-\frac{5}{7}$

13. (–9, –11), (6, 3)

ANSWER:
$\frac{14}{15}$

14. (–1.5, 3.5), (4.5, 6)

ANSWER:
$\frac{5}{12}$

15. (–4.5, 9.5), (–1, 2.5)

ANSWER:
–2

16. (–8, –0.5), (–4, 5)

ANSWER:

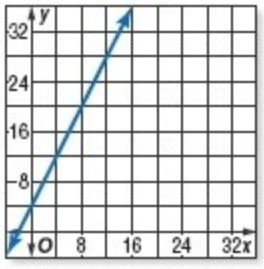
$\frac{11}{8}$

17. (–6, –2), (–1.5, 5.5)

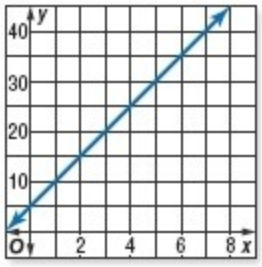
ANSWER:

$\frac{5}{3}$

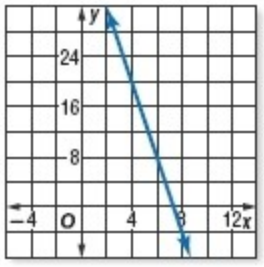
Determine the rate of change of each graph.

18. 

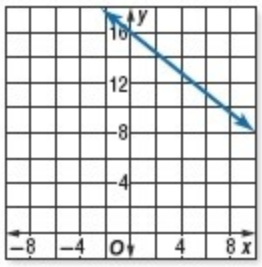
ANSWER:
2

19. 

ANSWER:
5

20. 

ANSWER:
–6

21. 

ANSWER:
–0.8

22. **CCSS REASONING** The table shows your height on a water slide at various time intervals.

Time (s)	Height (ft)
0	120
1	90
2	60
3	30
4	0
5	0

a. Graph the height versus the time on the water slide.

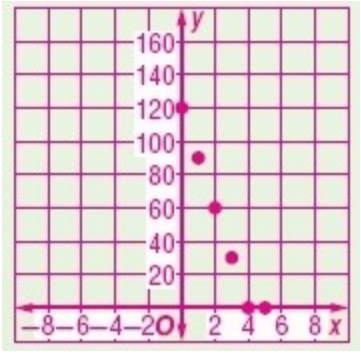
b. Find the average rate of change of a rider between 1 and 3 seconds.

c. Find the average rate of change of a rider between 0 and 5 seconds.

d. What is another word for *rate of change* in this situation?

ANSWER:

a.



b. –30 ft/s
c. –24 ft/s
d. speed or velocity

2-3 Rate of Change and Slope

Determine the rate of change for each equation.

23. $6y = 8x - 40$

ANSWER:

$\frac{4}{3}$

24. $-2y - 16x = 41$

ANSWER:

–8

25. $12x - 4y + 5 = 18$

ANSWER:

3

26. $20x + 85y = 120$

ANSWER:

$-\frac{4}{17}$

27. $\frac{3}{2}x - \frac{5}{4}y = 15$

ANSWER:

$\frac{6}{5}$

28. $\frac{1}{6}y + \frac{3}{8}x = 24$

ANSWER:

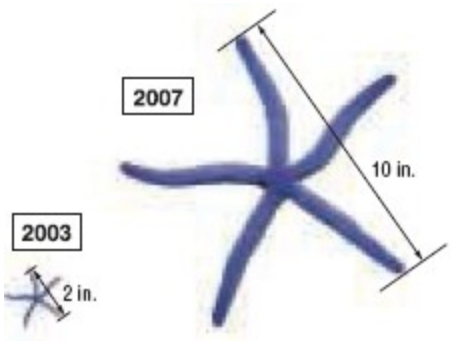
$-\frac{9}{4}$

29. **WASHINGTON MONUMENT** The Washington Monument is 555 feet $5\frac{1}{8}$ inches tall and weighs 90,854 tons. The monument is topped by an aluminum square pyramid. The sides of the pyramid's base measure 5.6 inches, and the pyramid is 8.9 inches tall. Estimate the slope that a face of the pyramid makes with its base.

ANSWER:

about 3.2

30. **MARINE LIFE** The illustrations show the growth of a starfish over time.



a. Find the average rate of change in the measure over time.

b. Predict the size of the starfish in 2009.

ANSWER:

a. 2 in./yr
b. 14 in.

Find the value of r so that the line that passes through each pair of points has the given slope.

31. (6, *r*), (3, 3), $m = 2$

ANSWER:

9

32. (8,1), (5, *r*), $m = \frac{1}{3}$

ANSWER:

0

33. (10, *r*), (4, –3), $m = \frac{4}{3}$

ANSWER:

5

34. (8, –2), (*r*, –6), $m = -4$

ANSWER:

9

35. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the rate of change for the function $f(x) = x^2$.

a. GRAPHICAL Graph $f(x) = x^2$.

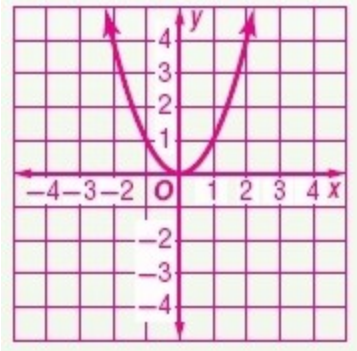
b. TABULAR Complete the table.

x	−4	−3	−2	−1	0	1	2	3	4
$f(x)$	16	9							
slope		−7							

c. VERBAL Describe what happens to the rate of change for $f(x) = x^2$ as x increases.

ANSWER:

a.

