4-1 Graphing Quadratic Functions

Complete parts a–c for each quadratic function.

a. Find the *y*-intercept, the equation of the axis of symmetry, and the *x*-coordinate of the vertex.

b. Make a table of values that includes the vertex.

c. Use this information to graph the function.

1. $f(x) = 3x^2$

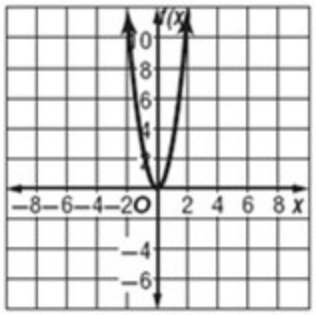
ANSWER:

a. *y*-int = 0; axis of symmetry: $x = 0$; *x*-coordinate = 0

b.

x	f(x)
–2	12
–1	3
0	0
1	3
2	12

c.



2. $f(x) = -6x^2$

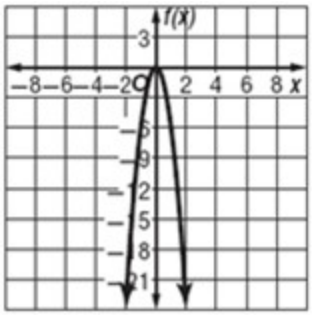
ANSWER:

a. *y*-int = 0; axis of symmetry: $x = 0$; *x*-coordinate = 0

b.

x	f(x)
–2	–24
–1	–6
0	0
1	–6
2	–24

c.



3. $f(x) = x^2 - 4x$

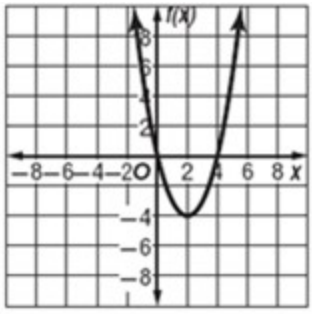
ANSWER:

a. y-int = 0; axis of symmetry: $x = 2$; x-coordinate = 2

b.

x	f(x)
0	0
1	–3
2	–4
3	–3
4	0

c.



4. $f(x) = -x^2 - 3x + 4$

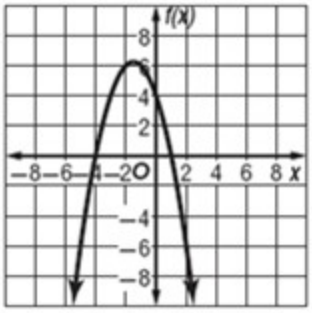
ANSWER:

a. y-int = 4; axis of symmetry: $x = –1.5$; x-coordinate = –1.5

b.

x	f(x)
–3	4
–2	6
–1.5	6.25
–1	6
0	4

c.



5. $f(x) = 4x^2 - 6x - 3$

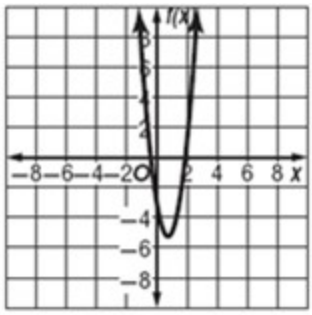
ANSWER:

a. y-int = –3; axis of symmetry: $x = 0.75$; x-coordinate = 0.75

b.

x	f(x)
–1	7
0	–3
0.75	–5.25
1.5	–3
2.5	7

c.



6. $f(x) = 2x^2 - 8x + 5$

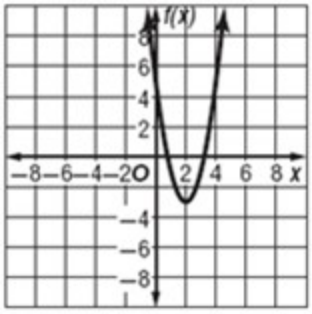
ANSWER:

a. y-int = 5; axis of symmetry: $x = 2$; x-coordinate = 2

b.

x	f(x)
0	5
1	–1
2	–3
3	–1
4	5

c.



Determine whether each function has a *maximum* or *minimum* value, and find that value. Then state the domain and range of the function.

