4-4 Complex Numbers

Simplify.

1. $\sqrt{-81}$

ANSWER:
$9i$

2. $\sqrt{-32}$

ANSWER:
$4i\sqrt{2}$

3. $(4i)(-3i)$

ANSWER:
12

4. $3\sqrt{-24} \cdot 2\sqrt{-18}$

ANSWER:
$-72\sqrt{3}$

5. i^{40}

ANSWER:
1

6. i^{63}

ANSWER:
$-i$

Solve each equation.

7. $4x^2 + 32 = 0$

ANSWER:
$\pm 2i\sqrt{2}$

8. $x^2 + 1 = 0$

ANSWER:
$\pm i$

Find the values of *a* and *b* that make each equation true.

9. $3a + (4b + 2)i = 9 - 6i$

ANSWER:
3, –2

10. $4b - 5 + (-a - 3)i = 7 - 8i$

ANSWER:
5, 3

Simplify.

11. $(-1 + 5i) + (-2 - 3i)$

ANSWER:
$-3 + 2i$

12. $(7 + 4i) - (1 + 2i)$

ANSWER:
$6 + 2i$

13. $(6 - 8i)(9 + 2i)$

ANSWER:
$70 - 60i$

14. $(3 + 2i)(-2 + 4i)$

ANSWER:
$-14 + 8i$

15. $\frac{3-i}{4+2i}$

ANSWER:
$\frac{1}{2} - \frac{1}{2}i$

16. $\frac{2+i}{5+6i}$

ANSWER:
$\frac{16}{61} - \frac{7}{61}i$

17. **ELECTRICITY** The current in one part of a series circuit is $5 - 3j$ amps. The current in another part of the circuit is $7 + 9j$ amps. Add these complex numbers to find the total current in the circuit.

ANSWER:
$12 + 6j$ amps

CCSS STRUCTURE Simplify.

18. $\sqrt{-121}$

ANSWER:
$11i$

19. $\sqrt{-169}$

ANSWER:
$13i$

20. $\sqrt{-100}$

ANSWER:
$10i$

21. $\sqrt{-81}$

ANSWER:
$9i$

22. $(-3i)(-7i)(2i)$

ANSWER:
$-42i$

23. $4i(-6i)^2$

ANSWER:
$-144i$

24. i^{11}

ANSWER:
$-i$

25. i^{25}

ANSWER:
i

26. $(10 - 7i) + (6 + 9i)$

ANSWER:
$16 + 2i$

27. $(-3 + i) + (-4 - i)$

ANSWER:
-7

28. $(12 + 5i) - (9 - 2i)$

ANSWER:
$3 + 7i$

29. $(11 - 8i) - (2 - 8i)$

ANSWER:
9

30. $(1 + 2i)(1 - 2i)$

ANSWER:
5

31. $(3 + 5i)(5 - 3i)$

ANSWER:
$30 + 16i$

32. $(4 - i)(6 - 6i)$

ANSWER:
$18 - 30i$

33. $\frac{2i}{1+i}$

ANSWER:
$1 + i$

34. $\frac{5}{2+4i}$

ANSWER:
$\frac{1}{2} - i$

35. $\frac{5+i}{3i}$

ANSWER:
$\frac{1}{3} - \frac{5}{3}i$

Solve each equation.

36. $4x^2 + 4 = 0$

ANSWER:
$\pm i$

37. $3x^2 + 48 = 0$

ANSWER:
$\pm 4i$

38. $2x^2 + 50 = 0$

ANSWER:
$\pm 5i$

39. $2x^2 + 10 = 0$

ANSWER:
$\pm i\sqrt{5}$

40. $6x^2 + 108 = 0$

ANSWER:
$\pm 3i\sqrt{2}$

41. $8x^2 + 128 = 0$

ANSWER:
$\pm 4i$

Find the values of x and y that make each equation true.