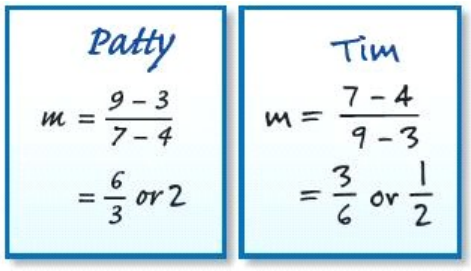


b.

x	−4	−3	−2	−1	0	1	2	3	4
$f(x)$	16	9	4	1	0	1	4	9	16
slope		−7	−5	−3	−1	1	3	5	7

c. Sample answer: The rate of change is not constant. The rate of change increases as x approaches infinity.

36. **CCSS CRITIQUE** Patty and Tim are asked to find the slope of the line passing through the points (4, 3) and (7, 9). Is either of them correct? Explain.



ANSWER:

Patty; Tim calculated slope as the ratio of the change in x to the change in y.

37. **CHALLENGE** The graph of a line passes through the points (2, 3) and (5, 8). Explain how you would find the y-coordinate of the point $(11, y)$ on the same line. Then find y.

ANSWER:

Sample answer: Because the slope from (2, 3) to (5, 8) is the same as the slope from (5, 8) to $(11, y)$, find the slope between each pair of points and set them equal to each other. Then solve for y.

$$\frac{8-3}{5-2} = \frac{y-8}{11-5}$$
$$\frac{5}{3} = \frac{y-8}{6}$$
$$30 = 3(y-8)$$
$$10 = y-8$$
$$18 = y$$

38. **OPEN ENDED** Write an example of a function with a rate of change four times as large as its x-intercept.

ANSWER:

Sample answer: $y = 12x - 36$

39. **REASONING** Determine whether the statement *A line has a slope that is a real number is sometimes, always,* or *never* true. Explain your reasoning.

ANSWER:

Sometimes; the slope of a vertical line is undefined.

40. **WRITING IN MATH** Describe the process of finding the rate of change for each.

a. a table of values

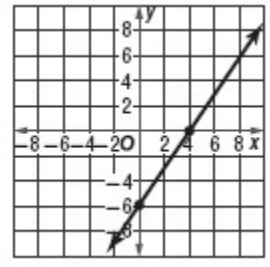
b. a graph

c. an equation

ANSWER:

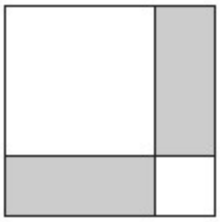
a. Sample answer: Pick two sets of values in the table and determine the difference of the outputs (y) and the difference of the inputs (x). Divide the difference of the outputs by the difference of the inputs. This is the rate of change. Follow this process with the rest of the data to confirm the rate of change is constant and, thus, the function is linear.
b. Sample answer: Pick two separate coordinates that fall on the graph of the straight line. Determine the difference of the outputs (y) and the difference of the inputs (x). Divide the difference of the outputs by the difference of the inputs. This is the rate of change.
c. Sample answer: Pick two random x-values and substitute them into the equation to determine the corresponding y-values. Determine the difference of the outputs (y) and the difference of the inputs (x). Divide the difference of the outputs by the difference of the inputs. This is the rate of change.

41. **GRIDDED RESPONSE** What is the slope of the line shown in the graph?



ANSWER:
3/2 or 1.5

42. **ACT/SAT** In the figure below, the large square contains two smaller squares. If the areas of the two smaller squares are 4 and 25, what is the sum of the perimeters of the two shaded rectangles?



A 14

B 20

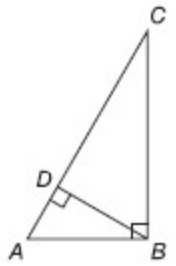
C 24

D 28

E 49

ANSWER:
D

43. **GEOMETRY** In ΔABC shown, $AC = 16$ and $m\angle DAB = 60$. What is the measure of $\overline{BD}$?



F $9\sqrt{2}$

G 9

H $4\sqrt{3}$

J 4

ANSWER:
H

44. The table shows the cost of bananas depending on the amount purchased. Which conclusion can be made based on information in the table?

Cost of Bananas	
Number of Pounds	Cost ($)
5	1.45
20	4.60
50	10.50
100	19.00

A The cost of 10 pounds of bananas would be more than $4.

B The cost of 200 pounds of bananas would be at most $38
.

C The cost of bananas is always more than $0.20 per pound.

D The cost of bananas is always less than $0.28 per pound.

ANSWER:
B

State whether each equation or function is a linear function. Write *yes* or *no*. Explain.

45. $6y - 8x = 19$

ANSWER:
Yes; it can be written in $f(x) = mx + b$ form.

46. $4x^2 = 2y - 9$

ANSWER:
No; it cannot be written in $f(x) = mx + b$ form.

47. $18 = 2xy + 6$

ANSWER:
No; it cannot be written in $f(x) = mx + b$ form.

Evaluate each function.

48. $f(-9)$ if $f(x) = -7x + 8$

ANSWER:
71

49. $g(-4)$ if $g(x) = -3x^2 + 2$

ANSWER:
–46

50. $h(12)$ if $h(x) = 4x^2 - 10x$

ANSWER:
456

51. **RACING** There are 8 contestants in a 400-meter race. In how many different ways can the top three runners finish?

ANSWER:
336

Determine the quadrant of the coordinate plane where each point is located.

52. (–4, –8)

ANSWER:
III

53. (–2, 6)

ANSWER:
II

54. (3, –1)

ANSWER:
IV

Solve each equation.

55. $8 = 4m - 6$

ANSWER:
3.5

56. $-6 = 3(8) + b$

ANSWER:
–30

57. $-2 = -3x + 5$

ANSWER:
$\frac{7}{3}$