7. $f(x) = -x^2 + 6x - 1$

ANSWER:

max = 8; D = {all real numbers},
$\mathrm{R} = \{f(x) \mid f(x) \le 8\}$

8. $f(x) = x^2 + 3x - 12$

ANSWER:

min = –14.25; D = {all real numbers},
$\mathrm{R} = \{f(x) \mid f(x) \ge -14.25\}$

9. $f(x) = 3x^2 + 8x + 5$

ANSWER:

$\min = -\frac{1}{3}$; D = {all real numbers},
$R = \left\{ f(x) \mid f(x) \geq -\frac{1}{3} \right\}$

10. $f(x) = -4x^2 + 10x - 6$

ANSWER:

max = 0.25; D = {all real numbers},
$R = \{ f(x) \mid f(x) \leq 0.25 \}$

11. **BUSINESS** A store rents 1400 videos per week at $2.25 per video. The owner estimates that they will rent 100 fewer videos for each $0.25 increase in price. What price will maximize the income of the store?

ANSWER:

$2.88

Complete parts *a*–*c* for each quadratic function.

a. Find the *y*-intercept, the equation of the axis of symmetry, and the *x*-coordinate of the vertex.

b. Make a table of values that includes the vertex.

c. Use this information to graph the function.

12. $f(x) = 4x^2$

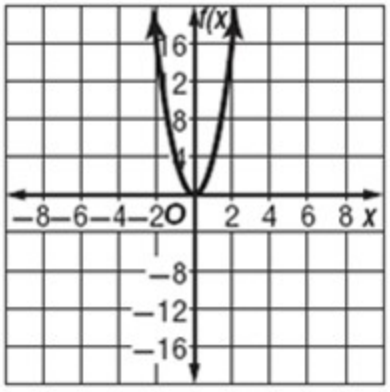
ANSWER:

a. *y*-int = 0; axis of symmetry: $x = 0$; *x*-coordinate = 0

b.

x	*f*(*x*)
–2	16
–1	4
0	0
1	4
2	16

c.



13. $f(x) = -2x^2$

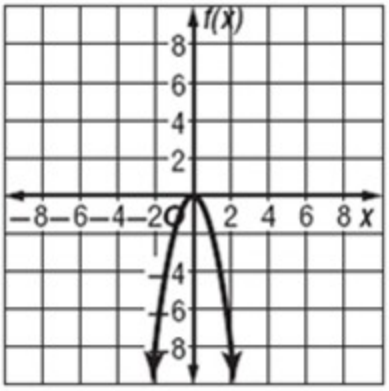
ANSWER:

a. y-int = 0; axis of symmetry: $x = 0$; x-coordinate = 0

b.

x	f(x)
–2	–8
–1	–2
0	0
1	–2
2	–8

c.



14. $f(x) = x^2 - 5$

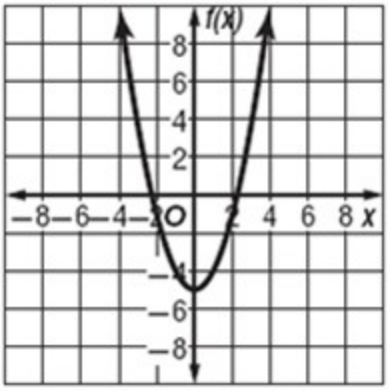
ANSWER:

a. y-int = –5; axis of symmetry: $x = 0$; x-coordinate = 0

b.

x	f(x)
–2	–1
–1	–4
0	–5
1	–4
2	–1

c.



15. $f(x) = 4x^2 - 3$

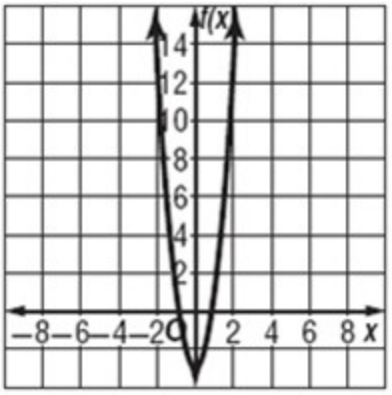
ANSWER:

a. y-int = –3; axis of symmetry: $x = 0$; x-coordinate = 0

b.

x	f(x)
–2	13
–1	1
0	–3
1	1
2	13

c.



16. $f(x) = x^2 + 3$

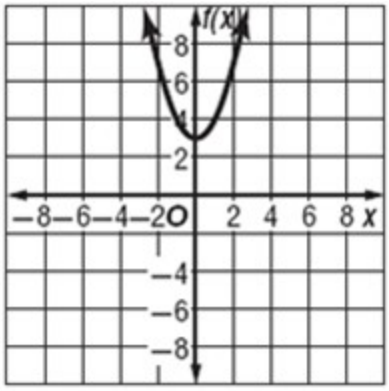
ANSWER:

a. y-int = 3; axis of symmetry: $x = 0$; x-coordinate = 0

b.

x	f(x)
–2	7
–1	4
0	3
1	4
2	7

c.



