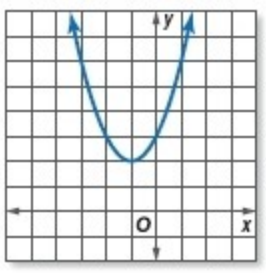
4-2 Solving Quadratic Equations by Graphing

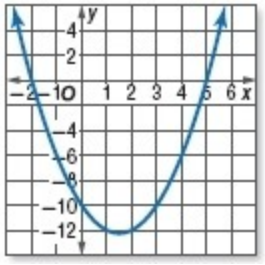
Use the related graph of each equation to determine its solutions.

1. $x^2 + 2x + 3 = 0$



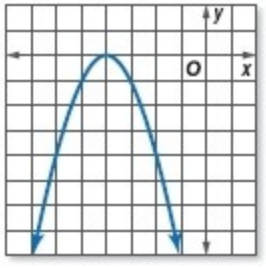
ANSWER:
no real solution

2. $x^2 - 3x - 10 = 0$



ANSWER:
–2, 5

3. $-x^2 - 8x - 16 = 0$

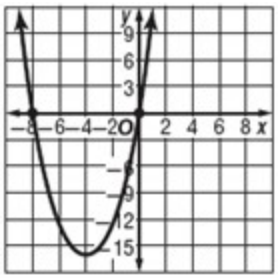


ANSWER:
–4

CCSS PRECISION Solve each equation. If exact roots cannot be found, state the consecutive integers between which the roots are located.

4. $x^2 + 8x = 0$

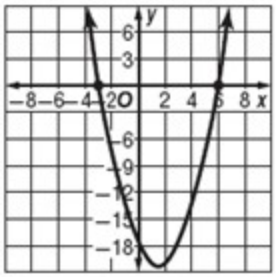
ANSWER:



0, –8

5. $x^2 - 3x - 18 = 0$

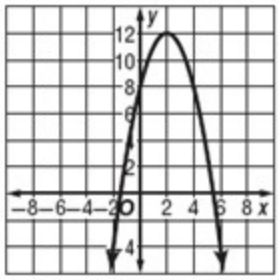
ANSWER:



– 3, 6

6. $4x - x^2 + 8 = 0$

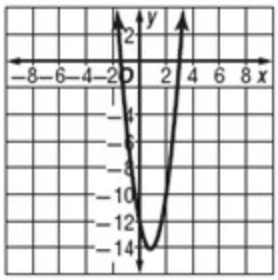
ANSWER:



between –2 and –1, between 5 and 6

7. $-12 - 5x + 3x^2 = 0$

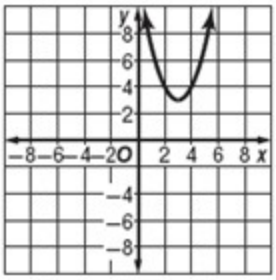
ANSWER:



between –2 and –1, 3

8. $x^2 - 6x + 4 = -8$

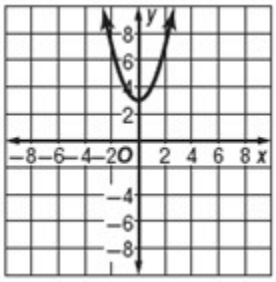
ANSWER:



no real solution

9. $9 - x^2 = 12$

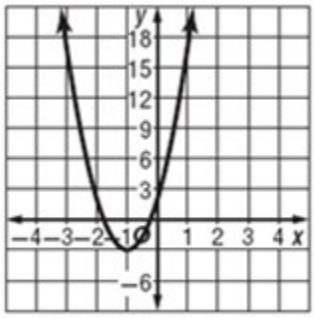
ANSWER:



no real solution

10. $5x^2 + 10x - 4 = -6$

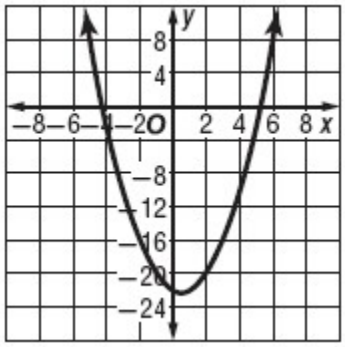
ANSWER:



between –2 and –1, between –1 and 0

11. $x^2 - 20 = 2 + x$

ANSWER:



between –5 and –4, between 5 and 6

12. **NUMBER THEORY** Use a quadratic equation to find two real numbers with a sum of 2 and a product of –24.

