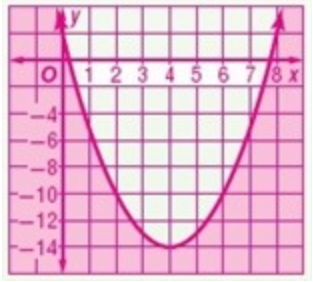
4-8 Quadratic Inequalities

Graph each inequality.

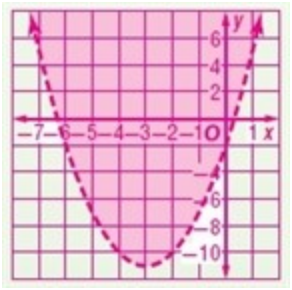
1. $y \leq x^2 - 8x + 2$

ANSWER:



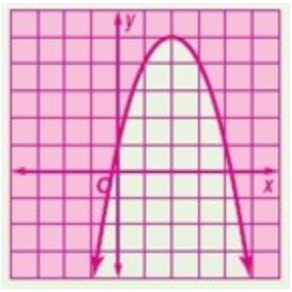
2. $y > x^2 + 6x - 2$

ANSWER:



3. $y \geq -x^2 + 4x + 1$

ANSWER:



CCSS SENSE-MAKING Solve each inequality by graphing.

4. $0 < x^2 - 5x + 4$

ANSWER:

$\{x \mid x < 1 \text{ or } x > 4\}$

5. $x^2 + 8x + 15 < 0$

ANSWER:

$\{x \mid -5 < x < -3\}$

6. $-2x^2 - 2x + 12 \geq 0$

ANSWER:

$\{x \mid -3 \leq x \leq 2\}$

7. $0 \geq 2x^2 - 4x + 1$

ANSWER:

$\{x \mid 0.29 \leq x \leq 1.71\}$

8. **SOCCER** A midfielder kicks a ball toward the goal during a match. The height of the ball in feet above the ground $h(t)$ at time t can be represented by $h(t) = -0.1t^2 + 2.4t + 1.5$. If the height of the goal is 8 feet, at what time during the kick will the ball be able to enter the goal?

ANSWER:

$\{t \mid 0 < t < 3.11\}$ or $\{t \mid 20.89 < t \leq 24.61\}$

4-8 Quadratic Inequalities

Solve each inequality algebraically.

9. $x^2 + 6x - 16 < 0$

ANSWER:
$\{x \mid -8 < x < 2\}$

10. $x^2 - 14x > -49$

ANSWER:
$\{x \mid x < 7 \text{ or } x > 7\}$

11. $-x^2 + 12x \geq 28$

ANSWER:
$\{x \mid 3.17 \leq x \leq 8.83\}$

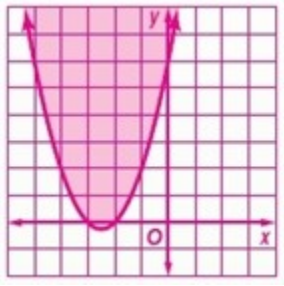
12. $x^2 - 4x \leq 21$

ANSWER:
$\{x \mid -3 \leq x \leq 7\}$

Graph each inequality.

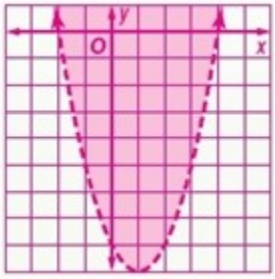
13. $y \geq x^2 + 5x + 6$

ANSWER:



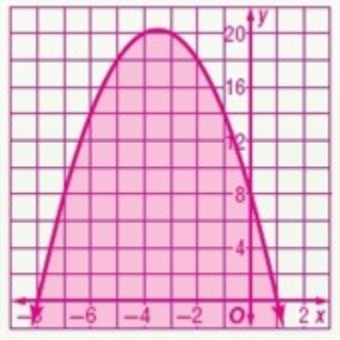
14. $x^2 - 2x - 8 < y$

ANSWER:



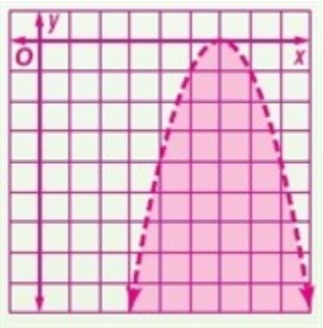
15. $y \leq -x^2 - 7x + 8$

ANSWER:



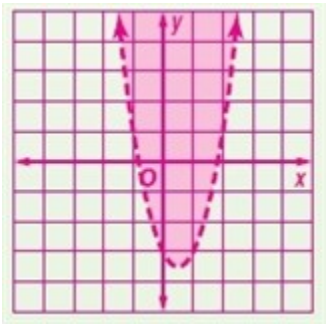
16. $-x^2 + 12x - 36 > y$

ANSWER:



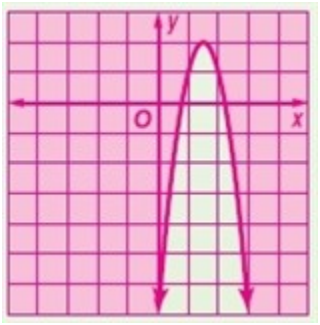
17. $y > 2x^2 - 2x - 3$

ANSWER:



18. $y \geq -4x^2 + 12x - 7$

ANSWER:



Solve each inequality by graphing.

19. $x^2 - 9x + 9 < 0$

ANSWER:
$\{x \mid 1.1 < x < 7.9\}$

20. $x^2 - 2x - 24 \leq 0$

ANSWER:
$\{x \mid -4 \leq x \leq 6\}$

21. $x^2 + 8x + 16 \geq 0$

ANSWER:
$\{x \mid$ all real numbers$\}$

22. $x^2 + 6x + 3 > 0$

ANSWER:
$\{x \mid x < -5.45 \text{ or } x > -0.55\}$

23. $0 > -x^2 + 7x + 12$

ANSWER:
$\{x \mid x < -1.42 \text{ or } x > 8.42\}$

24. $-x^2 + 2x - 15 < 0$

ANSWER:
$\{x \mid$ all real numbers$\}$

25. $4x^2 + 12x + 10 \leq 0$

ANSWER:
$\varnothing$

26. $-3x^2 - 3x + 9 > 0$

ANSWER:
$\{x \mid -2.30 < x < 1.30\}$

27. $0 > -2x^2 + 4x + 4$

ANSWER:
$\{x \mid x < -0.73 \text{ or } x > 2.73\}$

28. $3x^2 + 12x + 36 \leq 0$

ANSWER:
$\varnothing$

29. $0 \leq -4x^2 + 8x + 5$

ANSWER:
$\{x \mid -0.5 \leq x \leq 2.5\}$

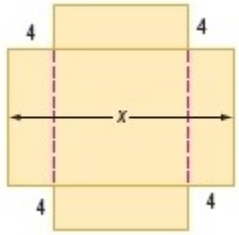
30. $-2x^2 + 3x + 3 \leq 0$

ANSWER:
$\{x \mid x \leq -0.69 \text{ or } x \geq 2.19\}$

31. **ARCHITECTURE** An arched entry of a room is shaped like a parabola that can be represented by the equation $f(x) = -x^2 + 6x + 1$. How far from the sides of the arch is its height at least 7 feet?

ANSWER:
about 1.26 ft to 4.73 ft

32. **MANUFACTURING** A box is formed by cutting 4-inch squares from each corner of a square piece of cardboard and then folding the sides. If $V(x) = 4x^2 - 64x + 256$ represents the volume of the box, what should the dimensions of the original piece of cardboard be if the volume of the box cannot exceed 750 cubic inches?



ANSWER:
greater than 8 in. but no more than 21.69 in.

Solve each inequality algebraically.