17. $f(x) = -3x^2 + 5$

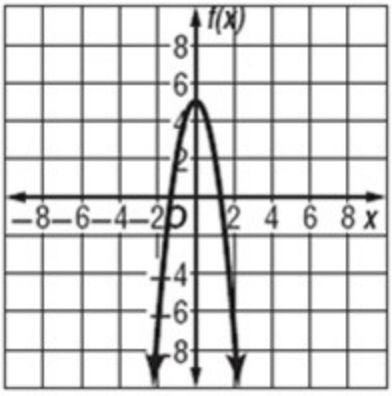
ANSWER:

a. y-int = 5; axis of symmetry: $x = 0$; x-coordinate = 0

b.

x	f(x)
–2	–7
–1	2
0	5
1	2
2	–7

c.



18. $f(x) = x^2 - 6x + 8$

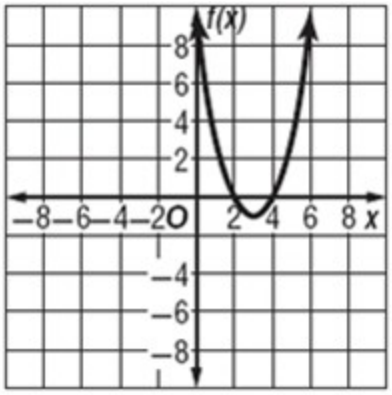
ANSWER:

a. y-int = 8; axis of symmetry: $x = 3$; x-coordinate = 3

b.

x	f(x)
1	3
2	0
3	–1
4	0
5	3

c.



19. $f(x) = x^2 - 3x - 10$

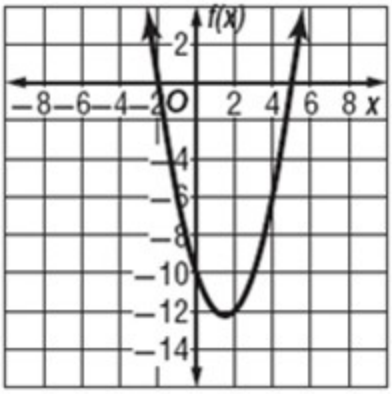
ANSWER:

a. y-int = –10; axis of symmetry: x = 1.5; x-coordinate =1.5

b.

x	f(x)
0	–10
1	–12
1.5	–12.25
2	–12
3	–10

c.



20. $f(x) = -x^2 + 4x - 6$

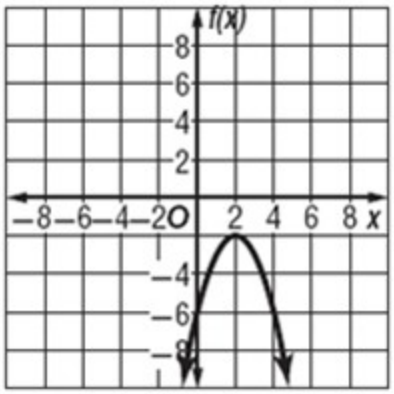
ANSWER:

a. y-int = –6; axis of symmetry: x = 2; x-coordinate = 2

b.

x	f(x)
0	–6
1	–3
2	–2
3	–3
4	–6

c.



21. $f(x) = -2x^2 + 3x + 9$

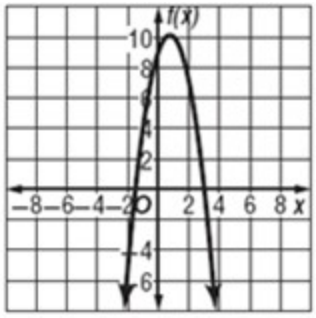
ANSWER:

a. y-int = 9; axis of symmetry: $x = 0.75$; x-coordinate = 0.75

b.

x	f(x)
–1	4
0	9
0.75	10.125
1.5	9
2.5	4

c.



Determine whether each function has a *maximum* or *minimum* value, and find that value. Then state the domain and range of the function.

