

4-7 Transformations of Quadratic Graphs

Write each function in vertex form.

1. $y = x^2 + 6x + 2$

ANSWER:

$$y = (x + 3)^2 - 7$$

2. $y = -2x^2 + 8x - 5$

ANSWER:

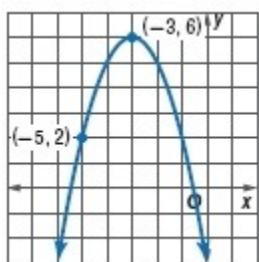
$$y = -2(x - 2)^2 + 3$$

3. $y = 4x^2 + 24x + 24$

ANSWER:

$$y = 4(x + 3)^2 - 12$$

4. **MULTIPLE CHOICE** Which function is shown in the graph?



A $y = -(x + 3)^2 + 6$

B $y = -(x - 3)^2 - 6$

C $y = -2(x + 3)^2 + 6$

D $y = -2(x - 3)^2 - 6$

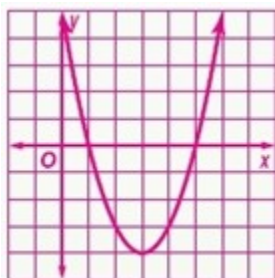
ANSWER:

A

Graph each function.

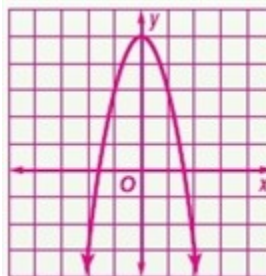
5. $y = (x - 3)^2 - 4$

ANSWER:



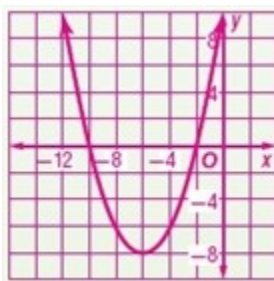
6. $y = -2x^2 + 5$

ANSWER:



7. $y = \frac{1}{2}(x + 6)^2 - 8$

ANSWER:



Write each function in vertex form.

8. $y = x^2 + 9x + 8$

ANSWER:

$$y = \left(x + \frac{9}{2}\right)^2 - \frac{49}{4}$$

9. $y = x^2 - 6x + 3$

ANSWER:

$$y = (x - 3)^2 - 6$$

10. $y = -2x^2 + 5x$

ANSWER:

$$y = -2\left(x - \frac{5}{4}\right)^2 + \frac{25}{8}$$

11. $y = x^2 + 2x + 7$

ANSWER:

$$y = (x + 1)^2 + 6$$

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12. $y = -3x^2 + 12x - 10$

ANSWER:

$$y = -3(x - 2)^2 + 2$$

13. $y = x^2 + 8x + 16$

ANSWER:

$$y = (x + 4)^2$$

14. $y = 2x^2 - 4x - 3$

ANSWER:

$$y = 2(x - 1)^2 - 5$$

15. $y = 3x^2 + 10x$

ANSWER:

$$y = 3\left(x + \frac{5}{3}\right)^2 - \frac{25}{3}$$

16. $y = x^2 - 4x + 9$

ANSWER:

$$y = (x - 2)^2 + 5$$

17. $y = -4x^2 - 24x - 15$

ANSWER:

$$y = -4(x + 3)^2 + 21$$

18. $y = x^2 - 12x + 36$

ANSWER:

$$y = (x - 6)^2$$

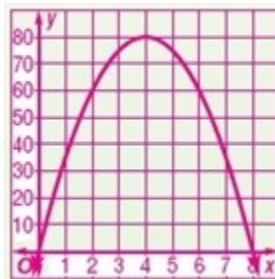
19. $y = -x^2 - 4x - 1$

ANSWER:

$$y = -(x + 2)^2 + 3$$

20. **FIREWORKS** During an Independence Day fireworks show, the height h in meters of a specific rocket after t seconds can be modeled by $h = -4.9(t - 4)^2 + 80$. Graph the function.

