

### 4-3 Solving Quadratic Equations by Factoring

Write a quadratic equation in standard form with the given root(s).

1.  $-8, 5$

ANSWER:

$$x^2 + 3x - 40 = 0$$

2.  $\frac{3}{2}, \frac{1}{4}$

ANSWER:

$$8x^2 - 14x + 3 = 0$$

3.  $-\frac{2}{3}, \frac{5}{2}$

ANSWER:

$$6x^2 - 11x - 10 = 0$$

Factor each polynomial.

4.  $35x^2 - 15x$

ANSWER:

$$5x(7x - 3)$$

5.  $18x^2 - 3x + 24x - 4$

ANSWER:

$$(6x - 1)(3x + 4)$$

6.  $x^2 - 12x + 32$

ANSWER:

$$(x - 8)(x - 4)$$

7.  $x^2 - 4x - 21$

ANSWER:

$$(x - 7)(x + 3)$$

8.  $2x^2 + 7x - 30$

ANSWER:

$$(2x - 5)(x + 6)$$

9.  $16x^2 - 16x + 3$

ANSWER:

$$(4x - 3)(4x - 1)$$

Solve each equation.

10.  $x^2 - 36 = 0$

ANSWER:

$$-6, 6$$

11.  $12x^2 - 18x = 0$

ANSWER:

$$0, \frac{3}{2}$$

12.  $12x^2 - 2x - 2 = 0$

ANSWER:

$$-\frac{1}{3}, \frac{1}{2}$$

13.  $x^2 - 9x = 0$

ANSWER:

$$0, 9$$

14.  $x^2 - 3x - 28 = 0$

ANSWER:

$$-4, 7$$

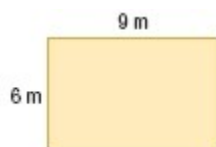
### 4-3 Solving Quadratic Equations by Factoring

15.  $2x^2 - 24x = -72$

ANSWER:

6

16. **CCSS SENSE-MAKING** Tamika wants to double the area of her garden by increasing the length and width by the same amount. What will be the dimensions of her garden then?



ANSWER:

9 m by 12 m

**Write a quadratic equation in standard form with the given root(s).**

17. 7

ANSWER:

$$x^2 - 14x + 49 = 0$$

18.  $-5, \frac{1}{2}$

ANSWER:

$$2x^2 + 9x - 5 = 0$$

19.  $\frac{1}{5}, 6$

ANSWER:

$$5x^2 - 31x + 6 = 0$$

**Factor each polynomial.**

20.  $40a^2 - 32a$

ANSWER:

$$8a(5a - 4)$$

21.  $51c^3 - 34c$

ANSWER:

$$17c(3c^2 - 2)$$

22.  $32xy + 40bx - 12ay - 15ab$

ANSWER:

$$(8x - 3a)(4y + 5b)$$

23.  $3x^2 - 12$

ANSWER:

$$3(x + 2)(x - 2)$$

24.  $15y^2 - 240$

ANSWER:

$$15(y + 4)(y - 4)$$

25.  $48cg + 36cf - 4dg - 3df$

ANSWER:

$$(12c - d)(4g + 3f)$$

26.  $x^2 + 13x + 40$

ANSWER:

$$(x + 8)(x + 5)$$

### 4-3 Solving Quadratic Equations by Factoring

27.  $x^2 - 9x - 22$

ANSWER:

$(x - 11)(x + 2)$

28.  $3x^2 + 12x - 36$

ANSWER:

$3(x + 6)(x - 2)$

29.  $15x^2 + 7x - 2$

ANSWER:

$(5x - 1)(3x + 2)$

30.  $4x^2 + 29x + 30$

ANSWER:

$(4x + 5)(x + 6)$

31.  $18x^2 + 15x - 12$

ANSWER:

$3(2x - 1)(3x + 4)$

32.  $8x^2z^2 - 4xz^2 - 12z^2$

ANSWER:

$4z^2(2x - 3)(x + 1)$

33.  $9x^2 - 25$

ANSWER:

$(3x + 5)(3x - 5)$

34.  $18x^2y^2 - 24xy^2 + 36y^2$

ANSWER:

$6y^2(3x^2 - 4x + 6)$

35.  $15x^2 - 84x - 36$

ANSWER:

$3(5x + 2)(x - 6)$

36.  $12x^2 + 13x - 14$

ANSWER:

$(4x + 7)(3x - 2)$

37.  $12xy^2 - 108x$

ANSWER:

$12x(y + 3)(y - 3)$

**Solve each equation by factoring.**

38.  $x^2 + 4x - 45 = 0$

ANSWER:

5, -9

39.  $x^2 - 5x - 24 = 0$

ANSWER:

8, -3

40.  $x^2 = 121$

ANSWER:

11, -11

### 4-3 Solving Quadratic Equations by Factoring

41.  $x^2 + 13 = 17$

ANSWER:

2, -2

42.  $-3x^2 - 10x + 8 = 0$

ANSWER:

$-4, \frac{2}{3}$

43.  $-8x^2 + 46x - 30 = 0$

ANSWER:

$5, \frac{3}{4}$

44. **GEOMETRY** The hypotenuse of a right triangle is 1 centimeter longer than one side and 4 centimeters longer than three times the other side. Find the dimensions of the triangle.

ANSWER:

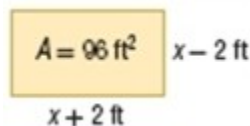
7 cm, 24 cm, 25 cm

45. **NUMBER THEORY** Find two consecutive even integers with a product of 624.

ANSWER:

24 and 26 or -24 and -26

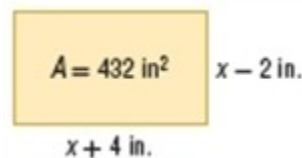
**GEOMETRY** Find  $x$  and the dimensions of each rectangle.



46.

ANSWER:

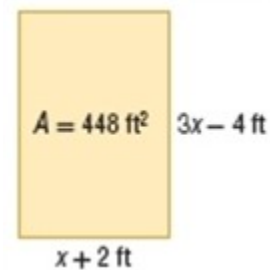
$x = 10$ ; 8 ft by 12 ft



47.

ANSWER:

$x = 20$ ; 24 in. by 18 in.



48.

ANSWER:

$x = 12$ ; 14 ft by 32 ft

**Solve each equation by factoring.**

49.  $12x^2 - 4x = 5$

ANSWER:

$-\frac{1}{2}, \frac{5}{6}$

### 4-3 Solving Quadratic Equations by Factoring

50.  $5x^2 = 15x$

ANSWER:

0, 3

51.  $16x^2 + 36 = -48x$

ANSWER:

$-\frac{3}{2}$

52.  $75x^2 - 60x = -12$

ANSWER:

$\frac{2}{5}$

53.  $4x^2 - 144 = 0$

ANSWER:

6, -6

54.  $-7x + 6 = 20x^2$

ANSWER:

$\frac{2}{5}, -\frac{3}{4}$

55. **MOVIE THEATER** A company plans to build a large multiplex theater. The financial analyst told her manager that the profit function for their theater was  $P(x) = -x^2 + 48x - 512$ , where  $x$  is the number of movie screens, and  $P(x)$  is the profit earned in thousands of dollars. Determine the range of production of movie screens that will guarantee that the company will not lose money.

ANSWER:

16 to 32 screens

**Write a quadratic equation in standard form with the given root(s).**

56.  $-\frac{4}{7}, \frac{3}{8}$

ANSWER:

$$56x^2 + 11x - 12 = 0$$

57. 3.4, 0.6

ANSWER:

$$25x^2 - 100x + 51 = 0$$

58.  $\frac{2}{11}, \frac{5}{9}$

ANSWER:

$$99x^2 - 73x + 10 = 0$$

**Solve each equation by factoring.**

59.  $10x^2 + 25x = 15$

ANSWER:

$-3, \frac{1}{2}$

### 4-3 Solving Quadratic Equations by Factoring

60.  $27x^2 + 5 = 48x$

ANSWER:

$$\frac{5}{3}, \frac{1}{9}$$

61.  $x^2 + 0.25x = 1.25$

ANSWER:

$$1, -\frac{5}{4}$$

62.  $48x^2 - 15 = -22x$

ANSWER:

$$\frac{3}{8}, -\frac{5}{6}$$

63.  $3x^2 + 2x = 3.75$

ANSWER:

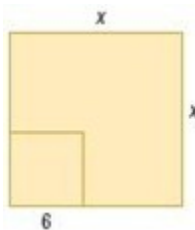
$$-\frac{3}{2}, \frac{5}{6}$$

64.  $-32x^2 + 56x = 12$

ANSWER:

$$\frac{1}{4}, \frac{3}{2}$$

65. **DESIGN** A square is cut out of the figure at the right. Write an expression for the area of the figure that remains, and then factor the expression.



ANSWER:

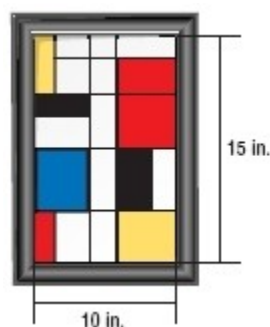
$$x^2 - 6^2; (x + 6)(x - 6)$$

66. **CCSS PERSEVERANCE** After analyzing the market, a company that sells Web sites determined the profitability of their product was modeled by  $P(x) = -16x^2 + 368x - 2035$ , where  $x$  is the price of each Web site and  $P(x)$  is the company's profit. Determine the price range of the Web sites that will be profitable for the company.

ANSWER:

\$9.25 to \$13.75

67. **PAINTINGS** Enola wants to add a border to her painting, distributed evenly, that has the same area as the painting itself. What are the dimensions of the painting with the border included?



ANSWER:

20 in. by 15 in.

68. **MULTIPLE REPRESENTATIONS** In this problem, you will consider  $a(x - p)(x - q) = 0$ .

### 4-3 Solving Quadratic Equations by Factoring

**a. GRAPHICAL** Graph the related function for  $a = 1, p = 2$ , and  $q = -3$ .

**b. ANALYTICAL** What are the solutions of the equation?

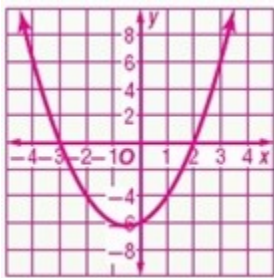
**c. GRAPHICAL** Graph the related functions for  $a = 4, -3$ , and  $\frac{1}{2}$  on the same graph.

**d. VERBAL** What are the similarities and differences between the graphs?

**e. VERBAL** What conclusion can you make about the relationship between the factored form of a quadratic equation and its solutions?

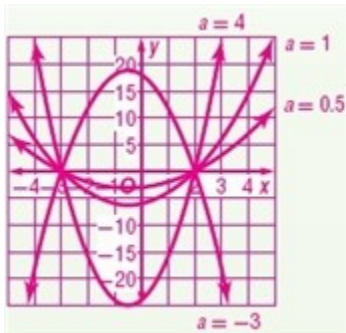
**ANSWER:**

**a.**



**b.** 2 and  $-3$

**c.**

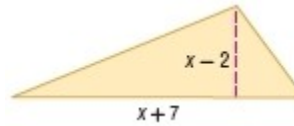


**d.** Sample answer: They all have the same roots,  $p$  and  $q$ . Therefore, they all have the same solutions. The graphs are shaped differently due to the value of  $a$ . The graph with  $a = -3$  is flipped due to the negative.

**e.** When quadratic equations have the same factors, they will have the same solutions, regardless of the

value of  $a$ , which only affects the shape of the graphs.

**69. GEOMETRY** The area of the triangle is 26 square centimeters. Find the length of the base.



**ANSWER:**

13 cm

**70. SOCCER** When a ball is kicked in the air, its height in meters above the ground can be modeled by  $h(t) = -4.9t^2 + 14.7t$  and the distance it travels can be modeled by  $d(t) = 16t$ , where  $t$  is the time in seconds.

