4-6 The Quadratic Formula and the Discriminant

Solve each equation by using the Quadratic Formula.

1. $x^2 + 12x - 9 = 0$

ANSWER:

$-6 \pm 3\sqrt{5}$

2. $x^2 + 8x + 5 = 0$

ANSWER:

$-4 \pm \sqrt{11}$

3. $4x^2 - 5x - 2 = 0$

ANSWER:

$\frac{5 \pm \sqrt{57}}{8}$

4. $9x^2 + 6x - 4 = 0$

ANSWER:

$\frac{-1 \pm \sqrt{5}}{3}$

5. $10x^2 - 3 = 13x$

ANSWER:

(1.5, –0.2)

6. $22x = 12x^2 + 6$

ANSWER:

$\left(1.5, \frac{1}{3}\right)$

7. $-3x^2 + 4x = -8$

ANSWER:

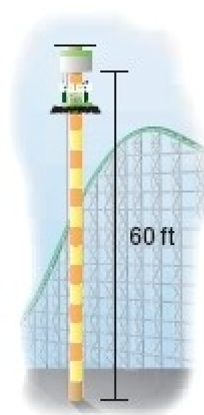
$\frac{2 \pm 2\sqrt{7}}{3}$

8. $x^2 + 3 = -6x + 8$

ANSWER:

$-3 \pm \sqrt{14}$

9. **CCSS MODELING** An amusement park ride takes riders to the top of a tower and drops them at speeds reaching 80 feet per second. A function that models this ride is $h = -16t^2 - 64t - 60$, where h is the height in feet and t is the time in seconds. About how many seconds does it take for riders to drop from 60 feet to 0 feet?



ANSWER:

about 0.78 second

Complete parts *a* and *b* for each quadratic equation.

a. Find the value of the discriminant.

b. Describe the number and type of roots.

10. $3x^2 + 8x + 2 = 0$

ANSWER:
a. 40

b. 2 irrational roots

11. $2x^2 - 6x + 9 = 0$

ANSWER:
a. –36

b. 2 complex roots

12. $-16x^2 + 8x - 1 = 0$

ANSWER:
a. 0

b. 1 rational root

13. $5x^2 + 2x + 4 = 0$

ANSWER:
a. –76

b. 2 complex roots

Solve each equation by using the Quadratic Formula.

14. $x^2 + 45x = -200$

ANSWER:
–5, –40

15. $4x^2 - 6 = -12x$

ANSWER:
$\frac{-3 \pm \sqrt{15}}{2}$

16. $3x^2 - 4x - 8 = -6$

ANSWER:
$\frac{2 \pm \sqrt{10}}{3}$

17. $4x^2 - 9 = -7x - 4$

ANSWER:
$\frac{-7 \pm \sqrt{129}}{8}$

18. $5x^2 - 9 = 11x$

ANSWER:
$\frac{11 \pm \sqrt{301}}{10}$

19. $12x^2 + 9x - 2 = -17$

ANSWER:

$\dfrac{-3 \pm i\sqrt{71}}{8}$

20. **DIVING** Competitors in the 10-meter platform diving competition jump upward and outward before diving into the pool below. The height *h* of a diver in meters above the pool after *t* seconds can be approximated by the equation $h = -4.9t^2 + 3t + 10$.

a. Determine a domain and range for which this function makes sense.

b. When will the diver hit the water?

ANSWER:

a. D: $\{t \mid 0 \le t \le 2\}$,
R: $\{h \mid 0 \le h \le 10\}$

b. about 1.77 seconds

Complete parts *a–c* for each quadratic equation.
a. Find the value of the discriminant.
b. Describe the number and type of roots.
c. Find the exact solutions by using the Quadratic Formula.

21. $2x^2 + 3x - 3 = 0$

ANSWER:

a. 33

b. 2 irrational

c. $\dfrac{-3 \pm \sqrt{33}}{4}$

22. $4x^2 - 6x + 2 = 0$

ANSWER:

a. 4
b. 2 rational
c. $\dfrac{1}{2}, 1$

23. $6x^2 + 5x - 1 = 0$

ANSWER:

a. 49

b. 2 rational

c. $\dfrac{1}{6}, -1$

24. $6x^2 - x - 5 = 0$

ANSWER:

a. 121

b. 2 rational

c. $1, -\dfrac{5}{6}$

25. $3x^2 - 3x + 8 = 0$

ANSWER:

a. –87

b. 2 complex

c. $\dfrac{3 \pm i\sqrt{87}}{6}$

4-6 The Quadratic Formula and the Discriminant

26. $2x^2 + 4x + 7 = 0$

ANSWER:
a. –40

b. 2 complex

c. $\frac{-2 \pm i\sqrt{10}}{2}$

27. $-5x^2 + 4x + 1 = 0$

ANSWER:
a. 36

b. 2 rational

c. $1, -\frac{1}{5}$

28. $x^2 - 6x = -9$

ANSWER:
a. 0

b. 1 rational

c. 3

29. $-3x^2 - 7x + 2 = 6$

ANSWER:
a. 1

b. 2 rational

c. $-1, -\frac{4}{3}$

30. $-8x^2 + 5 = -4x$

ANSWER:
a. 176

b. 2 irrational

c. $\frac{1 \pm \sqrt{11}}{4}$

31. $x^2 + 2x - 4 = -9$

ANSWER:
a. –16

b. 2 complex

c. $-1 \pm 2i$

32. $-6x^2 + 5 = -4x + 8$

ANSWER:
a. –56

b. 2 complex

c. $\frac{2 \pm i\sqrt{14}}{6}$

33. **VIDEO GAMES** While Darnell is grounded his friend Jack brings him a video game. Darnell stands at his bedroom window, and Jack stands directly below the window. If Jack tosses a game cartridge to Darnell with an initial velocity of 35 feet per second, an equation for the height h feet of the cartridge after t seconds is $h = -16t^2 + 35t + 5$.

a. If the window is 25 feet above the ground, will Darnell have 0, 1, or 2 chances to catch the video game cartridge?

b. If Darnell is unable to catch the video game cartridge, when will it hit the ground?

ANSWER:

a. 0

b. about 2.3 seconds

34. **CCSS SENSE-MAKING** Civil engineers are designing a section of road that is going to dip below sea level. The road's curve can be modeled by the equation $y = 0.00005x^2 - 0.06x$, where x is the horizontal distance in feet between the points where the road is at sea level and y is the elevation. The engineers want to put stop signs at the locations where the elevation of the road is equal to sea level. At what horizontal distances will they place the stop signs?

ANSWER:

0 ft and 1200 ft

Complete parts *a–c* for each quadratic equation.
a. Find the value of the discriminant.
b. Describe the number and type of roots.
c. Find the exact solutions by using the Quadratic Formula.

35. $5x^2 + 8x = 0$

ANSWER:

a. 64

b. 2 rational

c. $0, -\frac{8}{5}$

36. $8x^2 = -2x + 1$

ANSWER:

a. 36

b. 2 rational

c. $\frac{1}{4}, -\frac{1}{2}$

37. $4x - 3 = -12x^2$

ANSWER:

a. 160

b. 2 irrational

c. $\frac{-1 \pm \sqrt{10}}{6}$

38. $0.8x^2 + 2.6x = -3.2$

ANSWER:

a. –3.48

b. 2 imaginary roots

c. $\dfrac{-1.3 \pm i\sqrt{0.87}}{0.8}$

39. $0.6x^2 + 1.4x = 4.8$

ANSWER:

a. 13.48
b. 2 irrational
c. $\dfrac{-0.7 \pm \sqrt{3.37}}{0.6}$

40. $-4x^2 + 12 = -6x - 8$

ANSWER:

a. 356
b. 2 irrational
c. $\dfrac{3 \pm \sqrt{89}}{4}$

41. **SMOKING** A decrease in smoking in the United States has resulted in lower death rates caused by lung cancer. The number of deaths per 100,000 people y can be approximated by $y = -0.26x^2 - 0.55x + 91.81$, where x represents the number of years after 2000.

a. Calculate the number of deaths per 100,000 people for 2015 and 2017.

b. Use the Quadratic Formula to solve for x when $y = 50$.

c. According to the quadratic function, when will the death rate be 0 per 100,000? Do you think that this prediction is reasonable? Why or why not?

Year	Deaths per 100,000
2000	91.8
2002	89.7
2004	85.5
2010	60.3
2015	?
2017	?

ANSWER:

a. 25.1, 7.3

b. 11.7

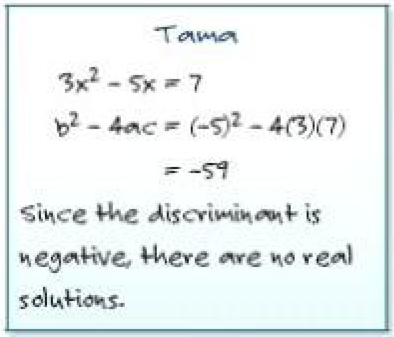
c. 2018; Sample answer: no; the death rate from cancer will never be 0 unless a cure is found. If and when a cure will be found cannot be predicted.

