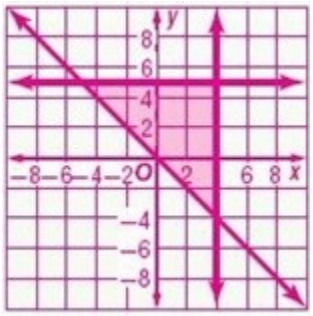
3-3 Optimization with Linear Programming

Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.

1. $y \le 5$
$x \le 4$
$y \ge -x$
$f(x, y) = 5x - 2y$

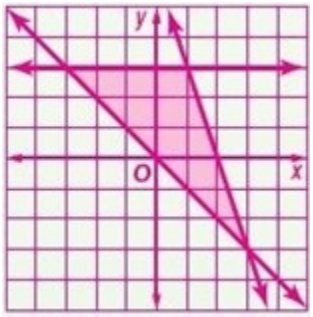
ANSWER:



$(4,5), (4,-4), (-5,5)$; max $= 28$, min $= -35$

2. $y \le -3x + 6$
$-y \le x$
$y \le 3$
$f(x, y) = 8x + 4y$

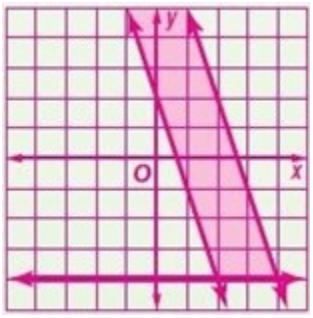
ANSWER:



(1, 3), (3, –3), (–3, 3); max = 20, min = –12

3. $y \ge -3x + 2$
$9x + 3y \le 24$
$y \ge -4$
$f(x, y) = 2x + 14y$

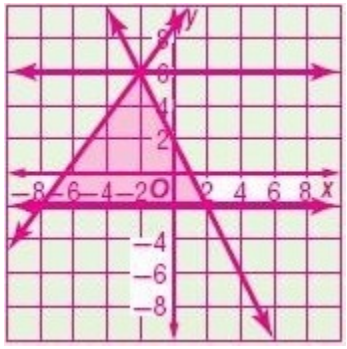
ANSWER:



(2, –4), (4, –4); max does not exist, min = –52

4. $-2 \le y \le 6$
$3y \le 4x + 26$
$y \le -2x + 2$
$f(x, y) = -3x - 6y$

ANSWER:

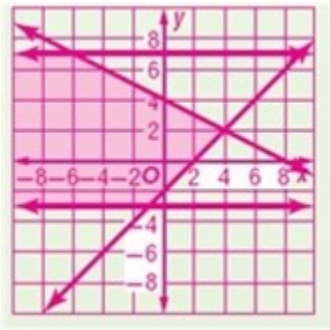


(2, –2), (–8, –2), (–2, 6); max = 36 min = –30

3-3 Optimization with Linear Programming

5. $-3 \le y \le 7$
$4y \ge 4x - 8$
$6y + 3x \le 12$
$f(x, y) = -12x + 9y$

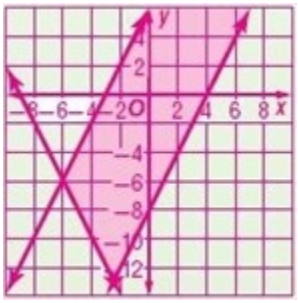
ANSWER:



(4, 2), (–1, –3), (–6, 7); max does not exist; min = –26

6. $y \le 2x + 6$
$y \ge 2x - 8$
$y \ge -2x - 18$
$f(x, y) = 5x - 4y$

ANSWER:



(–6, –6), (–2.5, –13), no min, max = 39.5

7. **CCSS PRECISION** The total number of workers' hours per day available for production in a skateboard factory is 85 hours. There are 40 hours available for finishing decks and quality control each day. The table shows the number of hours needed in each department for two different types of skateboards.