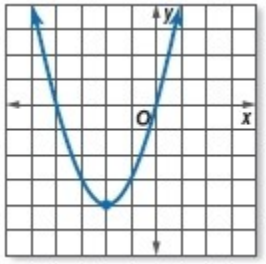
ANSWER:
6 and –4

13. **PHYSICS** How long will it take an object to fall from the roof of a building 400 feet above ground? Use the formula $h(x) = -16t^2 + h_0$, where t is the time in seconds and the initial height h_0 is in feet.

ANSWER:
5 seconds

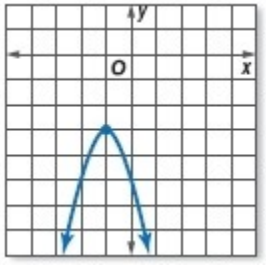
Use the related graph of each equation to determine its solutions.

14. $x^2 + 4x = 0$



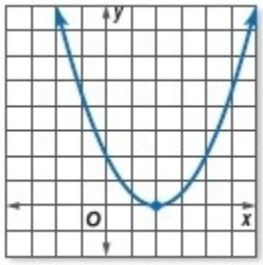
ANSWER:
–4, 0

15. $-2x^2 - 4x - 5 = 0$



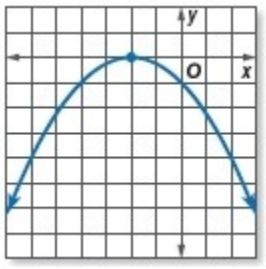
ANSWER:
no real solution

16. $0.5x^2 - 2x + 2 = 0$



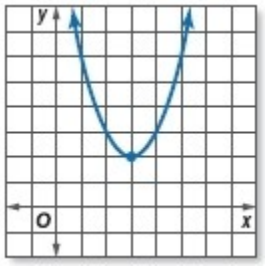
ANSWER:
2

17. $-0.25x^2 - x - 1 = 0$



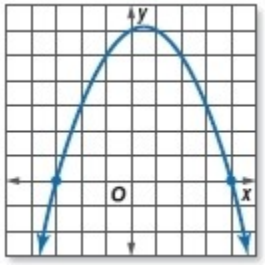
ANSWER:
–2

18. $x^2 - 6x + 11 = 0$



ANSWER:
no real solution

19. $-0.5x^2 + 0.5x + 6 = 0$

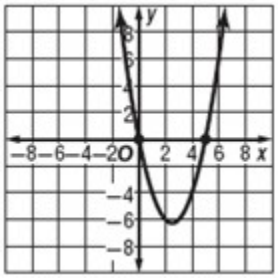


ANSWER:
–3, 4

Solve each equation. If exact roots cannot be found, state the consecutive integers between which the roots are located.