42. $9 + 12i = 3x + 4yi$

ANSWER:
3, 3

43. $x + 1 + 2yi = 3 - 6i$

ANSWER:
2, –3

44. $2x + 7 + (3 - y)i = -4 + 6i$

ANSWER:
$-\frac{11}{2}, -3$

45. $5 + y + (3x - 7)i = 9 - 3i$

ANSWER:
$\frac{4}{3}, 4$

46. $a + 3b + (3a - b)i = 6 + 6i$

ANSWER:
$\frac{12}{5}, \frac{6}{5}$

47. $(2a - 4b)i + a + 5b = 15 + 58i$

ANSWER:
25, –2

Simplify.

48. $\sqrt{-10} \cdot \sqrt{-24}$

ANSWER:
$-4\sqrt{15}$

49. $4i\left(\frac{1}{2}i\right)^2 (-2i)^2$

ANSWER:
$4i$

50. i^{41}

ANSWER:
i

4-4 Complex Numbers

51. $(4 - 6i) + (4 + 6i)$

ANSWER:
8

52. $(8 - 5i) - (7 + i)$

ANSWER:
$1 - 6i$

53. $(-6 - i)(3 - 3i)$

ANSWER:
$-21 + 15i$

54. $\frac{(5+i)^2}{3-i}$

ANSWER:
$\frac{31}{5} + \frac{27}{5}i$

55. $\frac{6-i}{2-3i}$

ANSWER:
$\frac{15}{13} + \frac{16}{13}i$

56. $(-4 + 6i)(2 - i)(3 + 7i)$

ANSWER:
$-118 + 34i$

57. $(1 + i)(2 + 3i)(4 - 3i)$

ANSWER:
$11 + 23i$

58. $\frac{4-i\sqrt{2}}{4+i\sqrt{2}}$

ANSWER:
$\frac{7}{9} - \frac{4i\sqrt{2}}{9}$

59. $\frac{2-i\sqrt{3}}{2+i\sqrt{3}}$

ANSWER:
$\frac{1}{7} - \frac{4\sqrt{3}}{7}i$

60. **ELECTRICITY** The impedance in one part of a series circuit is $7 + 8j$ ohms, and the impedance in another part of the circuit is $13 - 4j$ ohms. Add these complex numbers to find the total impedance in the circuit.

ANSWER:
$20 + 4j$ ohms

ELECTRICITY Use the formula $V = C \cdot I$.

61. The current in a circuit is $3 + 6j$ amps, and the impedance is $5 - j$ ohms. What is the voltage?

ANSWER:
$21 + 27j$ Volts

62. The voltage in a circuit is $20 - 12j$ volts, and the impedance is $6 - 4j$ ohms. What is the current?

ANSWER:
$\frac{42}{13} + \frac{2}{13}j$ Amps

63. Find the sum of $ix^2 - (4 + 5i)x + 7$ and $3x^2 + (2 + 6i)x - 8i$.

ANSWER:

$(3 + i)x^2 + (-2 + i)x - 8i + 7$

64. Simplify $[(2 + i)x^2 - ix + 5 + i] - [(-3 + 4i)x^2 + (5 - 5i)x - 6]$.

ANSWER:

$(5 - 3i)x^2 + (-5 + 4i)x + i + 11$

65. **MULTIPLE REPRESENTATIONS** In this problem, you will explore quadratic equations that have complex roots. Use a graphing calculator.

a. Algebraic Write a quadratic equation in standard form with 3i and -3i as its roots.

b. Graphical Graph the quadratic equation found in part **a** by graphing its related function.

c. Algebraic Write a quadratic equation in standard form with 2 + i and 2 - i as its roots.

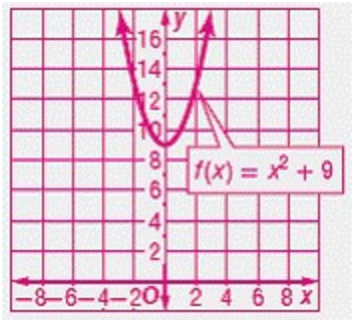
d. Graphical Graph the related function of the quadratic equation you found in part **c**. Use the graph to find the roots if possible. Explain.

e. Analytical How do you know when a quadratic equation will have only complex solutions?