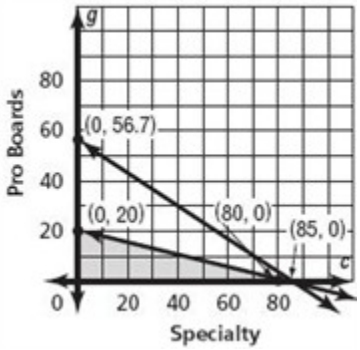
Skateboard Manufacturing Time		
Board Type	Production Time	Deck Finishing/Quality control
Pro Boards	1.5 hours	2 hours
Specialty Boards	1 hour	0.5 hour

a. Write a system of inequalities to represent the situation.
b. Draw the graph showing the feasible region.
c. List the coordinates of the vertices of the feasible region.
d. If the profit on a pro board is $50 and the profit on a specialty board is $65, write a function for the total profit on the skateboards.
e. Determine the number of each type of skateboard that needs to be made to have a maximum profit. What is the maximum profit?

ANSWER:

a.
$g \ge 0$
$c \ge 0$
$1.5g + c \le 85$
$2g + 0.5c \le 40$

b.

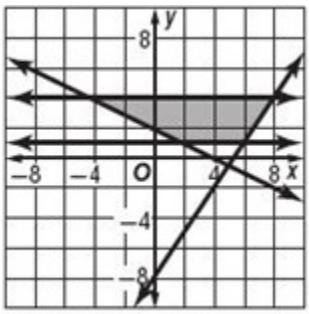


c. (0, 0), (0, 20), (80, 0)
d. $f(c, g) = 65c + 50g$
e. 80 specialty boards, 0 pro boards; $5200

Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.

8. $1 \le y \le 4$
$4y - 6x \ge -32$
$2y \ge -x + 4$
$f(x, y) = -6x + 3y$

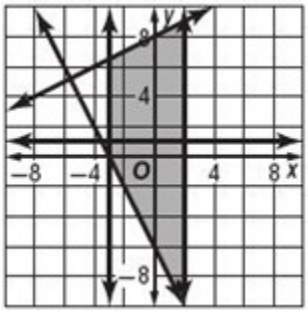
ANSWER:



(8, 4), (6, 1), (2, 1), (–4, 4); max = 36, min = –36

9. $2 \ge x \ge -3$
$y \ge -2x - 6$
$4y \le 2x + 32$
$f(x, y) = -4x - 9y$

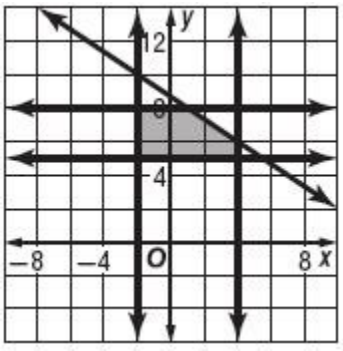
ANSWER:



(2, –10), (–3, 0), (–3, 6.5), (2, 9); max = 82, min = –89

10. $-2 \le x \le 4$
$5 \le y \le 8$
$2x + 3y \le 26$
$f(x, y) = 8x - 10y$

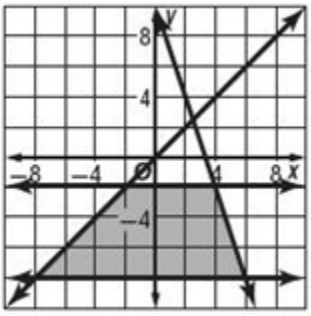
ANSWER:



(1, 8), (4, 6), (4, 5), (–2, 5), (–2, 8); max = –18, min = –96

11. $-8 \le y \le -2$
$y \le x$
$y \le -3x + 10$
$f(x, y) = 5x + 14y$

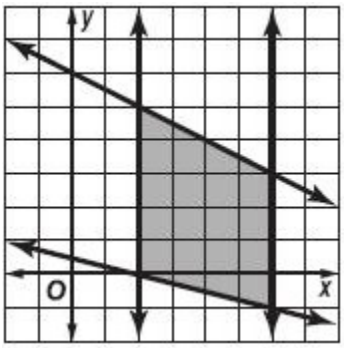
ANSWER:



(6, –8), (4, –2), (–2, –2), (–8, –8);max = –8, min = –152

12. $x + 4y \geq 2$
$2x + 4y \leq 24$
$2 \leq x \leq 6$
$f(x, y) = 6x + 7y$

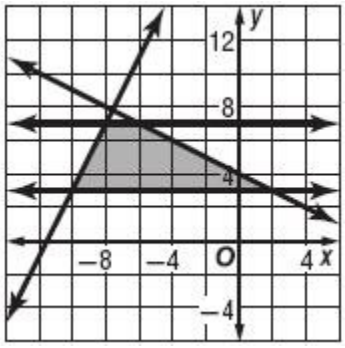
ANSWER:



(2, 0), (6, –1), (6, 3), (2, 5); max = 57, min = 12

13. $3 \leq y \leq 7$
$2y + x \leq 8$
$y - 2x \leq 23$
$f(x, y) = -3x + 5y$

ANSWER:

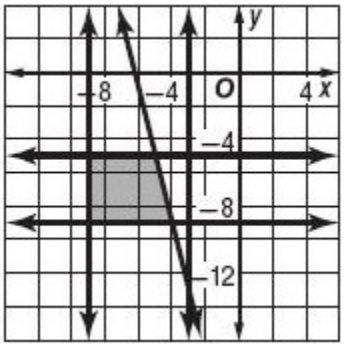


(–10, 3), (2, 3), (–6, 7), (–8, 7); max = 59, min = 9

Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.

14. $-9 \leq x \leq -3$
$-9 \leq y \leq -5$
$3y + 12x \leq -75$
$f(x, y) = 20x + 8y$

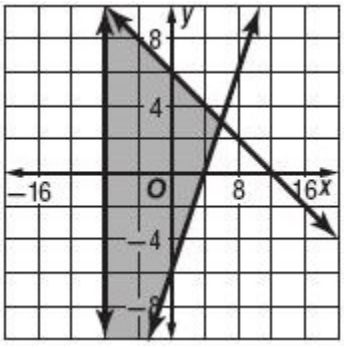
ANSWER:



(–9, –9), (–4, –9), (–5, –5), (–9, –5); max = –140, min = –252

15. $x \geq -8$
$3x + 6y \leq 36$
$2y + 12 \geq 3x$
$f(x, y) = 10x - 6y$

ANSWER:

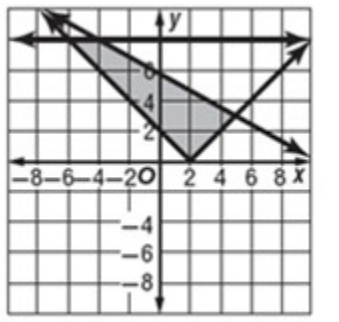


(6, 3), (–8, 10), (–8, –18); max = 42, min = –140

3-3 Optimization with Linear Programming

16. $y \geq |x-2|$
$y \leq 8$
$8y + 5x \leq 49$
$f(x,y) = -5x - 15y$

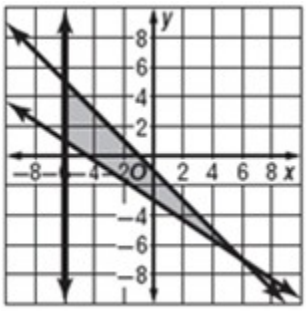
ANSWER:



(2, 0), (5, 3), (–3, 8), (–6, 8), max = –10, min = –105

17. $x \geq -6$
$y + x \leq -1$
$2x + 3y \geq -9$
$f(x,y) = -10x - 12y$

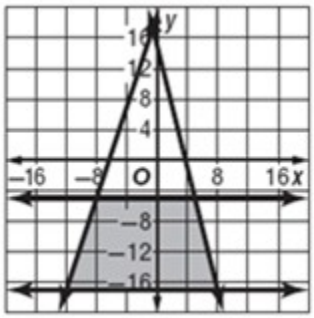
ANSWER:



(–6, 1), (6, –7), (–6, 5); max = 48, min = 0

18. $-5 \geq y \geq -17$
$y \leq 3x + 19$
$y \leq -4x + 15$
$f(x,y) = 8x - 3y$

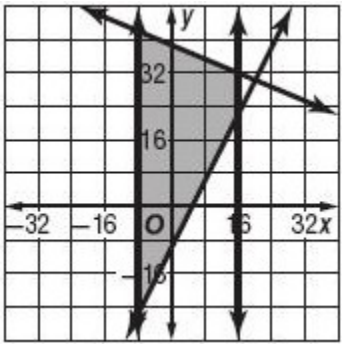
ANSWER:



(5, –5), (8, –17), (-12,-17), (-8,-5) : max =115, min = -49

19. $-8 \leq x \leq 16$
$y \geq 2x - 10$
$2y + x \leq 80$
$f(x,y) = 12x + 15y$

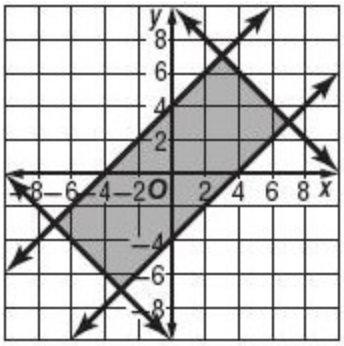
ANSWER:



(–8, 44), (16, 32), (–8, –26), (16, 22); max = 672, min = –486

20. $y \le x + 4$
$y \ge x - 4$
$y \le -x + 10$
$y \ge -x - 10$
$f(x, y) = -10x + 9y$

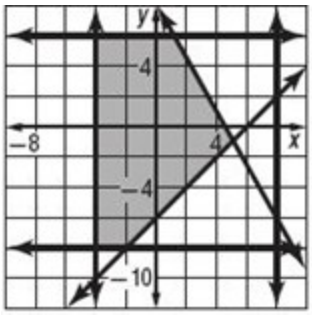
ANSWER:



(3, 7), (7, 3), (–3, –7), (–7, –3); max = 43, min = –43

21. $-4 \le x \le 8$
$-8 \le y \le 6$
$y \ge x - 6$
$4y + 7x \le 31$
$f(x, y) = 12x + 8y$

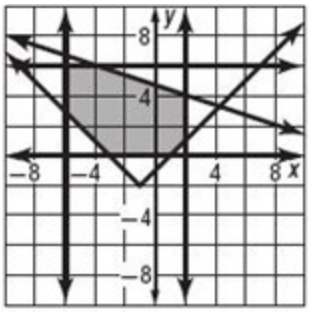
ANSWER:



(5, –1), (1, 6), (–2, –8), (–4, –8), (–4, 6), max = 60, min = –112

22. $y \ge |x + 1| - 2$
$0 \le y \le 6$
$-6 \le x \le 2$
$x + 3y \le 14$
$f(x, y) = 5x + 4y$

ANSWER:



(–4, 6), (2, 4), (2, 1), (1, 0), (–3, 0), (–6, 3), (–6, 6); max = 26, min = –18

23. **COOKING** Jenny's Bakery makes two types of birthday cakes: yellow cake, which sells for $25, and strawberry cake, which sells for $35. Both cakes are the same size, but the decorating and assembly time required for the yellow cake is 2 hours, while the time is 3 hours for the strawberry cake. There are 450 hours of labor available for production. How many of each type of cake should be made to maximize revenue?

ANSWER:

225 yellow cakes, 0 strawberry cakes

24. **BUSINESS** The manager of a travel agency is printing brochures and fliers to advertise special discounts on vacation spots during the summer months. Each brochure costs $0.08 to print, and each flier costs $0.04 to print. A brochure requires 3 pages, and a flier requires 2 pages. The manager does not want to use more than 600 pages, and she needs at least 50 brochures and 150 fliers. How many of each should she print to minimize the cost?