22. $f(x) = 5x^2$

ANSWER:

min = 0; D = {all real numbers},
$R = \{f(x) \mid f(x) \geq 0\}$

23. $f(x) = -x^2 - 12$

ANSWER:

max = –12; D = {all real numbers},
$R = \{f(x) \mid f(x) \leq -12\}$

24. $f(x) = x^2 - 6x + 9$

ANSWER:

min = 0; D = {all real numbers},
$R = \{f(x) \mid f(x) \geq 0\}$

25. $f(x) = -x^2 - 7x + 1$

ANSWER:

max = 13.25; D = {all real numbers},
$R = \{f(x) \mid f(x) \leq 13.25\}$

26. $f(x) = 8x - 3x^2 + 2$

ANSWER:

max = $\frac{22}{3}$; D = {all real numbers},
$R = \left\{f(x) \mid f(x) \leq \frac{22}{3}\right\}$

27. $f(x) = 5 - 4x - 2x^2$

ANSWER:

max = 7; D = {all real numbers},
$R = \{f(x) \mid f(x) \leq 7\}$

28. $f(x) = 15 - 5x^2$

ANSWER:

max = 15; D = {all real numbers},
$R = \{f(x) \mid f(x) \leq 15\}$

29. $f(x) = x^2 + 12x + 27$

ANSWER:
min = –9; D = {all real numbers},
$R = \{f(x) \mid f(x) \geq -9\}$

30. $f(x) = -x^2 + 10x + 30$

ANSWER:
max = 55; D = {all real numbers},
$R = \{f(x) \mid f(x) \leq 55\}$

31. $f(x) = 2x^2 - 16x - 42$

ANSWER:
min = –74; D = {all real numbers},
$R = \{f(x) \mid f(x) \geq -74\}$

32. **CCSS MODELING** A financial analyst determined that the cost, in thousands of dollars, of producing bicycle frames is $C = 0.000025f^2 - 0.04f + 40$, where f is the number of frames produced.

a. Find the number of frames that minimizes cost.

b. What is the total cost for that number of frames?

ANSWER:
a. 800
b. $24,000

Complete parts a–c for each quadratic function.

a. Find the *y*-intercept, the equation of the axis of symmetry, and the *x*-coordinate of the vertex.

b. Make a table of values that includes the vertex.

c. Use this information to graph the function.

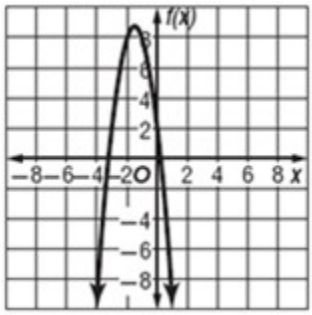
33. $f(x) = -3x^2 - 9x + 2$

ANSWER:
a. *y*-int = 2; axis of symmetry: $x = -1.5$; *x*-coordinate of vertex = –1.5

b.

x	f(x)
–3	2
–2	8
–1.5	8.75
–1	8
0	2

c.



34. $f(x) = 2x^2 - 6x - 9$

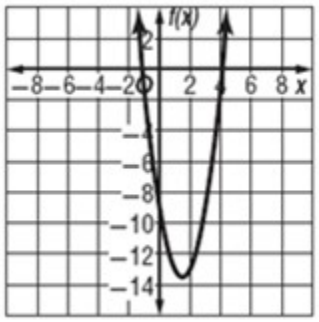
ANSWER:

a. y-int = –9; axis of symmetry: x = 1.5; x-coordinate of vertex = 1.5

b.

x	f(x)
0	–9
1	–13
1.5	–13.5
2	–13
3	–9

c.



35. $f(x) = -4x^2 + 5x$

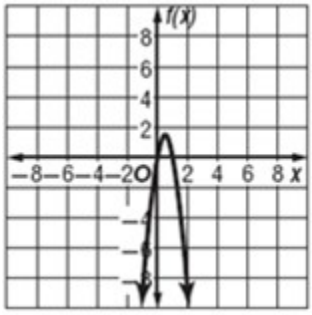
ANSWER:

a. y -int = 0; axis of symmetry: $x = \frac{5}{8}$; x-coordinate of vertex = $\frac{5}{8}$

b.

x	f(x)
$-\frac{3}{4}$	–6
$\frac{1}{4}$	1
$\frac{5}{8}$	1.5625
1	1
2	–6

c.



36. $f(x) = 2x^2 + 11x$

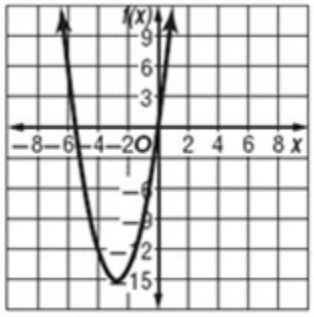
ANSWER:

a. y-int = 0; axis of symmetry: $x = -2.75$; x-coordinate of vertex = –2.75

b.

x	f(x)
–4	–12
–3	–15
–2.75	–15.125
–2.5	–15
–1.5	–12

c.