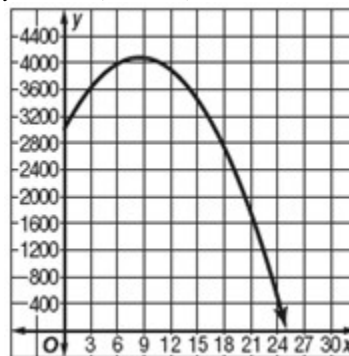
ANSWER:



21. **FINANCIAL LITERACY** A bicycle rental shop rents an average of 120 bicycles per week and charges \$25 per day. The manager estimates that there will be 15 additional bicycles rented for each \$1 reduction in the rental price. The maximum income the manager can expect can be modeled by $y = -15x^2 + 255x + 3000$. Write this function in vertex form. Then graph.

ANSWER:

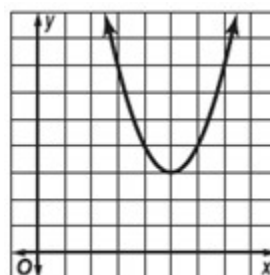
$$y = -15(x - 8.5)^2 + 4083.75$$



Graph each function.

22. $y = (x - 5)^2 + 3$

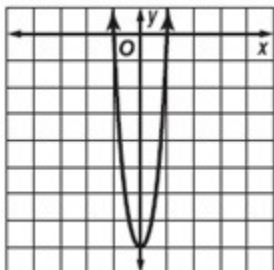
ANSWER:



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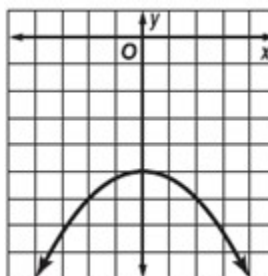
23. $y = 9x^2 - 8$

ANSWER:



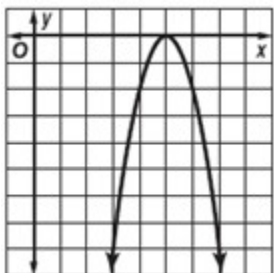
27. $y = -\frac{1}{4}x^2 - 5$

ANSWER:



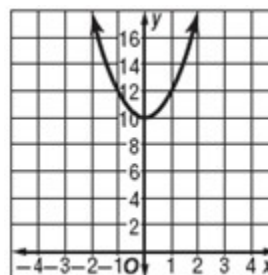
24. $y = -2(x - 5)^2$

ANSWER:



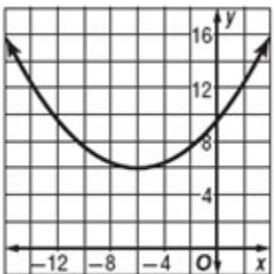
28. $y = 2x^2 + 10$

ANSWER:



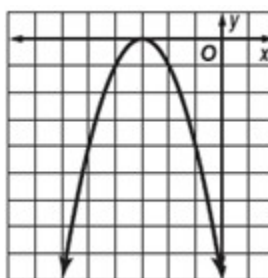
25. $y = \frac{1}{10}(x + 6)^2 + 6$

ANSWER:



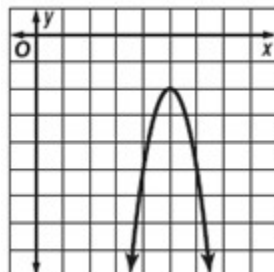
29. $y = -(x + 3)^2$

ANSWER:



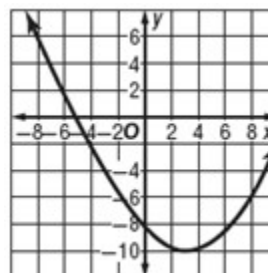
26. $y = -3(x - 5)^2 - 2$

ANSWER:



30. $y = \frac{1}{6}(x - 3)^2 - 10$

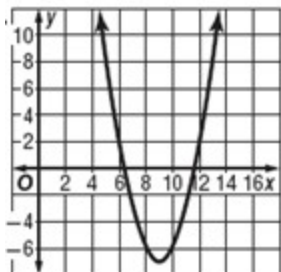
ANSWER:



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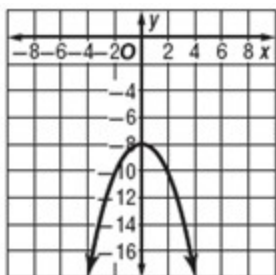
31. $y = (x - 9)^2 - 7$

ANSWER:



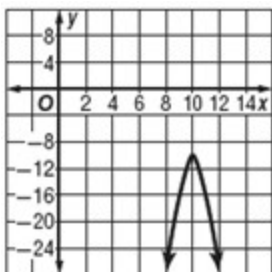
32. $y = -\frac{5}{8}x^2 - 8$

ANSWER:



33. $y = -4(x - 10)^2 - 10$

ANSWER:



34. **CCSS MODELING** A sailboard manufacturer uses an automated process to manufacture the masts for

its sailboards. The function $f(x) = \frac{1}{250}x^2 + \frac{3}{5}x$ is

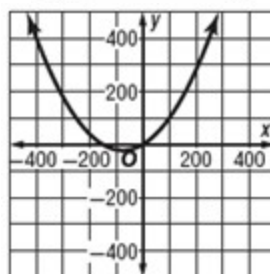
programmed into a computer to make one such mast.

a. Write the quadratic function in vertex form. Then graph the function.

b. Describe how the manufacturer can adjust the function to make its masts with a greater or smaller curve.

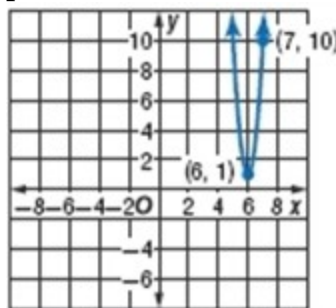
ANSWER:

a. $\frac{1}{250}(x + 75)^2 - \frac{45}{2}$



b. They can adjust the coefficient of x^2 .

Write an equation in vertex form for each parabola.

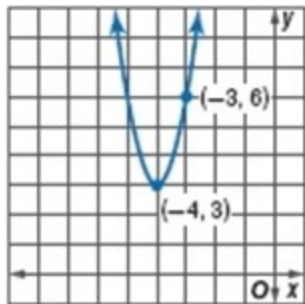


35.

ANSWER:

$$y = 9(x - 6)^2 + 1$$

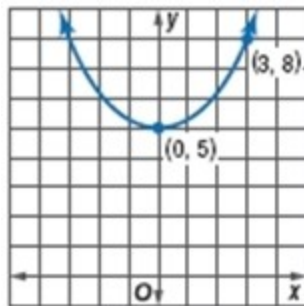
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36.

ANSWER:

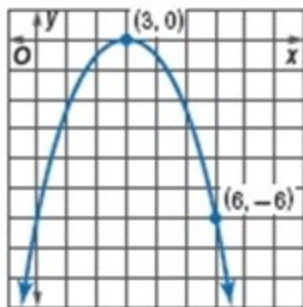
$$y = 3(x + 4)^2 + 3$$



39.

ANSWER:

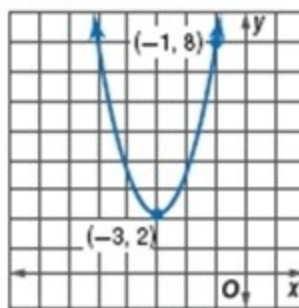
$$y = \frac{1}{3}x^2 + 5$$



37.

ANSWER:

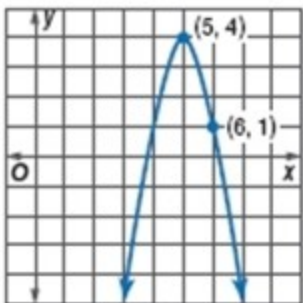
$$y = -\frac{2}{3}(x - 3)^2$$



40.

ANSWER:

$$y = \frac{3}{2}(x + 3)^2 + 2$$



38.

ANSWER:

$$y = -3(x - 5)^2 + 4$$

Write each function in vertex form. Then identify the vertex, axis of symmetry, and direction of opening.

41. $3x^2 - 4x = 2 + y$

ANSWER:

$$y = 3\left(x - \frac{2}{3}\right)^2 - \frac{10}{3}; \left(\frac{2}{3}, -\frac{10}{3}\right), x = \frac{2}{3}, \text{ open up}$$

42. $-2x^2 + 7x = y - 12$

ANSWER:

$$y = -2\left(x - \frac{7}{4}\right)^2 + \frac{145}{8}; \left(\frac{7}{4}, \frac{145}{8}\right), x = \frac{7}{4}, \text{ opens down}$$

43. $-x^2 - 4.7x = y - 2.8$

ANSWER:

$$y = -(x + 2.35)^2 + 8.3225; (-2.35, 8.3225), x = -2.35, \text{ opens down}$$

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44. $x^2 + 1.4x - 1.2 = y$

ANSWER:

$y = (x + 0.7)^2 - 1.69$; $(-0.7, -1.69)$, $x = -0.7$, opens up

45. $x^2 - \frac{2}{3}x - \frac{26}{9} = y$

ANSWER:

$y = \left(x - \frac{1}{3}\right)^2 - 3$; $\left(\frac{1}{3}, -3\right)$, $x = \frac{1}{3}$, open up

46. $x^2 + 7x + \frac{49}{4} = y$

ANSWER:

$y = (x + 3.5)^2$; $(-3.5, 0)$, $x = -3.5$, opens up

47. **CARS** The formula $S(t) = \frac{1}{2}at^2 + v_0t$ can be used

to determine the position $S(t)$ of an object after t seconds at a rate of acceleration a with initial velocity v_0 . Valerie's car can accelerate 0.002 miles per second squared.

a. Express $S(t)$ in vertex form as she accelerates from 35 miles per hour to enter highway traffic.

b. How long will it take Valerie to match the average speed of highway traffic of 68 miles per hour? (*Hint*: Use *acceleration* \cdot *time* = *velocity*.)

c. If the entrance ramp is $\frac{1}{8}$ mile long, will Valerie have sufficient time to match the average highway speed? Explain.

ANSWER:

a. $S(t) = 0.001(t + 4.861)^2 - 0.024$

b. 4.58 seconds

c. Yes; if we substitute $\frac{1}{8}$ for $S(t)$ and solve for t we get 7.35 seconds. This is how long Valerie will be on the ramp. Since it will take her 4.58 seconds to accelerate to 68 mph, she will be on the ramp long enough to accelerate to match the average expressway speed.

48. **OPEN ENDED** Write an equation for a parabola that has been translated, compressed, and reflected in the x -axis.

ANSWER:

Sample answer: $y = -\frac{1}{2}(x - 4)^2$

49. **CHALLENGE** Explain how you can find an equation of a parabola using the coordinates of three points on the graph.

ANSWER:

The equation of a parabola can be written in the form $y = ax^2 + bx + c$ with $a \neq 0$. For each of the three points, substitute the value of the x -coordinate for x in the equation and substitute the value of the y -coordinate for y in the equation. This will produce three equations in three variables a , b , and c . Solve the system of equations to find the values of a , b , and c . These values determine the quadratic equation.

50. **CHALLENGE** Write the standard form of a quadratic function $ax^2 + bx + c = y$ in vertex form. Identify the vertex and the axis of symmetry.

ANSWER:

$$a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right) = y; \left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right); x = -\frac{b}{2a}$$

51. **REASONING** Describe the graph of $f(x) = a(x - h)^2 + k$ when $a = 0$. Is the graph the same as that of $g(x) = ax^2 + bx + c$ when $a = 0$? Explain.

ANSWER:

Sample answer: The variable a represents different values for these functions, so making $a = 0$ will have a different effect on each function. For $f(x)$, when $a = 0$, the graph will be a horizontal line, $f(x) = k$. For $g(x)$, when $a = 0$, the graph will be linear, but not necessarily horizontal, $g(x) = bx + c$.

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52. **CCSS ARGUMENTS** Explain how the graph of $y = x^2$ can be used to graph any quadratic function. Include a description of the effects produced by changing a , h , and k in the equation $y = a(x - h)^2 + k$, and a comparison of the graph of $y = x^2$ and the graph of $y = a(x - h)^2 + k$ using values you choose for a , h , and k .

ANSWER:

All quadratic functions are transformations of the parent graph $y = x^2$. By identifying these transformations when a quadratic function is written in vertex form, you can redraw the graph of $y = x^2$ with its vertex translated to (h, k) , widened or narrowed as determined by a , opening downward if a is negative.

53. Flowering bushes need a mixture of 70% soil and 30% vermiculite. About how many buckets of vermiculite should you add to 20 buckets of soil?

- A** 6.0
B 8.0
C 14.0
D 24.0

ANSWER:

B

54. **SAT/ACT** The sum of the integers x and y is 495. The units digit of x is 0. If x is divided by 10, the result is equal to y . What is the value of x ?