ANSWER:

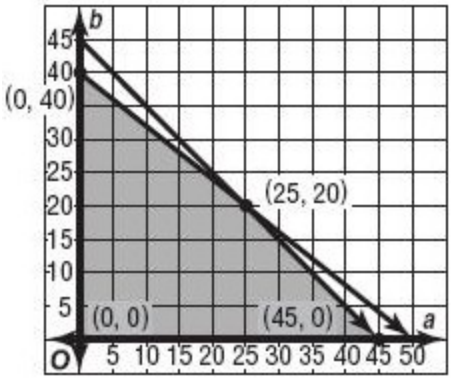
50 brochures, 150 fliers

25. **CCSS PRECISION** Sean has a maximum of 20 days each month to paint sheds and playhouses. He can paint 2.5 sheds per day or 2 playhouses per day. Each month he has access to at most 45 structures that need to be painted.
a. Write a system of inequalities to represent the constraints on the number of each structure Sean can paint each month.
b. Draw a graph showing the feasible region and list the coordinates of the vertices of the feasible region.
c. If Sean is paid $26 per shed and $30 per playhouse, how many of each should he paint to maximize his monthly earnings?
d. What is his maximum monthly earnings?

ANSWER:

no changea. Let a = the number of sheds and b = the number of playhouses.
$a \geq 0, b \geq 0, a + b \leq 45, \frac{2}{5}a + \frac{1}{2}b \leq 20$ (or $4a + 5b \leq 200$)

b.



c. 25 sheds, 20 playhouses
d. $1250

26. **MOVIES** Employees at a local movie theater work 8-hour shifts from noon to 8 P.M. or from 4 P.M. to midnight. The table below shows the number of employees needed and their corresponding pay. Find the numbers of day-shift workers and night-shift workers that should be scheduled to minimize the cost. What is the minimal cost?

Time	noon to 4 P.M.	4 P.M. to 8 P.M.	8 P.M. to midnight
Number of Employees Needed	at least 5	at least 14	6
Rate per Hour	$5.50	$7.50	$7.50

ANSWER:

8 day-shift and 6 night-shift workers; $776

27. **BUSINESS** Each car on a freight train can hold 4200 pounds of cargo and has a capacity of 480 cubic feet. The freight service handles two types of packages: small, which weigh 25 pounds and are 3 cubic feet each, and large, which are 50 pounds and are 5 cubic feet each. The freight service charges $5 for each small package and $8 for each large package.
a. Find the number of each type of package that should be placed on a train car to maximize revenue.
b. What is the maximum revenue per train car?
c. In this situation, is maximizing the revenue necessarily the best thing for the company to do? Explain.

ANSWER:

a. 160 small packages, 0 large packages
b. $800
c. No; if revenue is maximized, the company will not deliver any large packages, and customers with large packages to ship will probably choose another carrier.

28. **RECYCLING** A recycling plant processes used plastic into food or drink containers. The plant processes up to 1200 tons of plastic per week. At least 300 tons must be processed for food containers, while at least 450 tons must be processed for drink containers. The profit is $17.50 per ton for processing food containers and $20 per ton for processing drink containers. What is the profit if the plant maximizes processing?

ANSWER:

$23,250

29. **OPEN ENDED** Create a set of inequalities that forms a bounded region with an area of 20 units^2 and lies only in the fourth quadrant.

ANSWER:

Sample answer: $-2 \geq y \geq -6,\ 4 \leq x \leq 9$

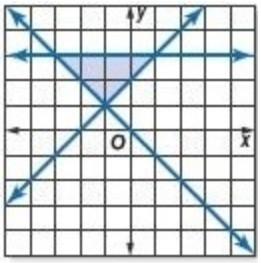
30. **CHALLENGE** Find the area of the bounded region formed by the following constraints:
$y \geq |x| - 3, y \leq -|x| + 3, \text{and } x \geq |y|.$

ANSWER:

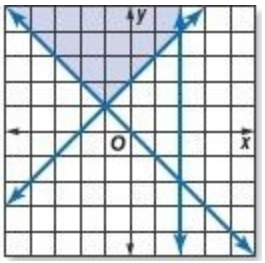
4.5 units^2

31. **CCSS ARGUMENTS** Identify the system of inequalities that is not the same as the other three. Explain your reasoning.

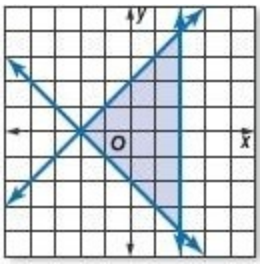
a.