37. $f(x) = 0.25x^2 + 3x + 4$

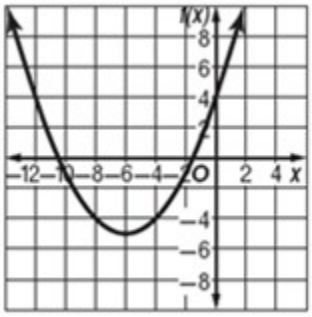
ANSWER:

a. y-int = 4; axis of symmetry: $x = -6$; x-coordinate of vertex = –6

b.

x	f(x)
–10	–1
–8	–4
–6	–5
–4	–4
–2	–1

c.



38. $f(x) = -0.75x^2 + 4x + 6$

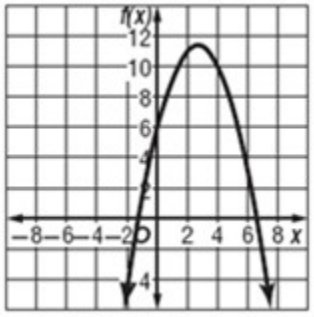
ANSWER:

a. y -int = 6; axis of symmetry: $x = \frac{8}{3}$; x-coordinate of vertex = $\frac{8}{3}$

b.

x	$f(x)$
$\frac{4}{3}$	10
$\frac{7}{3}$	11.25
$\frac{8}{3}$	$11\frac{1}{3}$
3	11.25
4	10

c.



39. $f(x) = \frac{3}{2}x^2 + 4x - \frac{5}{2}$

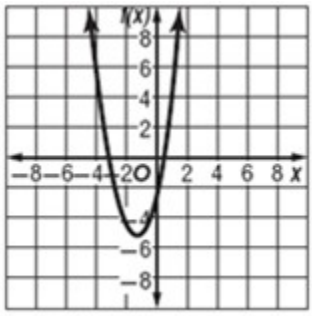
ANSWER:

a. y -int = –2.5; axis of symmetry: $x = -\frac{4}{3}$; x-coordinate of vertex = $-\frac{4}{3}$

b.

x	$f(x)$
$-\frac{11}{3}$	3
$-\frac{8}{3}$	–2.5
$-\frac{4}{3}$	$-5\frac{1}{6}$
0	–2.5
1	3

c.



40. $f(x) = \frac{2}{3}x^2 - \frac{7}{3}x + 9$

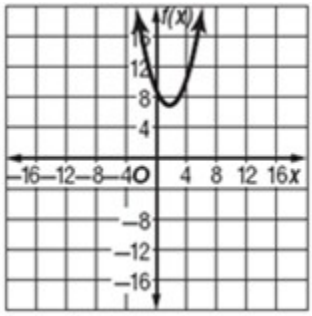
ANSWER:

a. y-int = 9; axis of symmetry: $x = 1.75$; x-coordinate of vertex = 1.75

b.

x	f(x)
0.5	8
1.5	7
1.75	$6\frac{23}{24}$
2	7
3	8

c.



41. **FINANCIAL LITERACY** A babysitting club sits for 50 different families. They would like to increase their current rate of $9.50 per hour. After surveying the families, the club finds that the number of families will decrease by about 2 for each $0.50 increase in the hourly rate.

a. Write a quadratic equation that models this situation.

b. State the domain and range of this function as it applies to the situation.

c. What hourly rate will maximize the club's income? Is this reasonable?

d. What is the maximum income the club can expect to make?

ANSWER:

a. $I(x) = -x^2 + 6x + 475$

b. $D = \{x \mid 0 \le x \le 25\}$; $R = \{y \mid 0 \le y \le 484\}$

c. $11; Because the function has a maximum at $x = 3$, it is in the domain. Therefore, three $0.50 increases is reasonable.

d. $484

42. **ACTIVITIES** Last year, 300 people attended the Franklin High School Drama Club's winter play. The ticket price was $8. The advisor estimates that 20 fewer people would attend for each $1 increase in ticket price.

a. What ticket price would give the greatest income for the Drama Club?

b. If the Drama Club raised its tickets to this price, how much income should it expect to bring in?

ANSWER:

a. $11.50

b. $2645

CCSS TOOLS Use a calculator to find the maximum or minimum of each function. Round to the nearest hundredth if necessary.