33. $x^2 - 9x < -20$

ANSWER:
$\{x \mid 4 < x < 5\}$

34. $x^2 + 7x \geq -10$

ANSWER:
$\{x \mid x \leq -5 \text{ or } x \geq -2\}$

35. $2 > x^2 - x$

ANSWER:
$\{x \mid -1 < x < 2\}$

36. $-3 \le -x^2 - 4x$

ANSWER:
$\{x \mid -4.65 \le x \le 0.65\}$

37. $-x^2 + 2x \le -10$

ANSWER:
$\{x \mid x \le -2.32 \text{ or } x \ge 4.32\}$

38. $-6 > x^2 + 4x$

ANSWER:
$\varnothing$

39. $2x^2 + 4 \ge 9$

ANSWER:
$\{x \mid x \le -1.58 \text{ or } x \ge 1.58\}$

40. $3x^2 + x \ge -3$

ANSWER:
$\{x \mid \text{all real numbers}\}$

41. $-4x^2 + 2x < 3$

ANSWER:
$\{x \mid \text{all real numbers}\}$

42. $-11 \ge -2x^2 - 5x$

ANSWER:
$\{x \mid x \le -3.91 \text{ or } x \ge 1.41\}$

43. $-12 < -5x^2 - 10x$

ANSWER:
$\{x \mid -2.84 < x < 0.84\}$

44. $-3x^2 - 10x > -1$

ANSWER:
$\{x \mid -3.43 < x < 0.10\}$

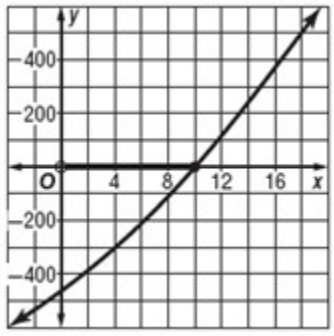
45. **CCSS PERSEVERANCE** The Sanchez family is adding a deck along two sides of their swimming pool. The deck width will be the same on both sides and the total area of the pool and deck cannot exceed 750 square feet.

a. Graph the quadratic inequality.

b. Determine the maximum width of the deck.

ANSWER:

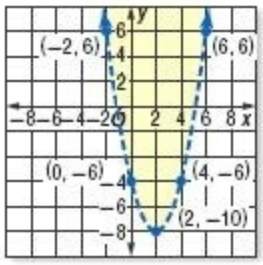
a.



b. greater than 0 ft but no more than 10.04 ft

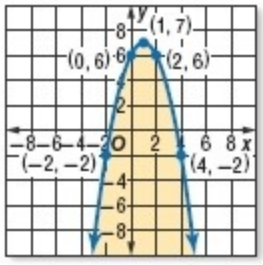
4-8 Quadratic Inequalities

Write a quadratic inequality for each graph.

46. 

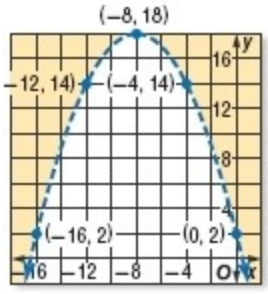
ANSWER:

$y > x^2 - 4x - 6$

47. 

ANSWER:

$y \leq -x^2 + 2x + 6$

48. 

ANSWER:

$y > -0.25x^2 - 4x + 2$

Solve each quadratic inequality by using a graph, a table, or algebraically.

49. $-2x^2 + 12x < -15$

ANSWER:

$\{x \mid x < -1.06 \text{ or } x > 7.06\}$

50. $5x^2 + x + 3 \geq 0$

ANSWER:

$\{x \mid \text{all real numbers}\}$

51. $11 \leq 4x^2 + 7x$

ANSWER:

$\{x \mid x \leq -2.75 \text{ or } x \geq 1\}$

52. $x^2 - 4x \leq -7$

ANSWER:

$\varnothing$

53. $-3x^2 + 10x < 5$

ANSWER:

$\{x \mid x < 0.61 \text{ or } x > 2.72\}$

54. $-1 \geq -x^2 - 5x$

ANSWER:

$\{x \mid x \leq -5.19 \text{ or } x \geq 0.19\}$

55. **BUSINESS** An electronics manufacturer uses the function $P(x) = x(-27.5x + 3520) + 20{,}000$ to model their monthly profits when selling x thousand digital audio players.

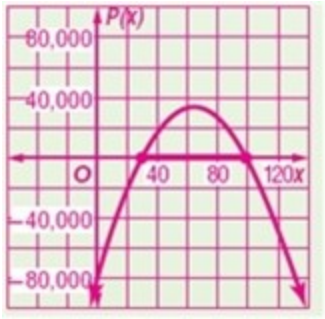
a. Graph the quadratic inequality for a monthly profit of at least $100,000.

b. How many digital audio players must the manufacturer sell to earn a profit of at least $100,000 in a month?

c. Suppose the manufacturer has an additional monthly expense of $25,000. Explain how this affects the graph of the profit function. Then determine how many digital audio players the manufacturer needs to sell to have at least $100,000 in profits.