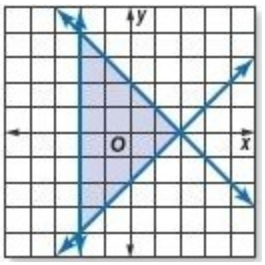
b.



c.



d.



ANSWER:

b; The feasible region of Graph b is unbounded while the other three are bounded.

32. **REASONING** Determine whether the following statement is sometimes, always, or never true. Explain your reasoning.
An unbounded region will not have both a maximum and minimum value.

ANSWER:

Sample answer: Always; if a point on the unbounded region forms a minimum, then a maximum cannot also be formed because of the unbounded region. There will always be a value in the solution that will produce a higher value than any projected maximum.

33. **WRITING IN MATH** Upon determining a bounded feasible region, Ayumi noticed that vertices $A(-3, 4)$ and $B(5, 2)$ yielded the same maximum value for $f(x, y) = 16y + 4x$. Kelvin confirmed that her constraints were graphed correctly and her vertices were correct. Then he said that those two points were not the only maximum values in the feasible region. Explain how this could have happened.

ANSWER:

Sample answer: Even though the region is bounded, multiple maximums occur at A and B and all of the points on the boundary of the feasible region containing both A and B. This happened because that boundary of the region has the same slope as the function.

34. Kelsey worked 350 hours during the summer and earned $2978.50. She earned $6.85 per hour when she worked at a video store and $11 per hour as an architectural intern. Let x represent the number of hours she worked at the video store and y represent the number of hours that she interned. Which system of equations represents this situation?

A $\begin{cases} x + y = 350 \\ 11x + 6.85y = 2978.50 \end{cases}$

B $\begin{cases} x + y = 350 \\ 6.85x + 11y = 2978.50 \end{cases}$

C $\begin{cases} x + y = 2978.50 \\ 6.85x + 11y = 350 \end{cases}$

D $\begin{cases} x + y = 2978.50 \\ 11x + 6.85y = 350 \end{cases}$

ANSWER:

B

35. **SHORT RESPONSE** A family of four went out to dinner. Their bill, including tax, was $60. They left a 17% tip on the total cost of their bill. What is the total cost of the dinner including tip?

ANSWER:

$70.20

36. **ACT/SAT** For a game she is playing, Liz must draw a card from a deck of 26 cards, one with each letter of the alphabet on it, and roll a die. What is the probability that Liz will draw a letter in her name and roll an odd number?

F $\frac{2}{3}$
G $\frac{1}{13}$
H $\frac{3}{52}$
J $\frac{1}{26}$
K $\frac{1}{52}$

ANSWER:

H

37. **GEOMETRY** Which of the following best describes the graphs of $y = 3x - 5$ and $4y = 12x + 16$?

A The lines have the same y-intercept.
B The lines have the same x-intercept.
C The lines are perpendicular.
D The lines are parallel.

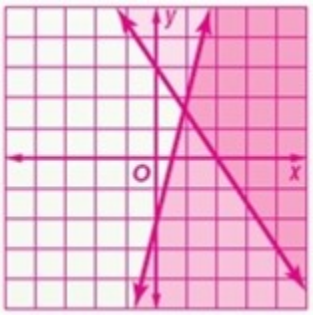
ANSWER:

D

Solve each system of inequalities by graphing.

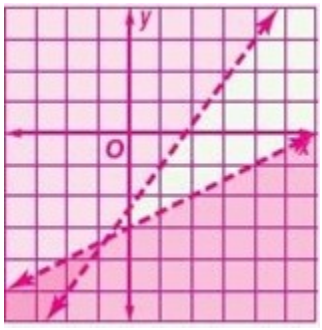
38. $\begin{cases} 3x + 2y \geq 6 \\ 4x - y \geq 2 \end{cases}$

ANSWER:



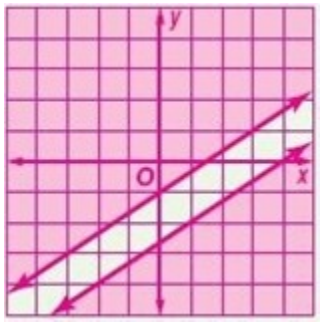
39. $4x - 3y < 7$
$2y - x < -6$

ANSWER:



40. $3y \le 2x - 8$
$y \ge \frac{2}{3}x - 1$

ANSWER:



no solution

41. **BUSINESS** Last year the chess team paid $7 per hat and $15 per shirt for a total purchase of $330. This year they spent $360 to buy the same number of shirts and hats because the hats now cost $8 and the shirts cost $16. Write and solve a system of two equations that represents the number of hats and shirts bought each year.