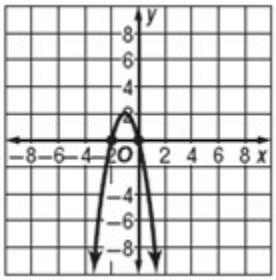
20. $x^2 = 5x$

ANSWER:
0, 5



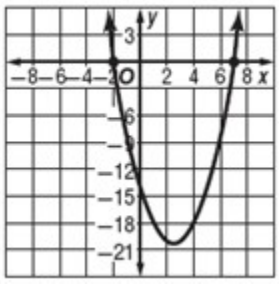
21. $-2x^2 - 4x = 0$

ANSWER:
–2, 0



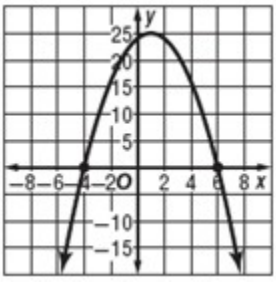
22. $x^2 - 5x - 14 = 0$

ANSWER:
−2, 7



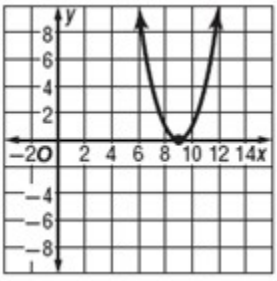
23. $-x^2 + 2x + 24 = 0$

ANSWER:
–4, 6



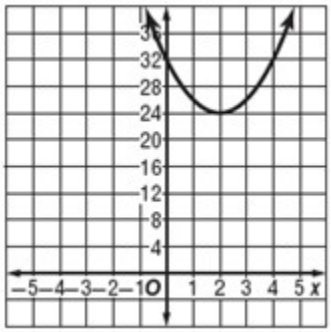
24. $x^2 - 18x = -81$

ANSWER:
9



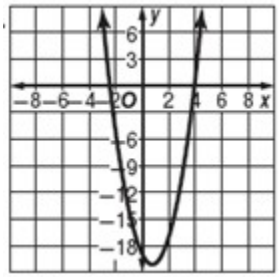
25. $2x^2 - 8x = -32$

ANSWER:
no real solution



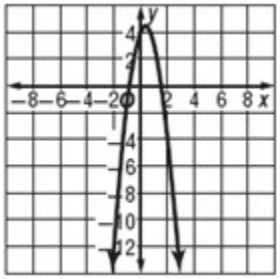
26. $2x^2 - 3x - 15 = 4$

ANSWER:
between –3 and –2 and between 3 and 4



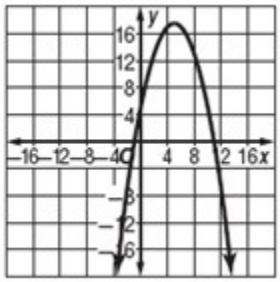
27. $-3x^2 - 7 + 2x = -11$

ANSWER:
between –1 and 0 and between 1 and 2



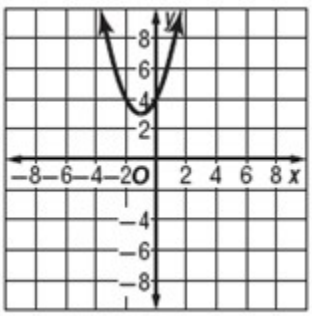
28. $-0.5x^2 + 3 = -5x - 2$

ANSWER:
between –1 and 0 and between 10 and 11



29. $-2x + 12 = x^2 + 16$

ANSWER:
no real solution



Use the tables to determine the location of the zeros of each quadratic function.

30.

x	–7	–6	–5	–4	–3	–2	–1	0
$f(x)$	–8	–1	4	4	–1	–8	–22	–48

ANSWER:
between –6 and –5;
between –4 and –3

31.

x	–2	–1	0	1	2	3	4	5
$f(x)$	32	14	2	–3	–3	2	14	32

ANSWER:
between 0 and 1;
between 2 and 3

32.

x	–6	–3	0	3	6	9	12	15
$f(x)$	–6	–1	3	5	3	–1	–6	–14

ANSWER:
between –3 and 0;
between 6 and 9

NUMBER THEORY Use a quadratic equation to find two real numbers that satisfy each situation, or show that no such numbers exist.

33. Their sum is –15, and their product is –54.

ANSWER:
3 and –18

34. Their sum is 4, and their product is –117.

ANSWER:
13 and –9

35. Their sum is 12, and their product is –84.

ANSWER:
about –5 and 17

36. Their sum is –13, and their product is 42.

ANSWER:
–6 and –7

37. Their sum is –8 and their product is –209.

ANSWER:
11 and –19

CCSS MODELING For Exercises 38–40, use the formula $h(t) = v_0t - 16t^2$, where $h(t)$ is the height of an object in feet, v_0 is the object's initial velocity in feet per second, and t is the time in seconds.

38. **BASEBALL** A baseball is hit directly upward with an initial velocity of 80 feet per second. Ignoring the height of the baseball player, how long does it take for the ball to hit the ground?