43. $f(x) = 12x^2 - 21x + 8$

ANSWER:

min = –1.19

44. $f(x) = -9x^2 - 12x + 19$

ANSWER:

max = 23

45. $f(x) = -8.3x^2 + 14x - 6$

ANSWER:

max = –0.01

46. $f(x) = 9.7x^2 - 13x - 9$

ANSWER:

min = –13.36

47. $f(x) = 28x - 15 - 18x^2$

ANSWER:

max = –4.11

48. $f(x) = -16 - 14x - 12x^2$

ANSWER:

max = –11.92

Determine whether each function has a *maximum* or *minimum* value, and find that value. Then state the domain and range of the function.

49. $f(x) = -5x^2 + 4x - 8$

ANSWER:

max = –7.2; D = {all real numbers},
$R = \{f(x) \mid f(x) \le -7.2\}$

50. $f(x) = -4x^2 - 3x + 2$

ANSWER:

max = 2.5625; D = {all real numbers},
$R = \{f(x) \mid f(x) \le 2.5625\}$

51. $f(x) = -9 + 3x + 6x^2$

ANSWER:

min = –9.375; D = {all real numbers},
$R = \{f(x) \mid f(x) \ge -9.375\}$

52. $f(x) = 2x - 5 - 4x^2$

ANSWER:

max = –4.75; D = {all real numbers},
$R = \{f(x) \mid f(x) \le -4.75\}$

53. $f(x) = \frac{2}{3}x^2 + 6x - 10$

ANSWER:

min = –23.5; D = {all real numbers},
$R = \{f(x) \mid f(x) \ge -23.5\}$

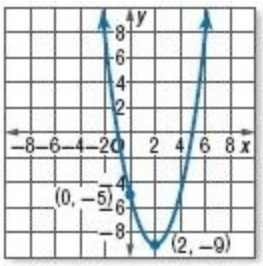
54. $f(x) = -\frac{3}{5}x^2 + 4x - 8$

ANSWER:

max = $-\frac{4}{3}$; D = {all real numbers},

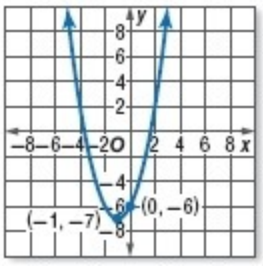
$R = \left\{ f(x) \mid f(x) \le -\frac{4}{3} \right\}$

Determine the function represented by each graph.

55. 

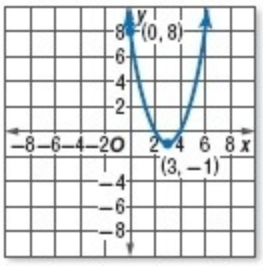
ANSWER:

$f(x) = x^2 - 4x - 5$

56. 

ANSWER:

$f(x) = x^2 + 2x - 6$

57. 

ANSWER:

$f(x) = x^2 - 6x + 8$

58. **MULTIPLE REPRESENTATIONS** Consider $f(x) = x^2 - 4x + 8$ and $g(x) = 4x^2 - 4x + 8$.

a. TABULAR Make a table of values for $f(x)$ and $g(x)$ if $-4 \le x \le 4$.

b. GRAPHICAL Graph $f(x)$ and $g(x)$.

c. VERBAL Explain the difference in the shapes of the graphs of $f(x)$ and $g(x)$. What value was changed to cause this difference?

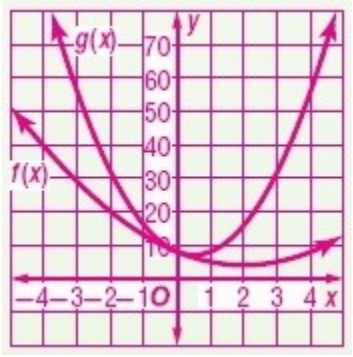
d. ANALYTICAL Predict the appearance of the graph of $h(x) = 0.25x^2 - 4x + 8$. Confirm your prediction by graphing all three functions if $-10 \le x \le 10$.

ANSWER:

a.

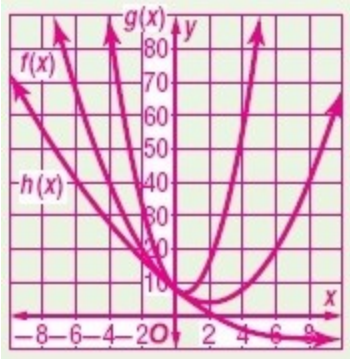
x	$f(x)$	$g(x)$
−4	40	88
−3	29	56
−2	20	32
−1	13	16
0	8	8
1	5	8
2	4	16
3	5	32
4	8	56

b.



c. Sample answer: $g(x)$ is much narrower than $f(x)$. The value of a changed from 1 to 4.

d. Sample answer: The graph of $h(x)$ will be wider than $f(x)$.