ANSWER:

a.



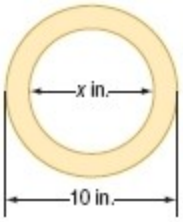
b. from 30,000 to 98,000 digital audio players

c. The graph is shifted down 25,000 units. The manufacturer must sell from 47,000 to 81,000 digital audio players.

56. **UTILITIES** A contractor is installing drain pipes for a shopping center's parking lot. The outer diameter of the pipe is to be 10 inches. The cross sectional area of the pipe must be at least 35 square inches and should not be more than 42 square inches.

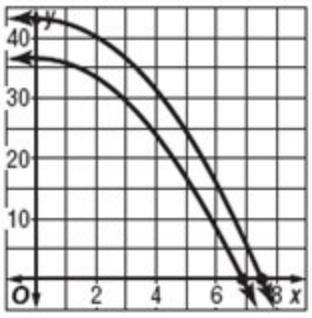
a. Graph the quadratic inequalities.

b. What thickness of drain pipe can the contractor use?



ANSWER:

a.



b. 1.28 in. to 1.59 in.

57. **OPEN ENDED** Write a quadratic inequality for each condition.

a. The solution set is all real numbers.

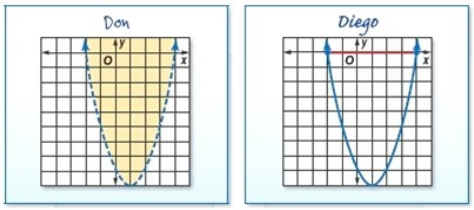
b. The solution set is the empty set.

ANSWER:

a. Sample answer: $x^2 + 2x + 1 \geq 0$

b. Sample answer: $x^2 - 4x + 6 < 0$

58. **CCSS CRITIQUE** Don and Diego used a graph to solve the quadratic inequality $x^2 - 2x - 8 > 0$. Is either of them correct? Explain.



ANSWER:

Neither; Don graphed the inequality in two variables, and Diego graphed the wrong interval.

59. **REASONING** Are the boundaries of the solution set of $x^2 + 4x - 12 \leq 0$ twice the value of the boundaries of $\frac{1}{2}x^2 + 2x - 6 \leq 0$? Explain.

ANSWER:

No; the graphs of the inequalities intersect the *x*-axis at the same points.

60. **REASONING** Determine if the following statement is sometimes, always, or never true. Explain your reasoning.

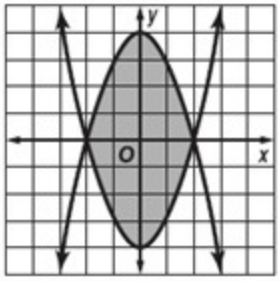
The intersection of $y \leq -ax^2 + c$ *and* $y \geq ax^2 - c$ *is the empty set*.

ANSWER:

Sample answer: Sometimes; when *a* is positive and *c* is negative, there is no solution and when *a* is negative and *c* is positive there is a solution set.

61. **CHALLENGE** Graph the intersection of the graphs of $y \leq -x^2 + 4$ and $y \geq x^2 - 4$..

ANSWER:



62. **WRITING IN MATH** How are the techniques used when solving quadratic inequalities and quadratic equations similar? different?

ANSWER:

For both quadratic and linear inequalities, you must first graph the related equation. You use the inequality symbol to determine if the line is dashed or solid. Then you use test points to determine where to shade. One difference is that one related equation is a straight line while the other related equation is a curve.

63. **]GRIDDED RESPONSE** You need to seed an area that is 80 feet by 40 feet. Each bag of seed can cover 25 square yards of land. How many bags of seed will you need?

ANSWER:

15

64. **SAT/ACT** The product of two integers is between 107 and 116. Which of the following cannot be one of the integers?

A 5

B 10

C 12

D 15

E 23

ANSWER:
D

65. **PROBABILITY** Five students are to be arranged side by side with the tallest student in the center and the two shortest students on the ends. If no two students are the same height, how many different arrangements are possible?