**a.** How long was the ball in the air?

**b.** How far did it travel before it hit the ground? (Hint: Ignore air resistance.)

**c.** What was the maximum height of the ball?

**ANSWER:**

**a.** 3 seconds

**b.** 48 m

**c.** 11.025 m

**Factor each polynomial.**

**71.**  $18a - 24ay + 48b - 64by$

**ANSWER:**

$2(3 - 4y)(3a + 8b)$

### 4-3 Solving Quadratic Equations by Factoring

72.  $3x^2 + 2xy + 10y + 15x$

ANSWER:

$$(3x + 2y)(x + 5)$$

73.  $6a^2b^2 - 12ab^2 - 18b^3$

ANSWER:

$$6b^2(a^2 - 2a - 3b)$$

74.  $12a^2 - 18ab + 30ab^3$

ANSWER:

$$6a(2a - 3b + 5b^3)$$

75.  $32ax + 12bx - 48ay - 18by$

ANSWER:

$$2(2x - 3y)(8a + 3b)$$

76.  $30ac + 80bd + 40ad + 60bc$

ANSWER:

$$10(a + 2b)(3c + 4d)$$

77.  $5ax^2 - 2by^2 - 5ay^2 + 2bx^2$

ANSWER:

$$(x + y)(x - y)(5a + 2b)$$

78.  $12c^2x + 4d^2y - 3d^2x - 16c^2y$

ANSWER:

$$(2c + d)(2c - d)(3x - 4y)$$

79. **ERROR ANALYSIS** Gwen and Morgan are solving  $-12x^2 + 5x + 2 = 0$ . Is either of them correct? Explain your reasoning.

Gwen

$$\begin{aligned} -12x^2 + 5x + 2 &= 0 \\ -12x^2 + 8x - 3x + 2 &= 0 \\ 4x(-3x + 2) - (3x + 2) &= 0 \\ (4x - 1)(3x + 2) &= 0 \\ x &= \frac{1}{4} \text{ or } -\frac{2}{3} \end{aligned}$$

Morgan

$$\begin{aligned} -12x^2 + 5x + 2 &= 0 \\ -12x^2 + 8x - 3x + 2 &= 0 \\ 4x(-3x + 2) + (-3x + 2) &= 0 \\ (4x + 1)(-3x + 2) &= 0 \\ x &= -\frac{1}{4} \text{ or } \frac{2}{3} \end{aligned}$$

ANSWER:

Sample answer: Morgan; Gwen did not have like terms in the parentheses in the third line.

80. **CHALLENGE** Solve  $3x^6 - 39x^4 + 108x^2 = 0$  by factoring.

ANSWER:

0, 3, -3, 2, or -2

81. **CHALLENGE** The rule for factoring a difference of cubes is shown below. Use this rule to factor  $40x^5 - 135x^2y^3$ .]

$$\begin{aligned} a^3 - b^3 \\ = (a - b)(a^2 + ab + b^2) \end{aligned}$$

ANSWER:

$$5x^2(2x - 3y)(4x^2 + 6xy + 9y^2)$$



### 4-3 Solving Quadratic Equations by Factoring

82. **OPEN ENDED** Choose two integers. Then write an equation in standard form with those roots. How would the equation change if the signs of the two roots were switched?

**ANSWER:**

Sample answer: 3 and 6  $\rightarrow x^2 - 9x + 18 = 0$ .  $-3$  and  $-6 \rightarrow x^2 + 9x + 18 = 0$ . The linear term changes sign.

83. **CHALLENGE** For a quadratic equation of the form  $(x - p)(x - q) = 0$ , show that the axis of symmetry of the related quadratic function is located halfway between the  $x$ -intercepts  $p$  and  $q$ .