ANSWER:
5 seconds

39. **CANNONS** A cannonball is shot directly upward with an initial velocity of 55 feet per second. Ignoring the height of the cannon, how long does it take for the cannonball to hit the ground?

ANSWER:
about 3.4375 seconds

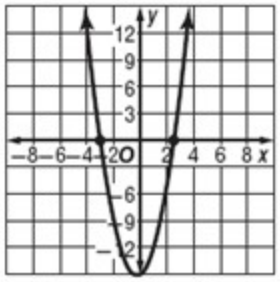
40. **GOLF** A golf ball is hit directly upward with an initial velocity of 100 feet per second. How long will it take for it to hit the ground?

ANSWER:
6.25 seconds

Solve each equation. If exact roots cannot be found, state the consecutive integers between which the roots are located.

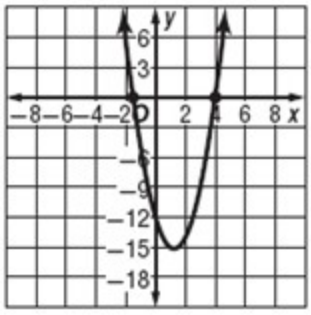
41. $2x^2 + x = 15$

ANSWER:
–3, between 2 and 3



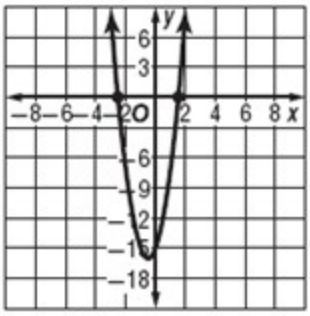
42. $-5x - 12 = -2x^2$

ANSWER:
$-\frac{3}{2}$, 4



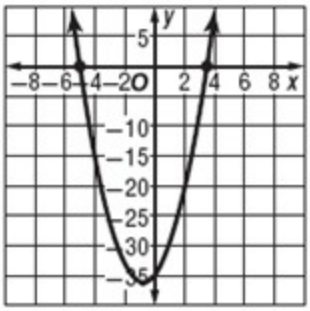
43. $4x^2 - 15 = -4x$

ANSWER:
between –3 and –2, between 1 and 2



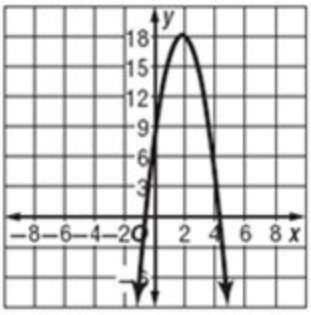
44. $-35 = -3x - 2x^2$

ANSWER:
–5, between 3 and 4.



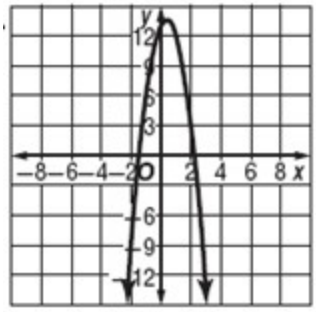
45. $-3x^2 + 11x + 9 = 1$

ANSWER:
between –1 and 0, between 4 and 5



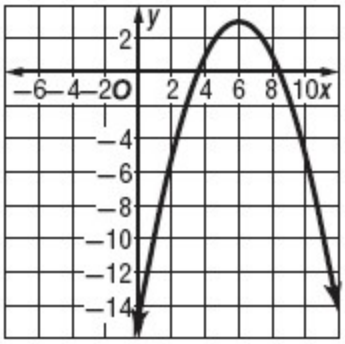
46. $13 - 4x^2 = -3x$

ANSWER:
between –2 and –1, between 2 and 3



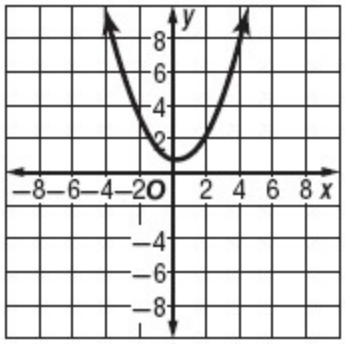
47. $-0.5x^2 + 18 = -6x + 33$

ANSWER:
between 3 and 4, between 8 and 9



48. $0.5x^2 + 0.75 = 0.25x$

ANSWER:
No real solution.



