3-4 Systems of Equations in Three Variables

Solve each system of equations.

1. $-3a-4b+2c=28$
   $a+3b-4c=-31$
   $2a+3c=11$

ANSWER:
(–2, –3, 5)

2. $3y-5z=-23$
   $4x+2y+3z=7$
   $-2x-y-z=-3$

ANSWER:
(4, –6, 1)

3. $3x+6y-2z=-6$
   $2x+y+4z=19$
   $-5x-2y+8z=62$

ANSWER:
(–4, 3, 6)

4. $-4r-s+3t=-9$
   $3r+2s-t=3$
   $r+3s-5t=29$

ANSWER:
(–2, 2, –5)

5. $3x+5y-z=12$
   $-2x-3y+5z=14$
   $4x+7y+3z=38$

ANSWER:
Infinite solutions

6. $2a-3b+5c=58$
   $-5a+b-4c=-51$
   $-6a-8b+c=22$

ANSWER:
(3, –4, 8)

7. **DOWNLOADING** Heather downloaded some television shows. A sitcom uses 0.3 gigabyte of memory; a drama, 0.6 gigabyte; and a talk show, 0.6 gigabyte. She downloaded 7 programs totaling 3.6 gigabytes. There were twice as many episodes of the drama as the sitcom.
a. Write a system of equations for the number of episodes of each type of show.
b. How many episodes of each show did she download?

ANSWER:
a. $s+d+t=7, d=2s, 0.3s+0.6d+0.6t=3.6$
b. 2 sitcoms, 4 dramas, 1 talk show

Solve each system of equations.

8. $-5x+y-4z=60$
   $2x+4y+3z=-12$
   $6x-3y-2z=-52$

ANSWER:
(–8, 4, –4)

9. $4a+5b-6c=2$
   $-3a-2b+7c=-15$
   $-a+4b+2c=-13$

ANSWER:
(–3, –2, –4)

10. $-2x+5y+3z=-25$
    $-4x-3y-8z=-39$
    $6x+8y-5z=14$

ANSWER:
(8, –3, 2)

11. $4r+6s-t=-18$
    $3r+2s-4t=-24$
    $-5r+3s+2t=15$

ANSWER:
(–2, –1, 4)

12. $-2x+15y+z=44$
    $4x+3y+3z=18$
    $-3x+6y-z=8$

ANSWER:
No solution

13. $4x + 2y + 6z = 13$
$-12x + 3y - 5z = 8$
$-4x + 7y + 7z = 34$

ANSWER:
Infinite solutions

14. $8x + 3y + 6z = 43$
$-3x + 5y + 2z = 32$
$5x - 2y + 5z = 24$

ANSWER:
(–1, 3, 7)

15. $-6x - 5y + 4z = 53$
$5x + 3y + 2z = -11$
$8x - 6y + 5z = 4$

ANSWER:
(–4, –1, 6)

16. $-9a + 3b - 2c = 61$
$8a + 7b + 5c = -138$
$5a - 5b + 8c = -45$

ANSWER:
(–8, –7, –5)

17. $2x - y + z = 1$
$x + 2y - 4z = 3$
$4x + 3y - 7z = -8$

ANSWER:
No solution

18. $x + 2y = 12$
$3y - 4z = 25$
$x + 6y + z = 20$

ANSWER:
(6, 3, –4)

19. $r - 3s + t = 4$
$3r - 6s + 9t = 5$
$4r - 9s + 10t = 9$

ANSWER:
Infinite solutions

20. **CCSS SENSE-MAKING** A friend e-mails you the results of a recent high school swim meet. The e-mail states that 24 individuals placed, earning a combined total of 53 points. First place earned 3 points, second place earned 2 points, and third place earned 1 point. There were as many first-place finishers as second- and third-place finishers combined.
a. Write a system of three equations that represents how many people finished in each place.
b. How many swimmers finished in first place, in second place, and in third place?
c. Suppose the e-mail had said that the athletes scored a combined total of 47 points. Explain why this statement is false and the solution is unreasonable.

ANSWER:
a. $x + y + z = 24$, $3x + 2y + z = 53$, $x = y + z$.
b. 7 swimmers placed third, 5 swimmers placed second, and 12 swimmers placed first.
c. The statement is false because when you solve for second place, you get a negative as an answer and you cannot have a negative person.

21. **AMUSEMENT PARKS** Nick goes to the amusement park to ride roller coasters, bumper cars, and water slides. The wait for the roller coasters is 1 hour, the wait for the bumper cars is 20 minutes long, and the wait for the water slides is only 15 minutes long. Nick rode 10 total rides during his visit. Because he enjoys roller coasters the most, the number of times he rode the roller coasters was the sum of the times he rode the other two rides. If Nick waited in line for a total of 6 hours and 20 minutes, how many of each ride did he go on?

