

4-5 Completing the Square

Solve each equation by using the Square Root Property. Round to the nearest hundredth if necessary.

1. $x^2 + 12x + 36 = 6$

ANSWER:

$\{-8.45, -3.55\}$

2. $x^2 - 8x + 16 = 13$

ANSWER:

$\{0.39, 7.61\}$

3. $x^2 + 18x + 81 = 15$

ANSWER:

$\{-12.87, -5.13\}$

4. $9x^2 + 30x + 25 = 11$

ANSWER:

$\{-2.77, -0.56\}$

5. **LASER LIGHT SHOW** The area A in square feet of a projected laser light show is given by $A = 0.16d^2$, where d is the distance from the laser to the screen in feet. At what distance will the projected laser light show have an area of 100 square feet?

ANSWER:

25 ft

Find the value of c that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

6. $x^2 - 10x + c$

ANSWER:

25; $(x - 5)^2$

7. $x^2 - 5x + c$

ANSWER:

6.25; $(x - 2.5)^2$

Solve each equation by completing the square.

8. $x^2 + 2x - 8 = 0$

ANSWER:

$\{-4, 2\}$

9. $x^2 - 4x + 9 = 0$

ANSWER:

$\{2 - i\sqrt{5}, 2 + i\sqrt{5}\}$

10. $2x^2 - 3x - 3 = 0$

ANSWER:

$\{-0.69, 2.19\}$

11. $2x^2 + 6x - 12 = 0$

ANSWER:

$\{-4.37, 1.37\}$

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12. $x^2 + 4x + 6 = 0$

ANSWER:

$$\{-2 - i\sqrt{2}, -2 + i\sqrt{2}\}$$

13. $x^2 + 8x + 10 = 0$

ANSWER:

$$\{-6.45, -1.55\}$$

Solve each equation by using the Square Root Property. Round to the nearest hundredth if necessary.

14. $x^2 + 4x + 4 = 10$

ANSWER:

$$\{-5.16, 1.16\}$$

15. $x^2 - 6x + 9 = 20$

ANSWER:

$$\{-1.47, 7.47\}$$

16. $x^2 + 8x + 16 = 18$

ANSWER:

$$\{-8.24, 0.24\}$$

17. $x^2 + 10x + 25 = 7$

ANSWER:

$$\{-7.65, -2.35\}$$

18. $x^2 + 12x + 36 = 5$

ANSWER:

$$\{-8.24, -3.76\}$$

19. $x^2 - 2x + 1 = 4$

ANSWER:

$$\{-1, 3\}$$

20. $x^2 - 5x + 6.25 = 4$

ANSWER:

$$\{0.5, 4.5\}$$

21. $x^2 - 15x + 56.25 = 8$

ANSWER:

$$\{4.67, 10.33\}$$

22. $x^2 + 32x + 256 = 1$

ANSWER:

$$\{-17, -15\}$$

23. $x^2 - 3x + \frac{9}{4} = 6$

ANSWER:

$$\{-0.95, 3.95\}$$

24. $x^2 + 7x + \frac{49}{4} = 4$

ANSWER:

$$\{-5.5, -1.5\}$$

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25. $x^2 - 9x + \frac{81}{4} = \frac{1}{4}$

ANSWER:

{4, 5}

Find the value of c that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

26. $x^2 + 8x + c$

ANSWER:

16; $(x + 4)^2$

27. $x^2 + 16x + c$

ANSWER:

64; $(x + 8)^2$

28. $x^2 - 11x + c$

ANSWER:

$\frac{121}{4}; \left(x - \frac{11}{2}\right)^2$

29. $x^2 + 9x + c$

ANSWER:

20.25; $(x + 4.5)^2$

Solve each equation by completing the square.

30. $x^2 - 4x + 12 = 0$

ANSWER:

$\{2 - 2i\sqrt{2}, 2 + 2i\sqrt{2}\}$

31. $x^2 + 2x - 12 = 0$

ANSWER:

{- 4.61, 2.61}

32. $x^2 + 6x + 8 = 0$

ANSWER:

{- 4, - 2}

33. $x^2 - 4x + 3 = 0$

ANSWER:

{1, 3}

34. $2x^2 + x - 3 = 0$

ANSWER:

$\left\{-\frac{3}{2}, 1\right\}$

35. $2x^2 - 3x + 5 = 0$

ANSWER:

$\left\{\frac{3 - i\sqrt{31}}{4}, \frac{3 + i\sqrt{31}}{4}\right\}$

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36. $2x^2 + 5x + 7 = 0$

ANSWER:

$$\left\{ \frac{-5 - i\sqrt{31}}{4}, \frac{-5 + i\sqrt{31}}{4} \right\}$$

37. $3x^2 - 6x - 9 = 0$

ANSWER:

$$\{-1, 3\}$$

38. $x^2 - 2x + 3 = 0$

ANSWER:

$$\{1 - i\sqrt{2}, 1 + i\sqrt{2}\}$$

39. $x^2 + 4x + 11 = 0$

ANSWER:

$$\{-2 - i\sqrt{7}, -2 + i\sqrt{7}\}$$

40. $x^2 - 6x + 18 = 0$

ANSWER:

$$\{3 - 3i, 3 + 3i\}$$

41. $x^2 - 10x + 29 = 0$

ANSWER:

$$\{5 - 2i, 5 + 2i\}$$

42. $3x^2 - 4x = 2$

ANSWER:

$$\{-0.39, 1.72\}$$

43. $2x^2 - 7x = -12$

ANSWER:

$$\left\{ \frac{7 - i\sqrt{47}}{4}, \frac{7 + i\sqrt{47}}{4} \right\}$$

44. $x^2 - 2.4x = 2.2$

ANSWER:

$$\{-0.71, 3.11\}$$

45. $x^2 - 5.3x = -8.6$

ANSWER:

$$\{2.65 - i\sqrt{1.5775}, 2.65 + i\sqrt{1.5775}\}$$

46. $x^2 - \frac{1}{5}x - \frac{11}{5} = 0$

ANSWER:

$$\{-1.39, 1.59\}$$

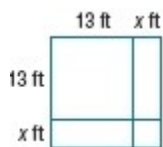
47. $x^2 - \frac{9}{2}x - \frac{24}{5} = 0$

ANSWER:

$$\{-0.89, 5.39\}$$

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48. **CCSS MODELING** An architect's blueprints call for a dining room measuring 13 feet by 13 feet. The customer would like the dining room to be a square, but with an area of 250 square feet. How much will this add to the dimensions of the room?



ANSWER:
about 2.81 ft

Solve each equation. Round to the nearest hundredth if necessary.

49. $4x^2 - 28x + 49 = 5$

ANSWER:
{2.38, 4.62}

50. $9x^2 + 30x + 25 = 11$

ANSWER:
{-2.77, -0.56}

51. $x^2 + x + \frac{1}{3} = \frac{2}{3}$

ANSWER:
{-1.26, 0.26}

52. $x^2 + 1.2x + 0.56 = 0.91$

ANSWER:
{-1.44, 0.24}

53. **FIREWORKS** A firework's distance d meters from the ground is given by $d = -1.5t^2 + 25t$, where t is the number of seconds after the firework has been lit.

- a. How many seconds have passed since the firework was lit when the firework explodes if it explodes at the maximum height of its path?
- b. What is the height of the firework when it explodes?

ANSWER:

a. $8\frac{1}{3}$ seconds

b. about 104.2 ft

Find the value of c that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

54. $x^2 + 0.7x + c$

ANSWER:
0.1225; $(x + 0.35)^2$

55. $x^2 - 3.2x + c$

ANSWER:
2.56; $(x - 1.6)^2$

56. $x^2 - 1.8x + c$

ANSWER:
 $\frac{81}{100}; \left(x - \frac{9}{10}\right)^2$

57. **MULTIPLE REPRESENTATIONS** In this

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problem, you will use quadratics to investigate golden rectangles and the golden ratio.

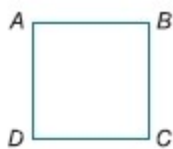
a. GEOMETRIC

- Draw square $ABCD$.
- Locate the midpoint of \overline{CD} . Label the midpoint P .
- Draw \overline{PB} .
- Construct an arc with a radius of \overline{PB} from B clockwise past the bottom of the square.
- Extend \overline{CD} until it intersects the arc. Label this point Q .
- Construct rectangle $ARQD$.

b. ALGEBRAIC Let $AD = x$ and $CQ = 1$. Use completing the square to solve $\frac{DQ}{AD} = \frac{QR}{CQ}$ for x .