59. **VENDING MACHINES** Omar owns a vending machine in a bowling alley. He currently sells 600 cans of soda per week at $0.65 per can. He estimates that he will lose 100 customers for every $0.05 increase in price and gain 100 customers for every $0.05 decrease in price. (*Hint*: The charge *must* be a multiple of 5.)

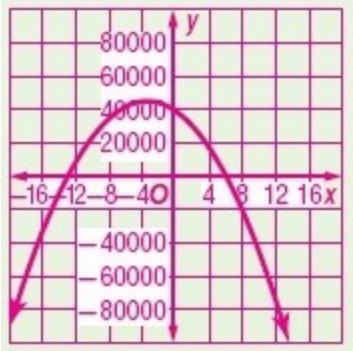
a. Write and graph the related quadratic equation for a price increase.

b. If Omar lowers the price, what price should he charge in order to maximize his income?

c. What will be his income per week from the vending machine?

ANSWER:

a. $f(x) = 39{,}000 - 3500x - 500x^2$



b. Omar can charge at 45 cents or 50 cents.
c. $450 per week

60. **BASEBALL** Lolita throws a baseball into the air and the height h of the ball in feet at a given time t in seconds after she releases the ball is given by the function

$h(t) = -16t^2 + 30t + 5$.

a. State the domain and range for this situation.

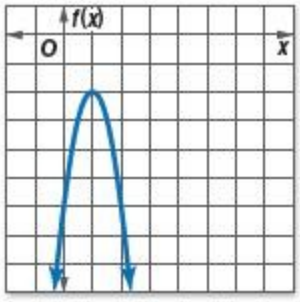
b. Find the maximum height the ball will reach.

ANSWER:

a. $D = \{t \mid 0 \le t \le 2.09\}$, $R = \{h(t) \mid 0 \le h(t) \le 19.0625\}$

b. 19.0625 ft

61. **CCSS CRITIQUE** Trent thinks that the function $f(x)$ graphed below, and the function $g(x)$ described next to it have the same maximum. Madison thinks that $g(x)$ has a greater maximum. Is either of them correct? Explain your reasoning.



$g(x)$ is a quadratic function with roots of 4 and 2 and a y-intercept of −8.

ANSWER:

Sample answer: Madison. When Trent found the x-coordinate of the vertex, he multiplied two negatives and mistakenly kept a negative.

62. **REASONING** Determine whether the following is sometimes, always, or never true. Explain your reasoning.
In a quadratic function, if two x-coordinates are equidistant from the axis of symmetry, then they will have the same y-coordinate.

ANSWER:
Sample answer: Always; the coordinates of a quadratic function are symmetrical, so x-coordinates equidistant from the vertex will have the same y-coordinate.

63. **CHALLENGE** The table at the right represents some points on the graph of a quadratic function.

a. Find the values of a, b, c, and d.

b. What is the x-coordinate of the vertex?

c. Does the function have a maximum or a minimum?

x	y
-20	-377
c	-13
-5	-2
-1	22
$d-1$	a
5	$a-24$
7	$-b$
15	-202
$14-c$	-377

ANSWER:
a. $a = 22$; $b = 26$; $c = -6$; $d = 2$
b. 0
c. maximum

64. **OPEN ENDED** Give an example of a quadratic function with a

a. maximum of 8.

b. minimum of –4.

c. vertex of (–2, 6).

ANSWER:
a. Sample answer: $f(x) = -x^2 + 8$
b. Sample answer: $f(x) = x^2 - 4$
c. Sample answer: $f(x) = x^2 + 4x + 10$

65. **WRITING IN MATH** Why can the discriminant be used to confirm the number and the type of solutions of a quadratic equation?

ANSWER:
Sample answer: If the discriminant is positive, the Quadratic Formula will result in two real solutions because you are adding and subtracting the square root of a positive number in the numerator of the expression. If the discriminant is zero, there will be one real solution because you are adding and subtracting the square root of zero. If the discriminant is negative, there will be two complex solutions because you are adding and subtracting the square root of a negative number in the numerator of the expression.

66. Which expression is equivalent to $\frac{8!}{5!}$

A $\frac{8}{5}$

B $8 \cdot 7 \cdot 6$

C $3!$

D $8 \cdot 7 \cdot 6 \cdot 5$

ANSWER:
B

67. **SAT/ACT** The price of coffee beans is d dollars for 6 ounces, and each ounce makes c cups of coffee. In terms of c and d, what is the cost of the coffee beans required to make 1 cup of coffee?