49. **WATER BALLOONS** Tony wants to drop a water balloon so that it splashes on his brother. Use the formula $h(t) = -16t^2 + h_0$, where t is the time in seconds and the initial height h_0 is in feet, to determine how far his brother should be from the target when Tony lets go of the balloon.

ANSWER:
about 8.5 ft

50. **WATER HOSES** A water hose can spray water at an initial velocity of 40 feet per second. Use the formula $h(t) = v_0t - 16t^2$, where $h(t)$ is the height of the water in feet, v_0 is the initial velocity in feet per second, and t is the time in seconds.

a. How long will it take the water to hit the nozzle on the way down?

b. Assuming the nozzle is 5 feet up, what is the maximum height of the water?

ANSWER:
a. 2.5 seconds

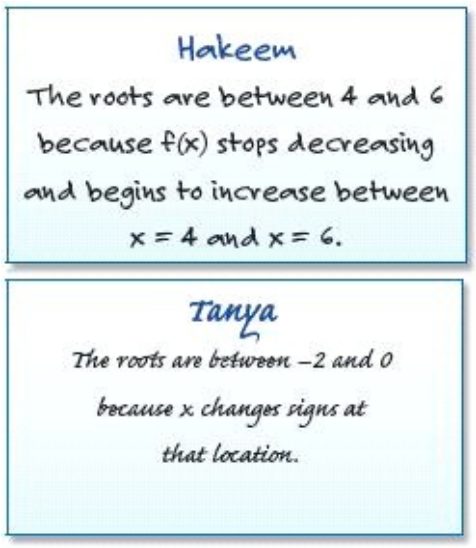
b. 30 ft

51. **SKYDIVING** In 2003, John Fleming and Dan Rossi became the first two blind skydivers to be in free fall together. They jumped from an altitude of 14,000 feet and free fell to an altitude of 4000 feet before their parachutes opened. Ignoring air resistance and using the formula $h(t) = -16t^2 + h_0$, where t is the time in seconds and the initial height h_0 is in feet, determine how long they were in free fall.

ANSWER:
25 seconds

52. **CCSS CRITIQUE** Hakeem and Tanya were asked to find the location of the roots of the quadratic function represented by the table. Is either of them correct? Explain.

x	–4	–2	0	2	4	6	8	10
$f(x)$	52	26	8	–2	–4	2	16	38



ANSWER:

Sample answer: No; roots are located where $f(x)$ changes signs.

53. **CHALLENGE** Find the value of a positive integer k such that $f(x) = x^2 - 2kx + 55$ has roots at $k + 3$ and $k - 3$.

ANSWER:

$k = 8$

54. **REASONING** If a quadratic function has a minimum at (–6, –14) and a root at $x = -17$, what is the other root? Explain your reasoning.

ANSWER:

Sample answer: The other root is at $x = 5$ because the x-coordinates of the roots need to be equidistant from the x-value of the vertex.

55. **OPEN ENDED** Write a quadratic function with a maximum at (3, 125) and roots at –2 and 8.

ANSWER:

$f(x) = -5x^2 + 30x + 80$

56. **WRITING IN MATH** Explain how to solve a quadratic equation by graphing its related quadratic function.

ANSWER:

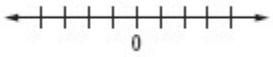
Sample answer: Graph the function using the axis of symmetry. Determine where the graph intersects the x-axis. The x-coordinates of those points are solutions to the quadratic equation.

57. **SHORT RESPONSE** A bag contains five different colored marbles. The colors of the marbles are black, silver, red, green, and blue. A student randomly chooses a marble. Then, without replacing it, chooses a second marble. What is the probability that the student chooses the red and then the green marble?