ANSWER:

$7x + 15y = 330, 8x + 16y = 360$; hats: 15, shirts: 15

Write an equation in slope-intercept form for the line that satisfies each set of conditions.

42. passes through (5, 1) and (8, –4)

ANSWER:

$y = -\frac{5}{3}x + \frac{28}{3}$

43. passes through (–3, 5) and (3, 2)

ANSWER:

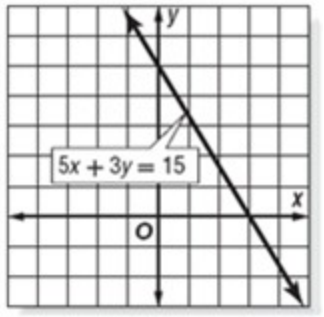
$y = -\frac{1}{2}x + \frac{7}{2}$

Find the x-intercept and the y-intercept of the graph of each equation. Then graph the equation.

44. $5x + 3y = 15$

ANSWER:

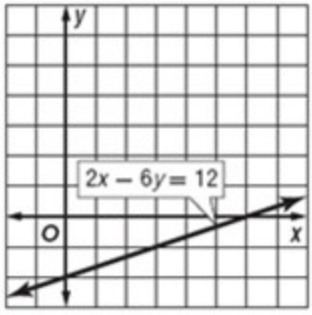
3; 5



45. $2x - 6y = 12$

ANSWER:

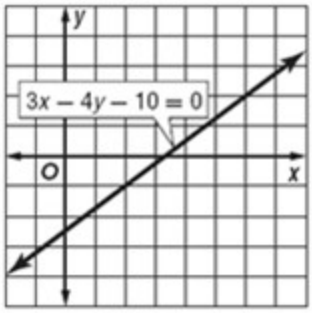
6; –2



46. $3x - 4y - 10 = 0$

ANSWER:

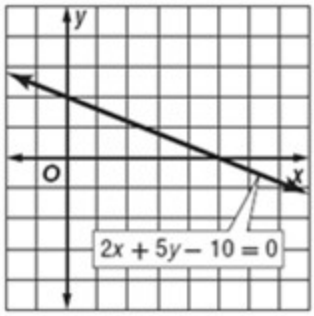
$\frac{10}{3}; -\frac{5}{2}$



47. $2x + 5y - 10 = 0$

ANSWER:

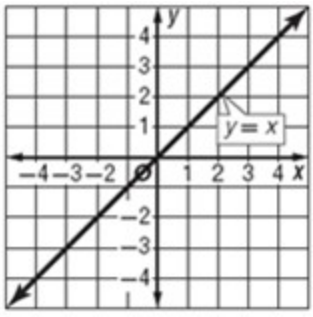
5; 2



48. $y = x$

ANSWER:

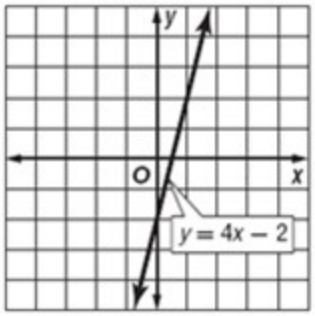
0; 0



49. $y = 4x - 2$

ANSWER:

$\frac{1}{2}; -2$



Evaluate each expression if $x = -1$, $y = 3$, and $z = 7$.

50. $x + y + z$

ANSWER:

9

51. $2x - y + 2z$

ANSWER:

9

52. $-x + 4y - 3z$

ANSWER:

–8

53. $4x + 2y - z$

ANSWER:

–5

54. $5x - y + 4z$

ANSWER:

20

55. $-3x - 3y + 3z$

ANSWER:

15