**ANSWER:**

Sample answer:

$$(x - p)(x - q) = 0 \quad \text{Original equation}$$

$$x^2 - px - qx + pq = 0 \quad \text{Multiply}$$

$$x^2 - (p + q)x + pq = 0 \quad \text{Simplify.}$$

$$x = -\frac{b}{2a} \quad \text{Formula for axis of symmetry}$$

$$x = -\frac{-(p + q)}{2(1)} \quad a = 1 \text{ and } b = -(p + q)$$

$$x = \frac{p + q}{2} \quad \text{Simplify.}$$

$x$  is midway between  $p$  and  $q$ . Definition of midpoint

84. **WRITE A QUESTION** A classmate is using the guess and check strategy to factor trinomials of the form  $x^2 + bx + c$ . Write a question to help him think of a way to use that strategy for  $ax^2 + bx + c$ .

**ANSWER:**

Sample answer: What do you know about  $a \cdot c$  to use guess and check to factor trinomials of the form  $ax^2 + bx + c$ ?

85. **CCSS ARGUMENTS** Determine whether the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

*In a quadratic equation in standard form where  $a$ ,  $b$ , and  $c$  are integers, if  $b$  is odd, then the quadratic cannot be a perfect square trinomial.*

**ANSWER:**

Sample answer: Always; in order to factor using perfect square trinomials, the coefficient of the linear term,  $bx$ , must be a multiple of 2, or even.

86. **WRITING IN MATH** Explain how to factor a trinomial in standard form with  $a > 1$ .

**ANSWER:**

Sample answer: In standard form, we have  $ax^2 + bx + c$ . Multiply  $a$  and  $c$ .

Then find a pair of integers,  $g$  and  $h$ , that multiply to equal  $ac$  and add to equal  $b$ .

Then write out the quadratic, turning the middle term,  $bx$ , into  $gx + hx$ .

We now have  $ax^2 + gx + hx + c$ .

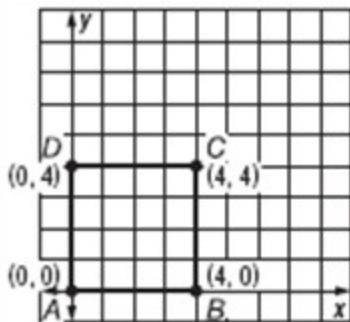
Now factor the GCF from the first two terms and then factor the GCF from the second two terms.

So we now have  $\text{GCF}(x - q) + \text{GCF}_2(x - q)$ .

Simplifying, we get  $(\text{GCF} + \text{GCF}_2)(x - q)$  or  $(x - p)(x - q)$ .

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87. **SHORT RESPONSE** If  $ABCD$  is transformed by  $(x, y) \rightarrow (3x, 4y)$ , determine the area of  $A'B'C'D'$ .



**ANSWER:**

192 square units

88. For  $y = 2|6 - 3x| + 4$ , which set describes  $x$  when  $y < 6$ ?

A  $\left\{ x \mid \frac{5}{3} < x < \frac{7}{3} \right\}$

B  $\left\{ x \mid x < \frac{5}{3} \text{ or } x > \frac{7}{3} \right\}$

C  $\left\{ x \mid x < \frac{5}{3} \right\}$

D  $\left\{ x \mid x > \frac{7}{3} \right\}$

**ANSWER:**

A

89. **PROBABILITY** A 5-character password can contain the numbers 0 through 9 and 26 letters of the alphabet. None of the characters can be repeated. What is the probability that the password begins with a consonant?

F  $\frac{21}{26}$

G  $\frac{21}{35}$

H  $\frac{21}{36}$

J  $\frac{5}{36}$

**ANSWER:**

H

90. **SAT/ACT** If  $c = \frac{8a^3}{b}$ , what happens to the value of  $c$  when both  $a$  and  $b$  are doubled?

A  $c$  is unchanged.

B  $c$  is halved.

C  $c$  is doubled.

D  $c$  is multiplied by 4.

E  $c$  is multiplied by 8.