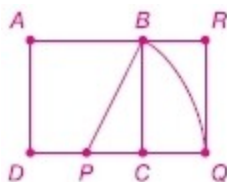
c. TABULAR Make a table of x and values for $CQ = 2, 3,$ and 4 .

d. VERBAL What do you notice about the x -values? Write an equation you could use to determine x for $CQ = n$, where n is a nonzero real number.



ANSWER:

a.



b. $x = \frac{1 + \sqrt{5}}{2}$

c.

CQ	x
2	$1 + \sqrt{5}$
3	$\frac{3 + 3\sqrt{5}}{2}$
4	$2 + 2\sqrt{5}$

d. Sample answer: the x -values are multiples of

$$\frac{1 + \sqrt{5}}{2}; x = \frac{n(1 + \sqrt{5})}{2}$$

58. **ERROR ANALYSIS** Alonso and Aida are solving $x^2 + 8x - 20 = 0$ by completing the square. Is either of them correct? Explain your reasoning.

Alonso

$$x^2 + 8x - 20 = 0$$

$$x^2 + 8x = 20$$

$$x^2 + 8x + 16 = 20 + 16$$

$$(x + 4)^2 = 36$$

$$x + 4 = \pm 6$$

$$x = -4 \pm 6$$

Aida

$$x^2 + 8x - 20 = 0$$

$$x^2 + 8x = 20$$

$$x^2 + 8x + 16 = 20$$

$$(x + 4)^2 = 20$$

$$x + 4 = \pm\sqrt{20}$$

$$x = -4 \pm \sqrt{20}$$

ANSWER:

Alonso; Aida did not add 16 to each side; she added it only to the left side.

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59. **CHALLENGE** Solve $x^2 + bx + c = 0$ by completing the square. Your answer will be an expression for x in terms of b and c .

ANSWER:

$$x = \frac{-b}{2} \pm \sqrt{\frac{b^2}{4} - c}$$

60. **CCSS ARGUMENTS** Without solving, determine how many unique solutions there are for each equation. Are they rational, real, or complex? Justify your reasoning.

a. $(x + 2)^2 = 16$

b. $(x - 2)^2 = 16$

c. $-(x - 2)^2 = 16$

d. $36 - (x - 2)^2 = 16$

e. $16(x + 2)^2 = 0$

f. $(x + 4)^2 = (x + 6)^2$

ANSWER:

a. 2; rational; 16 is a perfect square so $x + 2$ and x are rational.

b. 2; rational; 16 is a perfect square so $x - 2$ and x are rational.

c. 2; complex; If the opposite of square is positive, the square is negative. The square root of a negative number is complex.

d. 2; real; The square must equal 20. Since that is positive but not a perfect square, the solutions will be real but not rational.

e. 1; rational; The expression must be equal to 0 and only -2 makes the expression equal to 0.

f. 1; rational; The expressions $(x + 4)$ and $(x + 6)$ must either be equal or opposites. No value makes them equal, -5 makes them opposites. The only solution is -5.

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61. **OPEN ENDED** Write a perfect square trinomial equation in which the linear coefficient is negative and the constant term is a fraction. Then solve the equation.

ANSWER:

Sample answer: $x^2 - \frac{2}{3}x + \frac{1}{9} = \frac{1}{4}; \left\{ \frac{5}{6}, -\frac{1}{6} \right\}$

62. **WRITING IN MATH** Explain what it means to complete the square. Include a description of the steps you would take.

ANSWER:

Completing the square allows you to rewrite one side of a quadratic equation in the form of a perfect square. Once in this form, the equation can be solved by using the Square Root Property.

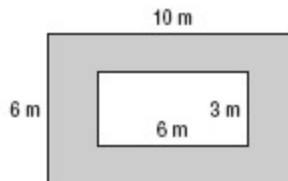
63. **SAT/ACT** If $x^2 + y^2 = 2xy$, then y must equal

- A -1
- B 0
- C 1
- D $-x$
- E x

ANSWER:

E

64. **GEOMETRY** Find the area of the shaded region.



F 14 m^2

G 18 m^2

H 42 m^2

J 60 m^2

ANSWER:

H

65. **SHORT RESPONSE** What value of c should be used to solve the following equation by completing the square?

$$5x^2 - 50x + c = 12 + c$$

ANSWER:

125

66. If $5 - 3i$ is a solution for $x^2 + ax + b = 0$, where a and b are real numbers, what is the value of b ?

A 10

B 14

C 34

D 40

ANSWER:

C

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Simplify.