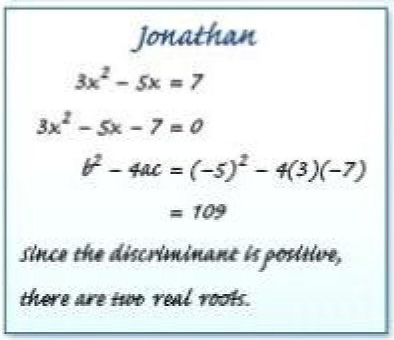
42. **NUMBER THEORY** The sum S of consecutive integers 1, 2, 3, …, n is given by the formula $S = \frac{1}{2}n(n+1)$. How many consecutive integers, starting with 1, must be added to get a sum of 666?

ANSWER:

36

43. **CCSS CRITIQUE** Tama and Jonathan are determining the number of solutions of $3x^2 - 5x = 7$. Is either of them correct? Explain your reasoning.


Tama
$3x^2 - 5x = 7$
$b^2 - 4ac = (-5)^2 - 4(3)(7)$
$= -59$
Since the discriminant is negative, there are no real solutions.


Jonathan
$3x^2 - 5x = 7$
$3x^2 - 5x - 7 = 0$
$b^2 - 4ac = (-5)^2 - 4(3)(-7)$
$= 109$
Since the discriminant is positive, there are two real roots.

ANSWER:

Jonathan is correct; you must first write the equation in the form $ax^2 + bx + c = 0$ to determine the values of a, b, and c. Therefore, the value of c is –7, not 7.

44. **CHALLENGE** Find the solutions of $4ix^2 - 4ix + 5i = 0$ by using the Quadratic Formula.

ANSWER:

$\frac{1 \pm 2i}{2}$

45. **REASONING** Determine whether each statement is *sometimes, always,* or *never* true. Explain your reasoning.

a. In a quadratic equation in standard form, if a and c are different signs, then the solutions will be real.

b. If the discriminant of a quadratic equation is greater than 1, the two roots are real irrational numbers.

ANSWER:

a. Sample answer: Always; when a and c are opposite signs, then ac will always be negative and $-4ac$ will always be positive. Since b^2 will also always be positive, then $b^2 - 4ac$ represents the addition of two positive values, which will never be negative. Hence, the discriminant can never be negative and the solutions can never be imaginary.

b. Sample answer: Sometimes; the roots will only be irrational if $b^2 - 4ac$ is not a perfect square.

46. **OPEN ENDED** Sketch the corresponding graph and state the number and type of roots for each of the following.

a. $b^2 - 4ac = 0$

b. A quadratic function in which $f(x)$ never equals zero.

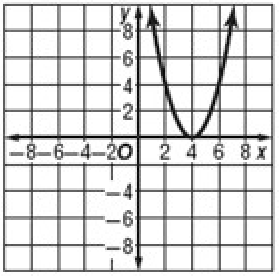
c. A quadratic function in which $f(a) = 0$ and $f(b) = 0$; $a \neq b$.

d. The discriminant is less than zero.

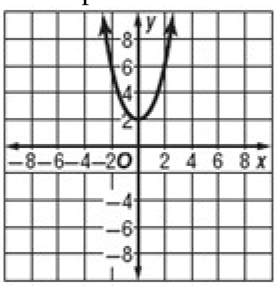
e. a and b are both solutions and can be represented as fractions.

ANSWER:

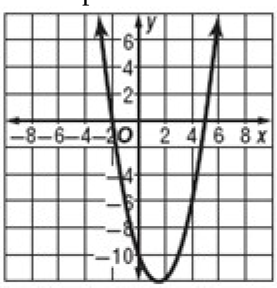
a. Sample answer: 1 rational root



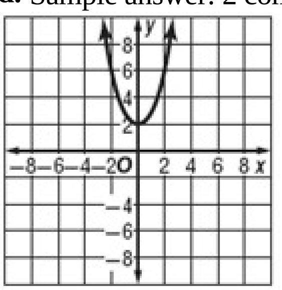
b. Sample answer: 2 complex roots



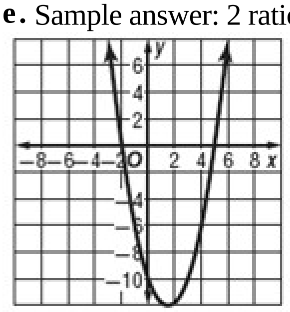
c. Sample answer: 2 real roots



d. Sample answer: 2 complex roots



e. Sample answer: 2 rational roots



47. **CHALLENGE** Find the value(s) of m in the quadratic equation $x^2 + x + m + 1 = 0$ such that it has one solution.

ANSWER:

–0.75

48. **WRITING IN MATH** Describe three different ways to solve $x^2 - 2x - 15 = 0$. Which method do you prefer, and why?

ANSWER:

Sample answer: (1) Factor $x^2 - 2x - 15$ as $(x + 3)(x - 5)$. Then according to the Zero Product Property, either $x + 3 = 0$ or $x - 5 = 0$. Solving these equations, $x = -3$ or $x = 5$.