- F** 40
G 45
H 245
J 250
K 450

ANSWER:

K

55. What is the solution set of the inequality

$$|4x - 1| < 9??$$

- A** $\{x \mid 2.5 < x \text{ or } x < -2\}$
B $\{x \mid x < 2.5\}$
C $\{x \mid x > -2\}$
D $\{x \mid -2 < x < 2.5\}$

ANSWER:

D

56. **SHORT RESPONSE** At your store, you buy wrenches for \$30.00 a dozen and sell them for \$3.50 each. What is the percent markup for the wrenches?

ANSWER:

40%

Solve each equation by using the method of your choice. Find exact solutions.

57. $4x^2 + 15x = 21$

ANSWER:

$$\frac{-15 \pm \sqrt{561}}{8}$$

58. $-3x^2 + 19 = 5x$

ANSWER:

$$\frac{-5 \pm \sqrt{253}}{6}$$

59. $6x - 5x^2 + 9 = 3$

ANSWER:

$$\frac{3 \pm \sqrt{39}}{5}$$

Find the value of c that makes each trinomial a perfect square.

60. $x^2 - 12x + c$

ANSWER:

36

61. $x^2 + 0.1x + c$

ANSWER:

0.0025

62. $x^2 - 0.45x + c$

ANSWER:

0.050625

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Determine whether each function has a maximum or minimum value, and find that value.

63. $f(x) = 6x^2 - 8x + 12$

ANSWER:

minimum, $9\frac{1}{3}$

64. $f(x) = -4x^2 + x - 18$

ANSWER:

maximum, $-17\frac{15}{16}$

65. $f(x) = 3x^2 - 9 + 6x$

ANSWER:

minimum, -12

66. **ARCHAEOLOGY** A coordinate grid is laid over an archaeology dig to identify the location of artifacts. Three corners of a building have been partially unearthed at $(-1, 6)$, $(4, 5)$, and $(-1, -2)$. If each square on the grid measures one square foot, estimate the area of the floor of the building.

ANSWER:

about 20 ft^2

67. **HOTELS** Use the costs for an overnight stay at a hotel provided at the right.

a. Write a 3×2 matrix that represents the cost of each room.

b. Write a 2×3 matrix that represents the cost of each room.

HOTEL		
	Weekday	Weekend
Single Room	\$60.00	\$79.00
Double Room	\$70.00	\$89.00
Suite	\$75.00	\$95.00

ANSWER:

a. Weekday Weekend

$$\begin{matrix} \text{Single} \\ \text{Double} \\ \text{Suite} \end{matrix} \begin{bmatrix} 60 & 79 \\ 70 & 89 \\ 75 & 95 \end{bmatrix}$$

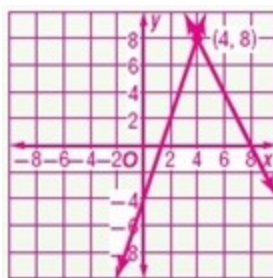
b. Single Double Suite

$$\begin{matrix} \text{Weekday} \\ \text{Weekend} \end{matrix} \begin{bmatrix} 60 & 70 & 75 \\ 79 & 89 & 95 \end{bmatrix}$$

Solve each system of equations by graphing.

68. $y = 3x - 4$
 $y = -2x + 16$

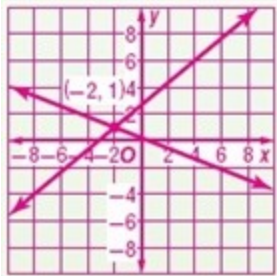
ANSWER:



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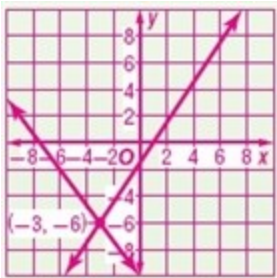
69. $2x + 5y = 1$
 $6y - 5x = 16$

ANSWER:



70. $4x + 3y = -30$
 $3x - 2y = 3$

ANSWER:



Evaluate each function.

71. $f(3)$ if $f(x) = x^2 - 4x + 12$

ANSWER:

9

72. $f(-2)$ if $f(x) = -4x^2 + x - 8$

ANSWER:

-26

73. $f(4)$ if $f(x) = 3x^2 + x$

ANSWER:

52

Determine whether the given value satisfies the inequality.

74. $3x^2 - 5 > 6$; $x = 2$

ANSWER:

yes

75. $-2x^2 + x - 1 < 4$; $x = -2$

ANSWER:

yes

76. $4x^2 + x - 3 \leq 36$; $x = 3$

ANSWER:

yes