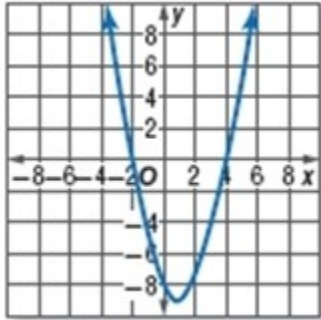
**ANSWER:**

D

### 4-3 Solving Quadratic Equations by Factoring

Use the related graph of each equation to determine its solutions.

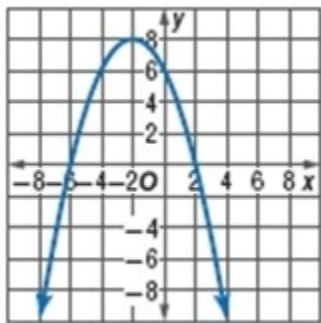
91.  $x^2 - 2x - 8 = 0$



ANSWER:

-2, 4

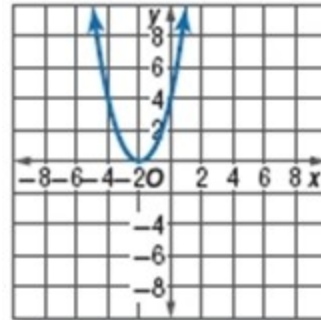
92.  $x^2 + 4x = 12$



ANSWER:

-6, 2

93.  $x^2 + 4x + 4 = 0$



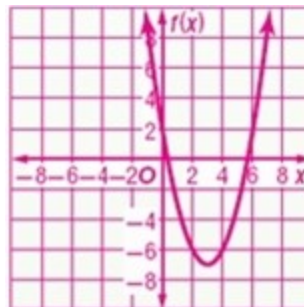
ANSWER:

-2

Graph each function.

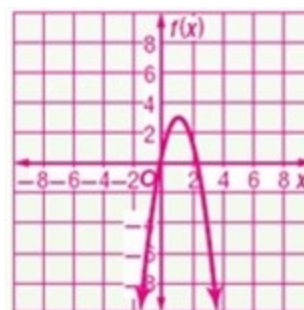
94.  $f(x) = x^2 - 6x + 2$

ANSWER:



95.  $f(x) = -2x^2 + 4x + 1$

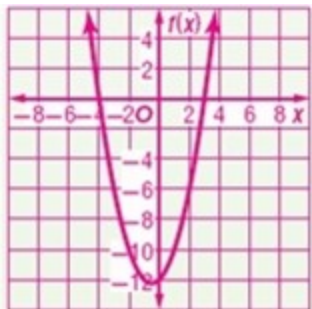
ANSWER:



### 4-3 Solving Quadratic Equations by Factoring

96.  $f(x) = (x - 3)(x + 4)$

ANSWER:



97. **FUNDRAISING** Lawrence High School sold wrapping paper and boxed cards for their fundraising event. The school gets \$1.00 for each roll of wrapping paper sold and \$0.50 for each box of cards sold.

Total Amounts for Each Class		
Class	Wrapping Paper	Cards
Freshmen	72	49
Sophomores	68	63
Juniors	90	56
Seniors	86	62

- Write a matrix that represents the amounts sold for each class and a matrix that represents the amount of money the school earns for each item sold.
- Write a matrix that shows how much each class earned.
- Which class earned the most money?
- What is the total amount of money the school made from the fundraiser?

ANSWER:

a.  $\begin{bmatrix} 72 & 49 \\ 68 & 63 \\ 90 & 56 \\ 86 & 62 \end{bmatrix}, \begin{bmatrix} 1.00 \\ 0.50 \end{bmatrix}$

b.  $\begin{bmatrix} 96.50 \\ 99.50 \\ 118 \\ 117 \end{bmatrix}$

- juniors
- \$431

### 4-3 Solving Quadratic Equations by Factoring

**Simplify.**

98.  $\sqrt{5} \cdot \sqrt{15}$

**ANSWER:**

$5\sqrt{3}$

99.  $\sqrt{8} \cdot \sqrt{32}$

**ANSWER:**

16

100.  $2\sqrt{3} \cdot \sqrt{27}$

**ANSWER:**

18