F $\frac{cd}{6}$

G $\frac{6c}{d}$

H $\frac{6}{cd}$

J $6cd$

K $\frac{d}{6c}$

ANSWER:
K

68. **SHORT RESPONSE** Each side of the square base of a pyramid is 20 feet, and the pyramid's height is 90 feet. What is the volume of the pyramid?

ANSWER:
12,000 ft^3

69. Which ordered pair is the solution of the following system of equations?

$3x - 5y = 11$
$3x - 8y = 5$

A (2, 1)

B (7, –2)

C (7, 2)

D $\left(\frac{1}{3}, -2\right)$

ANSWER:
C

Find the inverse of each matrix, if it exists.

70. $\begin{bmatrix} 3 & -4 \\ 2 & -1 \end{bmatrix}$

ANSWER:
$\begin{bmatrix} -\frac{1}{5} & \frac{4}{5} \\ -\frac{2}{5} & \frac{3}{5} \end{bmatrix}$

71. $\begin{bmatrix} -4 & -1 \\ 0 & 6 \end{bmatrix}$

ANSWER:
$\begin{bmatrix} -\frac{1}{4} & -\frac{1}{24} \\ 0 & \frac{1}{6} \end{bmatrix}$

72. $\begin{bmatrix} 2 & 8 \\ -3 & -5 \end{bmatrix}$

ANSWER:
$\begin{bmatrix} -\frac{5}{14} & -\frac{4}{7} \\ \frac{3}{14} & \frac{1}{7} \end{bmatrix}$

Evaluate each determinant.

73. $\begin{vmatrix} 6 & -3 \\ -1 & 8 \end{vmatrix}$

ANSWER:
45

74. $\begin{vmatrix} -3 & -5 \\ -1 & -9 \end{vmatrix}$

ANSWER:
22

75. $\begin{vmatrix} 8 & 6 \\ 4 & 3 \end{vmatrix}$

ANSWER:
0

76. **MANUFACTURING** The Community Service Committee is making canvas tote bags and leather tote bags for a fundraiser. They will line both types of bags with canvas and use leather handles on both. For the canvas bags, they need 4 yards of canvas and 1 yard of leather. For the leather bags, they need 3 yards of leather and 2 yards of canvas. The committee leader purchased 56 yards of leather and 104 yards of canvas.

a. Let c represent the number of canvas bags, and let ℓ represent the number of leather bags. Write a system of inequalities for the number of bags that can be made.

b. Draw the graph showing the feasible region.

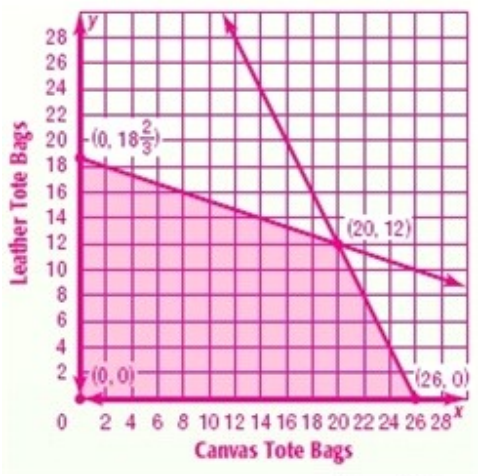
c. List the coordinates of the vertices of the feasible region.

d. If the club plans to sell the canvas bags at a profit of $20 each and the leather bags at a profit of $35 each, write a function for the total profit on the bags.

e. How can the club make the maximum profit?

f. What is the maximum profit?

ANSWER:
a. $c \geq 0, \ell \geq 0, c + 3\ell \leq 56, 4c + 2\ell \leq 104$
b.



c. (0, 0), (26, 0), (20, 12), $\left(0, 18\frac{2}{3}\right)$

d. $f(c, \ell) = 20c + 35\ell$

e. Make 20 canvas tote bags and 12 leather tote bags.
f. $820

State whether each function is a linear function. Write *yes* or *no*. Explain.

77. $y = 4x^2 - 3x$

ANSWER:
No; it cannot be written as $y = mx + b$.

78. $y = -2x - 4$

ANSWER:
Yes; it is written in $y = mx + b$ form.

79. $y = 4$

ANSWER:
Yes; it is written in $y = mx + b$ form, $m = 0$.

Evaluate each function for the given value.

80. $f(x) = 3x^2 - 4x + 6, x = -2$

ANSWER:
26

81. $f(x) = -2x^2 + 6x - 5, x = 4$

ANSWER:
–13

82. $f(x) = 6x^2 + 18, x = -5$

ANSWER:
168