ANSWER:
roller coasters: 5; bumper cars: 1; water slides: 4

22. **BUSINESS** Ramón usually gets one of the routine maintenance options at Annie's Garage. Today however, he needs a different combination of work than what is listed.

a. Assume that the price of an option is the same price as purchasing each item separately. Find the prices for an oil change, a radiator flush, and a brake pad replacement.
b. If Ramón wants his brake pads replaced and his radiator flushed, how much should he plan to spend?

ANSWER:

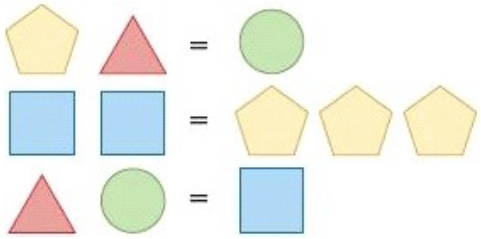
a. oil change: \$19.99; brake pad replacement: \$20; radiator flush: \$10
b. \$30

23. **FINANCIAL LITERACY** Kate invested \$100,000 in three different accounts. If she invested \$30,000 more in account A than account C and is expected to earn \$6300 in interest, how much did she invest in each account?

Account	Expected Interest
A	4%
B	8%
C	10%

ANSWER:
A: \$55,000; B: \$20,000; C: \$25,000

24. **CCSS REASONING** Write a system of equations to represent the three rows of figures below. Use the system to find the number of red triangles that will balance one green circle.



ANSWER:
$t + c = s, p + t = c, 2s = 3p$
where t represents triangle, c represents circle, s represents square, and p represents pentagon; 5 red triangles

25. **CHALLENGE** The general form of an equation for a parabola is $y = ax^2 + bx + c$, where (x, y) is a point on the parabola. If three points on a parabola are $(2, -10)$, $(-5, -101)$, and $(6, -90)$, determine the values of a, b, and c and write the general form of the equation.

ANSWER:

$$y = -3x^2 + 4x - 6; a = -3, b = 4, c = -6$$

26. **PROOF** Consider the following system and prove that if $b = c = -a$, then $ty = a$.

$rx + ty + vz = a$
$rx - ty + vz = b$
$rx + ty - vz = c$

ANSWER:

$$\begin{aligned}
a + b &= (rx + ty + vz) + (rx - ty + vz) && \\
a + b &= 2rx + 2vz && \text{Simplify.} \\
a + (-a) &= 2rx + 2vz && \text{Replace b with -a.} \\
0 &= 2rx + 2vz && \text{Simplify.} \\
0 &= rx + vz && \text{Divide by 2.} \\
rx + ty + vz &= a && \text{Given} \\
ty + (rx + vz) &= a && \text{Commutative and Associative Properties (+)} \\
ty + 0 &= a && \text{Substitution} \\
ty &= a && \text{Simplify.}
\end{aligned}$$

27. **OPEN ENDED** Write a system of three linear equations that has a solution of $(-5, -2, 6)$. Show that the ordered triple satisfies all three equations.

ANSWER:

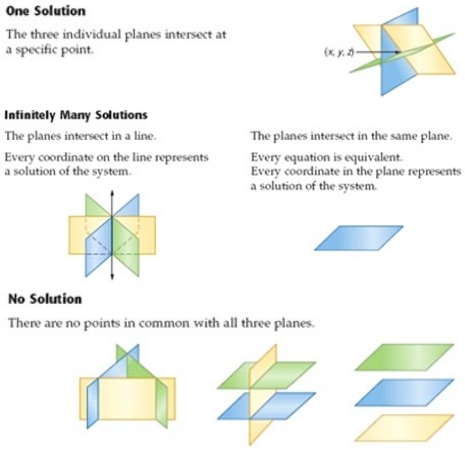
Sample answer:

$$3x + 4y + z = -17$$
$$3(-5) + 4(-2) + 6 \stackrel{?}{=} -17$$
$$-15 + (-8) + 6 \stackrel{?}{=} -17$$
$$-23 + 6 \stackrel{?}{=} -17$$
$$-17 = -17 \checkmark$$

$$2x - 5y - 3z = -18$$
$$2(-5) - 5(-2) - 3(6) \stackrel{?}{=} -18$$
$$-10 + 10 - 18 \stackrel{?}{=} -18$$
$$-18 = -18 \checkmark$$

$$-x + 3y + 8z = 47$$
$$-(-5) + 3(-2) + 8(6) \stackrel{?}{=} 47$$
$$5 - 6 + 48 \stackrel{?}{=} 47$$
$$-1 + 48 \stackrel{?}{=} 47$$
$$47 = 47 \checkmark$$

28. **REASONING** Use the diagram below of the solution of systems of equations to consider a system on inequalities in three variables. Describe the solution of such a system.