(2) Rewrite the equation as $x^2 - 2x = 15$. Then add 1 to each side of the equation to complete the square on the left side. Then $(x - 1)^2 = 16$. Taking the square root of each side, $x - 1 = \pm 4$. Therefore, $x = 1 \pm 4$ and $x = -3$ or $x = 5$.

(3) Use the Quadratic Formula. Thus, $x = \frac{2 \pm \sqrt{2^2 - 4(1)(-15)}}{2(1)}$ or $x = \frac{2 \pm \sqrt{64}}{2}$. Simplifying the expression, $x = -3$ or $x = 5$. See students' preferences.

49. A company determined that its monthly profit P is given by $P = -8x^2 + 165x - 100$, where x is the selling price for each unit of product. Which of the following is the best estimate of the maximum price per unit that the company can charge without losing money?

A \$10

B \$20

C \$30

D \$40

ANSWER:
B

50. **SAT/ACT** For which of the following sets of numbers is the mean greater than the median?

F {4, 5, 6, 7, 8}

G {4, 6, 6, 6, 8}

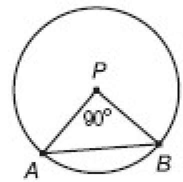
H {4, 5, 6, 7, 9}

J {3, 5, 6, 7, 8}

K {2, 6, 6, 6, 6}

ANSWER:
H

51. **SHORT RESPONSE** In the figure below, P is the center of the circle with radius 15 inches. What is the area of ΔAPB?



ANSWER:
112.5 in^2

52. 75% of 88 is the same as 60% of what number?

A 100

B 101

C 108

D 110

ANSWER:
D

Find the value of *c* that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

53. $x^2 + 13x + c$

ANSWER:
42.25; $(x + 6.5)^2$

54. $x^2 + 2.4x + c$

ANSWER:
1.44; $(x + 1.2)^2$

55. $x^2 + \frac{4}{5}x + c$

ANSWER:

$\frac{4}{25}; \left(x + \frac{2}{5}\right)^2$

Simplify.

56. i^{26}

ANSWER:

–1

57. $\sqrt{-16}$

ANSWER:

4i

58. $4\sqrt{-9} \cdot 2\sqrt{-25}$

ANSWER:

–120

59. **PILOT TRAINING** Evita is training for her pilot's license. Flight instruction costs $105 per hour, and the simulator costs $45 per hour. She spent 4 more hours in airplane training than in the simulator. If Evita spent $3870, how much time did she spend training in an airplane and in a simulator?

ANSWER:

27 hours of flight instruction and 23 hours in the simulator

60. **BUSINESS** Ms. Larson owns three fruit farms on which she grows apples, peaches, and apricots. She sells apples for $22 a case, peaches for $25 a case, and apricots for $18 a case.

a. Write an inventory matrix for the number of cases for each type of fruit for each farm and a cost matrix for the price per case for each type of fruit.

b. Find the total income of the three fruit farms expressed as a matrix.

c. What is the total income from all three fruit farms?

Number of Cases in Stock of Each Type of Fruit			
Fruit	Farm 1	Farm 2	Farm 3
apples	290	175	110
peaches	165	240	75
apricots	210	190	0

ANSWER:

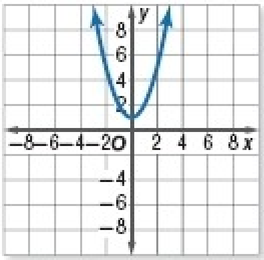
a. $I = \begin{bmatrix} 290 & 165 & 210 \\ 175 & 240 & 190 \\ 110 & 75 & 0 \end{bmatrix}, C = \begin{bmatrix} 22 \\ 25 \\ 18 \end{bmatrix}$

b. $\begin{bmatrix} 14,285 \\ 13,270 \\ 4295 \end{bmatrix}$

c. $31,850

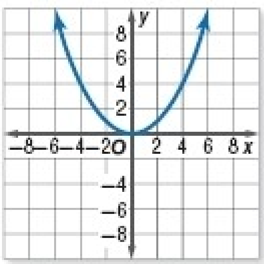
4-6 The Quadratic Formula and the Discriminant

Write an equation for each graph.

61. 

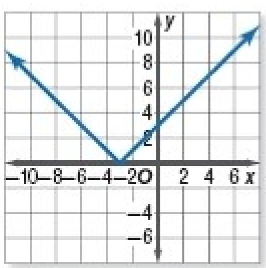
ANSWER:

$y = x^2 + 1$

62. 

ANSWER:

$y = 0.25x^2$

63. 

ANSWER:

$y = |x + 3|$