67. $(8 + 5i)^2$

ANSWER:

$$39 + 80i$$

68. $4(3 - i) + 6(2 - 5i)$

ANSWER:

$$24 - 34i$$

69. $\frac{5 - 2i}{6 + 9i}$

ANSWER:

$$\frac{4}{39} - \frac{19}{39}i$$

Write a quadratic equation in standard form with the given root(s).

70. $\frac{4}{5}, \frac{3}{4}$

ANSWER:

$$20x^2 - 31x + 12 = 0$$

71. $-\frac{2}{5}, 6$

ANSWER:

$$5x^2 - 28x - 12 = 0$$

72. $-\frac{1}{4}, -\frac{6}{7}$

ANSWER:

$$28x^2 + 31x + 6 = 0$$

73. **TRAVEL** Yoko is going with the Spanish Club to Costa Rica. She buys 10 traveler's checks in denominations of \$20, \$50, and \$100, totaling \$370. She has twice as many \$20 checks as \$50 checks. How many of each denomination of traveler's checks does she have?

ANSWER:

1 \$100, 3 \$50, and 6 \$20 checks

74. **SHOPPING** Main St. Media sells all DVDs for one price and all books for another price. Alex bought 4 DVDs and 6 books for \$170, while Matt bought 3 DVDs and 8 books for \$180. What is the cost of a DVD and the cost of a book?

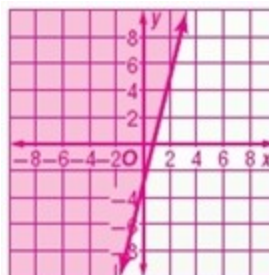
ANSWER:

DVD: \$20; book: \$15

Graph each inequality.

75. $y \geq 4x - 3$

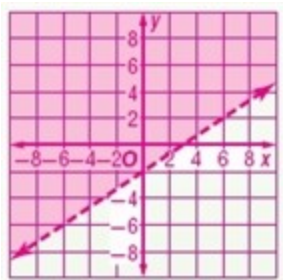
ANSWER:



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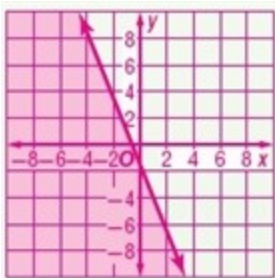
76. $2x - 3y < 6$

ANSWER:

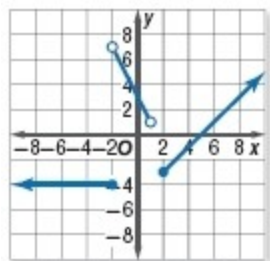


77. $5x + 2y + 3 \leq 0$

ANSWER:



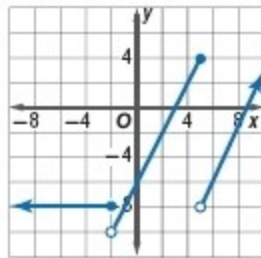
Write the piecewise function shown in each graph.



78.

ANSWER:

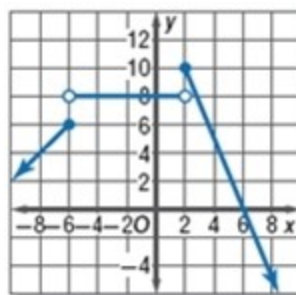
$$f(x) = \begin{cases} -4 & \text{if } x \leq -2 \\ -2x + 3 & \text{if } -2 < x < 1 \\ x - 5 & \text{if } x \geq 2 \end{cases}$$



79.

ANSWER:

$$f(x) = \begin{cases} -8 & \text{if } x \leq -2 \\ 2x - 6 & \text{if } -2 < x \leq 5 \\ 2x - 18 & \text{if } x > 5 \end{cases}$$



80.

ANSWER:

$$f(x) = \begin{cases} x + 12 & \text{if } x \leq -6 \\ 8 & \text{if } -6 < x < 2 \\ -2.5x + 15 & \text{if } x \geq 2 \end{cases}$$

Evaluate $b^2 - 4ac$ for the given values of a , b , and c .

81. $a = 5, b = 6, c = 2$

ANSWER:

-4

82. $a = -2, b = -7, c = 3$

ANSWER:

73

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83. $a = -5, b = -8, c = -10$

ANSWER:

-136