ANSWER:

Sample answer: The solution of an inequality in 3 variables would be the region of space on one side or the other of a plane, which the plane included if the inequality is $\leq$ or $\geq$. The solution of a system of inequalities in 3 variables would be the intersection of the regions of space that are solution to the individual inequalities in the system.

29. **WRITING IN MATH** Use your knowledge of solving a system of three linear equations with three variables to explain how to solve a system of four equations with four variables.

ANSWER:

Sample answer: First, combine two of the original equations using elimination to form a new equation with three variables. Next, combine a different pair of the original equations using elimination to eliminate the same variable and form a second equation with three variables. Do the same thing with a third pair of the original equations. You now have a system of three equations with three variables. Follow the same procedure you learned in this section. Once you find the three variables, you need to use them to find the eliminated variable.

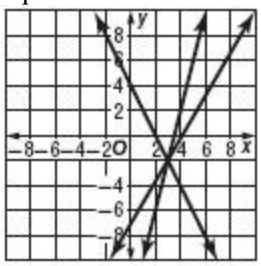
30. What is the solution of the system of equations shown below?

$$\begin{cases} x - y + z = 0 \\ -5x + 3y - 2z = -1 \\ 2x - y + 4z = 11 \end{cases}$$

A (0, 3, 3)
B (2, 5, 3)
C no solution
D infinitely many solutions

ANSWER:
B

31. **ACT/SAT** The graph shows which system of equations?



A
$y + 14 = 4x$
$y = 4 - 2x$

B
$-7 = y - \frac{5}{3}x$
$y - 14x = 4$
$2x = 4 + y$

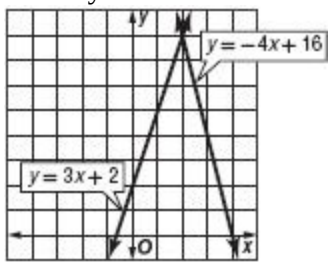
C
$7 = y - \frac{5}{3}x$
$y - 14 = 4x$
$y = 4 + 2x$

D
$-7 = y + \frac{5}{3}x$
$y - 4x = 14$
$y = 2x + 4$

E
$7 = y + \frac{5}{3}x$
$y + 14x = 4$
$-2y = 4 + y$
$-7 = y - \frac{5}{3}x$

ANSWER:
A

32. **EXTENDED RESPONSE** Use the graph to find the solution of the systems of equations. Describe one way to check the solution.



ANSWER:
(2, 8); Sample answer: You can substitute (2, 8) into each of the equations and make sure the equations are true.

33. Which of the following represents a correct procedure for solving each equation?

F
$-3(x - 7) = -16$
$-3x - 21 = -16$
$-3x = 5$
$x = -\frac{5}{3}$

G
$7 - 4x = 3x + 27$
$7 - 7x = 27$
$-7x = -\frac{20}{7}$
$x = 20$

H
$2(x - 4) = 20$
$2x - 8 = 20$
$2x = 12$
$x = 6$

J
$6(2x + 1) = 30$
$12x + 6 = 30$
$12x = 24$
$x = 2$

ANSWER:
J

A feasible region has vertices at (–3, 2), (1, 3), (6, 1), and (2, –2). Find the maximum and minimum values of each function.

34. $f(x, y) = 2x - y$

ANSWER:
11; –8

35. $f(x, y) = x + 5y$

ANSWER:
16; –8

36. $f(x, y) = y - 4x$

ANSWER:
14; –23

37. $f(x, y) = -x + 3y$

ANSWER:
9; –8

38. **SKI CLUB** The ski club's budget for the year is $4250. They are able to find skis for $75 per pair and boots for $40 per pair. They know they should buy more boots than skis because the skis are adjustable to several sizes of boots.
a. Give an example of three different purchases that the ski club can make.
b. Suppose the ski club wants to spend all of its budget. What combination of skis and boots should they buy? Explain.

ANSWER:
a. Sample answer: 40 boots, 35 skis; 45 boots, 32 skis; 50 boots, 30 skis
b. 50 boots and 30 skis cost exactly $4250.

Solve each system of equations.

39. $x = y + 5$
$3x + y = 19$

ANSWER:
(6, 1)

40. $3x - 2y = 1$
$4x + 2y = 20$

ANSWER:
(3, 4)

41. $5x + 3y = 25$
$4x + 7y = -3$

ANSWER:
(8, –5)

42. $y = x - 7$
$2x - 8y = 2$

ANSWER:
(9, 2)