F 2

G 4

H 5

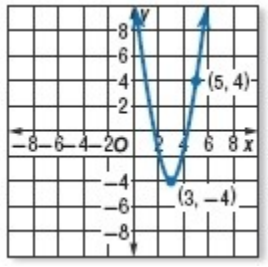
J 6

ANSWER:
G

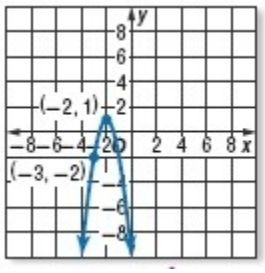
66. **SHORT RESPONSE** Simplify $\frac{5+i}{6-3i}$.

ANSWER:
$\frac{3}{5}+\frac{7}{15}i$

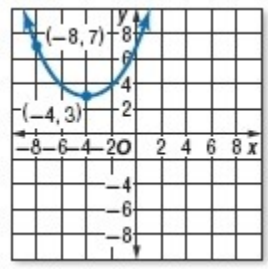
Write an equation in vertex form for each parabola.

67. 

ANSWER:
$y = 2(x - 3)^2 - 4$

68. 

ANSWER:
$y = -3(x + 2)^2 + 1$

69. 

ANSWER:
$y = 0.25(x + 4)^2 + 3$

Complete parts *a* and *b* for each quadratic equation

a. Find the value of the discriminant.

b. Describe the number and type of roots.

70. $4x^2 + 7x - 3 = 0$

ANSWER:
97; 2 irrational roots

71. $-3x^2 + 2x - 4 = 9$

ANSWER:
–152; 2 complex roots

72. $6x^2 + x - 4 = 12$

ANSWER:
385; 2 irrational roots

Perform the indicated operation. If the matrix does not exist, write *impossible*.

73. $4\begin{bmatrix} 3 & -6 \\ -5 & 2 \end{bmatrix} - 3\begin{bmatrix} 4 & -1 \\ -2 & 8 \end{bmatrix}$

ANSWER:
$\begin{bmatrix} 0 & -21 \\ -14 & -16 \end{bmatrix}$

74. $-2\begin{bmatrix} 5 & -9 \\ 5 & 11 \end{bmatrix} - 6\begin{bmatrix} 3 & -7 \\ -5 & 8 \end{bmatrix}$

ANSWER:
$\begin{bmatrix} -28 & 60 \\ 20 & -70 \end{bmatrix}$

75. $\begin{bmatrix} 2 & -6 \\ -4 & 6 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 & 1 \\ -1 & 6 & 4 \end{bmatrix}$

ANSWER:
$\begin{bmatrix} 10 & -38 & -22 \\ -14 & 40 & 20 \end{bmatrix}$

76. **EXERCISE** Refer to the graphic.

a. For each option, write an equation that represents the cost of belonging to the gym.

b. Graph the equations. Estimate the break-even point for the gym memberships.

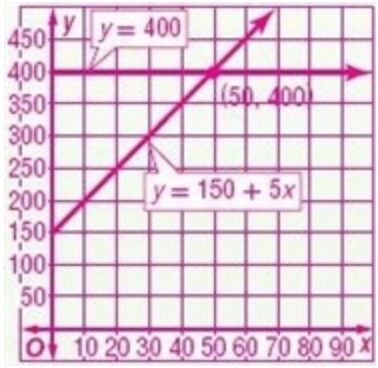
c. Explain what the break-even point means.

d. If you plan to visit the gym at least once per week during the year, which option should you chose?

ANSWER:

a. $y = 400;\ y = 150 + 5x$

b.



c. It means that the options cost the same if you visit 50 times in a year.

d. $400 per year

Use the Distributive Property to find each product.

77. $-6(x-4)$

ANSWER:
$-6x + 24$

78. $8(w+3x)$

ANSWER:
$8w + 24x$

79. $-4(-2y+3z)$

ANSWER:
$8y - 12z$

80. $-1(c-d)$

ANSWER:
$d - c$

81. $0.5(5x+6y)$

ANSWER:
$2.5x + 3y$

82. $-3(-6y-4z)$

ANSWER:
$18y + 12z$