ANSWER:

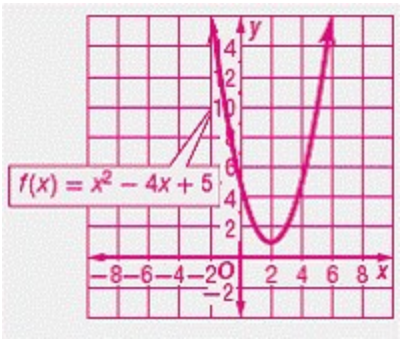
a. Sample answer: $x^2 + 9 = 0$

b.



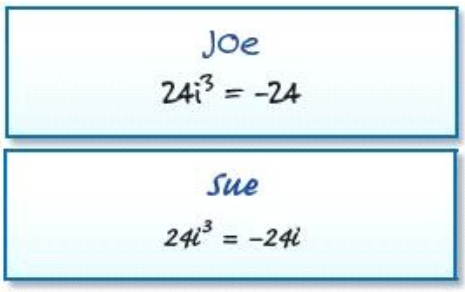
c. Sample answer: $x^2 - 4x + 5 = 0$

d.



e. Sample answer: A quadratic equation will have only complex solutions when the graph of the related function has no x-intercepts.

66. **CCSS CRITIQUE** Joe and Sue are simplifying $(2i)(3i)(4i)$. Is either of them correct? Explain your reasoning.



ANSWER:

Sue; $i^3 = -i$, not -1.

67. **CHALLENGE** Simplify $(1 + 2i)^3$.

ANSWER:
$-11 - 2i$

68. **REASONING** Determine whether the following statement is *always, sometimes,* or *never* true. Explain your reasoning.

Every complex number has both a real part and an imaginary part.

ANSWER:
Sample answer: Always. The value of 5 can be represented by $5 + 0i$, and the value of 3i can be represented by $0 + 3i$.

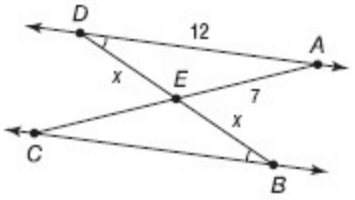
69. **OPEN ENDED** Write two complex numbers with a product of 20.

ANSWER:
Sample answer: $(4 + 2i)(4 - 2i)$

70. **WRITING IN MATH** Explain how complex numbers are related to quadratic equations.

ANSWER:
Some quadratic equations have complex solutions and cannot be solved using only the real numbers.

71. **EXTENDED RESPONSE** Refer to the figure to answer the following.



a. Name two congruent triangles with vertices in correct order.

b. Explain why the triangles are congruent.

c. What is the length of $\overline{EC}$? Explain your procedure.

ANSWER:
a. $\Delta CBE \cong \Delta ADE$
b. $\angle AED \cong \angle CEB$ (Vertical angles)
$\overline{DE} \cong \overline{BE}$ (Both have length x.)
$\angle ADE \cong \angle CBE$ (Given) Consecutive angles and the included side are all congruent, so the triangles are congruent by the ASA Property.
c. $\overline{EC} \cong \overline{EA}$ by CPCTC (corresponding parts of congruent triangles are congruent.) $EA = 7$, so $EC = 7$.)

72. $(3 + 6)^2 =$

A $2 \times 3 + 2 \times 6$

B 9^2

C $3^2 + 6^2$

D $3^2 \times 6^2$

ANSWER:
B

73. **SAT/ACT** A store charges $49 for a pair of pants. This price is 40% more than the amount it costs the store to buy the pants. After a sale, any employee is allowed to purchase any remaining pairs of pants at 30% off the store's cost.
How much would it cost an employee to purchase the pants after the sale?