ANSWER:

$\frac{1}{20}$

58. Which number would be closest to zero on the number line?



A –0.6

B $\frac{2}{5}$

C $\frac{\sqrt{2}}{2}$

D 0.5

ANSWER:

B

59. **SAT/ACT** A salesman's monthly gross pay consists of $3500 plus 20 percent of the dollar amount of his sales. If his gross pay for one month was $15,500, what was the dollar amount of his sales for that month?

F $12,000

G $16,000

H $60,000

J $70,000

K $77,000

ANSWER:
H

60. Find the next term in the sequence below.

$\frac{2x}{5}, \frac{3x}{5}, \frac{4x}{5}, \ldots$

A x

B $5x$

C $\frac{x}{5}$

D $\frac{5x}{4}$

ANSWER:
A

Determine whether each function has a *maximum* or *minimum* value, and find that value. Then state the domain and range of the function.

61. $f(x) = -4x^2 + 8x - 16$

ANSWER:
maximum, –12; D = {all real numbers}, R = $\{f(x) \mid f(x) \le -12\}$.

62. $f(x) = 3x^2 + 12x - 18$

ANSWER:
minimum, –30; D = {all real numbers}, $R = \{f(x) \mid f(x) \ge -30\}$

63. $f(x) = 4x + 13 - 2x^2$

ANSWER:
maximum, 15; D = {all real numbers}, $R = \{f(x) \mid f(x) \le 15\}$

Determine whether each pair of matrices are inverses of each other.

64. $\begin{bmatrix} 4 & -3 \\ -1 & -6 \end{bmatrix}$ and $\begin{bmatrix} \frac{3}{13} & -\frac{1}{18} \\ -\frac{1}{26} & -\frac{2}{13} \end{bmatrix}$

ANSWER:
no

65. $\begin{bmatrix} 6 & -3 \\ 4 & 8 \end{bmatrix}$ and $\begin{bmatrix} \frac{1}{10} & \frac{1}{20} \\ -\frac{1}{15} & \frac{2}{15} \end{bmatrix}$

ANSWER:
no

66. $\begin{bmatrix} 2 & 4 \\ -3 & -2 \end{bmatrix}$ and $\begin{bmatrix} -\frac{1}{4} & -\frac{1}{2} \\ \frac{3}{8} & \frac{1}{4} \end{bmatrix}$

ANSWER:
yes

Solve each system of equations.

67. $4x - 7y = -9$
$5x + 2y = -22$

ANSWER:
$(-4, -1)$

68. $3x + 8y = 24$
$-16y - 6x = 48$

ANSWER:
no solution

69. $8y - 2x = 38$
$5x - 3y = -27$

ANSWER:
$(-3, 4)$

70. **SALES** Alex is in charge of stocking shirts for the concession stand at the high school football game. The number of shirts needed for a regular season game is listed in the matrix. Alex plans to double the number of shirts stocked for a playoff game.

a. Write a matrix A to represent the regular season stock.

b. What scalar can be used to determine a matrix M to represent the new numbers? Find M.

c. What is $M - A$? What does this represent in this situation?

Size	small	medium	large
Child	10	10	15
Adult	25	35	45

ANSWER:
a. $\begin{bmatrix} 10 & 10 & 15 \\ 25 & 35 & 45 \end{bmatrix}$

b. $2\begin{bmatrix} 10 & 10 & 15 \\ 25 & 35 & 45 \end{bmatrix} = \begin{bmatrix} 20 & 20 & 30 \\ 50 & 70 & 90 \end{bmatrix}$

c. $\begin{bmatrix} 10 & 10 & 15 \\ 25 & 35 & 45 \end{bmatrix}$; The number of shirts that he needs to stock additionally.

Solve each inequality.

71. $3x - 6 \le -14$

ANSWER:
$x \le -\frac{8}{3}$

72. $-6x + 3 \ge 3x - 16$

ANSWER:
$x \le \frac{19}{9}$

73. $6 - 4x \le 2$

ANSWER:
$x \ge 1$

Find the GCF of each set of numbers.

74. 16, 48, 128

ANSWER:
16

75. 15, 21, 49

ANSWER:
1

76. 12, 28, 36

ANSWER:
4