F $10.50

G $12.50

H $13.72

J $24.50

K $35.00

ANSWER:
J

74. What are the values of x and y when $(5 + 4i) - (x + yi) = (-1 - 3i)$?

A $x = 6, y = 7$

B $x = 4, y = i$

C $x = 6, y = i$

D $x = 4, y = 7$

ANSWER:
A

Solve each equation by factoring.

75. $2x^2 + 7x = 15$

ANSWER:
$-5, \frac{3}{2}$

76. $4x^2 - 12 = 22x$

ANSWER:
$-\frac{1}{2}, 6$

77. $6x^2 = 5x + 4$

ANSWER:
$-\frac{1}{2}, \frac{4}{3}$

NUMBER THEORY Use a quadratic equation to find two real numbers that satisfy each situation, or show that no such numbers exist.

78. Their sum is –3, and their product is –40.

ANSWER:
–8, 5

79. Their sum is 19, and their product is 48.

ANSWER:
3, 16

80. Their sum is –15, and their product is 56.

ANSWER:
–7 and –8

81. Their sum is –21, and their product is 108.

ANSWER:
–9, –12

82. **RECREATION** Refer to the table.

a. Write a matrix that represents the cost of admission for residents and a matrix that represents the cost of admission for nonresidents.

b. Write the matrix that represents the additional cost for nonresidents.

c. Write a matrix that represents the difference in cost if a child or adult goes after 6:00 P.M. instead of before 6:00 P.M.

Daily Admission Fees

Residents		
Time of day	**Child**	**Adult**
Before 6:00 P.M.	$3.00	$4.50
After 6:00 P.M.	$2.00	$3.50

Nonresidents		
Time of day	**Child**	**Adult**
Before 6:00 P.M.	$4.50	$6.75
After 6:00 P.M.	$3.00	$5.25

ANSWER:

a.

$$\begin{matrix} & \text{Child} & \text{Adult} \\ \text{Residents:} & \end{matrix}\begin{bmatrix} 3.00 & 4.50 \\ 2.00 & 3.50 \end{bmatrix}$$

$$\begin{matrix} & \text{Child} & \text{Adult} \\ \text{Nonresidents:} & \end{matrix}\begin{bmatrix} 4.50 & 6.75 \\ 3.00 & 5.25 \end{bmatrix}$$

b.

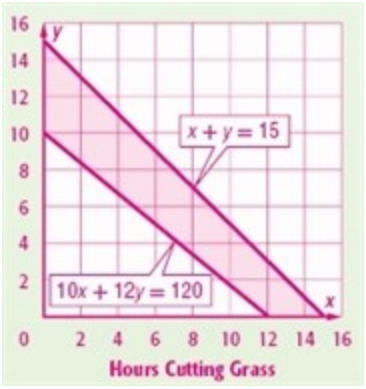
$$\begin{matrix} & \text{Child} & \text{Adult} \\ \text{Before 6} & \end{matrix}\begin{bmatrix} 1.50 & 2.25 \\ 1.00 & 1.75 \end{bmatrix}$$
$$\text{After 6}$$

c.

$$\begin{matrix} & \text{Child} & \text{Adult} \\ \text{Resident} & \end{matrix}\begin{bmatrix} 1.00 & 1.00 \\ 1.50 & 1.50 \end{bmatrix}$$
$$\text{Nonresident}$$

83. **PART-TIME JOBS** Terrell makes $10 an hour cutting grass and $12 an hour for raking leaves. He cannot work more than 15 hours per week. Graph two inequalities that Terrell can use to determine how many hours he needs to work at each job if he wants to earn at least $120 per week.

ANSWER:



Determine whether each trinomial is a perfect square trinomial. Write *yes* or *no*.

84. $x^2 + 16x + 64$

ANSWER:
yes

85. $x^2 - 12x + 36$

ANSWER:
yes

86. $x^2 + 8x - 16$

ANSWER:
no

87. $x^2 - 14x - 49$

ANSWER:
no

88. $x^2 + x + 0.25$

ANSWER:
yes

89. $x^2 + 5x + 6.25$

ANSWER:
yes