

5-1 Operations with Polynomials

Simplify. Assume that no variable equals 0.

1. $(2a^3b^{-2})(-4a^2b^4)$

ANSWER:

$$-8a^5b^2$$

2. $\frac{12x^4y^2}{2xy^5}$

ANSWER:

$$\frac{6x^3}{y^3}$$

3. $\left(\frac{2a^2}{3b}\right)^3$

ANSWER:

$$\frac{8a^6}{27b^3}$$

4. $(6g^5h^{-4})^3$

ANSWER:

$$\frac{216g^{15}}{h^{12}}$$

Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

5. $3x + 4y$

ANSWER:

yes, 1

6. $\frac{1}{2}x^2 - 7y$

ANSWER:

yes, 2

7. $x^2 + \sqrt{x}$

ANSWER:

no

8. $\frac{ab^3 - 1}{az^4 + 3}$

ANSWER:

no

Simplify.

9. $(x^2 - 5x + 2) - (3x^2 + x - 1)$

ANSWER:

$$-2x^2 - 6x + 3$$

10. $(3a + 4b) + (6a - 6b)$

ANSWER:

$$9a - 2b$$

11. $2a(4b + 5)$

ANSWER:

$$8ab + 10a$$

12. $3x^2(2xy - 3xy^2 + 4x^2y^3)$

ANSWER:

$$6x^3y - 9x^3y^2 + 12x^4y^3$$

13. $(n - 9)(n + 7)$

ANSWER:

$$n^2 - 2n - 63$$

14. $(a + 4)(a - 6)$

ANSWER:

$$a^2 - 2a - 24$$

15. **EXERCISE** Tara exercises 75 minutes a day. She does cardio, which burns an average of 10 Calories a minute, and weight training, which burns an average of 7.5 Calories a minute. Write a polynomial to represent the amount of Calories Tara burns in one day if she does x minutes of weight training.

ANSWER:

$$750 - 2.5x$$

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Simplify. Assume that no variable equals 0.

16. $(5x^3y^{-5})(4xy^3)$

ANSWER:

$$\frac{20x^4}{y^2}$$

17. $(-2b^3c)(4b^2c^2)$

ANSWER:

$$-8b^5c^3$$

18. $\frac{a^3n^7}{an^4}$

ANSWER:

$$a^2n^3$$

19. $\frac{-y^3z^5}{y^2z^3}$

ANSWER:

$$-yz^2$$

20. $\frac{-7x^5y^5z^4}{21x^7y^5z^2}$

ANSWER:

$$\frac{z^2}{-3x^2}$$

21. $\frac{9a^7b^5c^5}{18a^5b^9c^3}$

ANSWER:

$$\frac{a^2c^2}{2b^4}$$

22. $(n^5)^4$

ANSWER:

$$n^{20}$$

23. $(z^3)^6$

ANSWER:

$$z^{18}$$

Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

24. $2x^2 - 3x + 5$

ANSWER:

yes; 2

25. $a^3 - 11$

ANSWER:

yes; 3

26. $\frac{5np}{n^2} - \frac{2g}{h}$

ANSWER:

no

27. $\sqrt{m-7}$

ANSWER:

no

CCSS REGULARITY Simplify.

28. $(6a^2 + 5a + 10) - (4a^2 + 6a + 12)$

ANSWER:

$$2a^2 - a - 2$$

29. $(7b^2 + 6b - 7) - (4b^2 - 2)$

ANSWER:

$$3b^2 + 6b - 5$$

30. $3p(np - z)$

ANSWER:

$$3np^2 - 3pz$$

31. $4x(2x^2 + y)$

ANSWER:

$$8x^3 + 4xy$$

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32. $(x - y)(x^2 + 2xy + y^2)$

ANSWER:

$$x^3 + x^2y - xy^2 - y^3$$

33. $(a + b)(a^3 - 3ab - b^2)$

ANSWER:

$$a^4 + a^3b - 3a^2b - 4ab^2 - b^3$$

34. $4(a^2 + 5a - 6) - 3(2a^3 + 4a - 5)$

ANSWER:

$$-6a^3 + 4a^2 + 8a - 9$$

35. $5c(2c^2 - 3c + 4) + 2c(7c - 8)$

ANSWER:

$$10c^3 - c^2 + 4c$$

36. $5xy(2x - y) + 6y^2(x^2 + 6)$

ANSWER:

$$10x^2y - 5xy^2 + 6x^2y^2 + 36y^2$$

37. $3ab(4a - 5b) + 4b^2(2a^2 + 1)$

ANSWER:

$$12a^2b + 8a^2b^2 - 15ab^2 + 4b^2$$

38. $(x - y)(x + y)(2x + y)$

ANSWER:

$$2x^3 + x^2y - 2xy^2 - y^3$$

39. $(a + b)(2a + 3b)(2x - y)$

ANSWER:

$$4a^2x - 2a^2y + 10abx - 5aby + 6b^2x - 3b^2y$$

40. **PAINTING** Connor has hired two painters to paint his house. The first painter charges \$12 an hour and the second painter charges \$11 an hour. It will take 15 hours of labor to paint the house.

a. Write a polynomial to represent the total cost of the job if the first painter does x hours of the labor.

b. Write a polynomial to represent the total cost of the job if the second painter does y hours of the labor.

ANSWER:

a. $x + 165$

b. $180 - y$

Simplify. Assume that no variable equals 0.

41. $\left(\frac{8x^2y^3}{24x^3y^2}\right)^4$

ANSWER:

$$\frac{y^4}{81x^4}$$

42. $\left(\frac{12a^3b^5}{4a^6b^3}\right)^3$

ANSWER:

$$\frac{27b^6}{a^9}$$

43. $\left(\frac{4x^{-2}y^3}{xy^{-4}}\right)^{-2}$

ANSWER:

$$\frac{x^6}{16y^{14}}$$

44. $\left(\frac{5a^{-7}b^2}{ab^{-6}}\right)^{-3}$

ANSWER:

$$\frac{a^{24}}{125b^{24}}$$

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45. $(a^2b^3)(ab)^{-2}$

ANSWER:

$$b$$

46. $(-3x^3y)^2(4xy^2)$

ANSWER:

$$36x^7y^4$$

47. $\frac{3c^2d(2c^3d^5)}{15c^4d^2}$

ANSWER:

$$\frac{2}{5}cd^4$$

48. $\frac{-10g^6h^9(g^2h^3)}{30g^3h^3}$

ANSWER:

$$-\frac{1}{3}g^5h^9$$

49. $\frac{5x^4y^2(2x^5y^6)}{20x^3y^5}$

ANSWER:

$$\frac{1}{2}x^6y^3$$

50. $\frac{-12n^7p^5(n^2p^4)}{36n^6p^7}$

ANSWER:

$$-\frac{1}{3}n^3p^2$$

51. **ASTRONOMY** The light from the Sun takes approximately 8 minutes to reach Earth. So if you are outside right now you are basking in sunlight that the Sun emitted approximately 8 minutes ago.

Light travels very fast, at a speed of about 3×10^8 meters per second. How long would it take light to get here from the Andromeda galaxy, which is approximately 2.367×10^{21} meters away?

a. How long does it take light from Andromeda to reach Earth?

b. The average distance from the Sun to Mars is approximately 2.28×10^{11} meters. How long does it take light from the Sun to reach Mars?

ANSWER:

a. 7.89×10^{12} s or about 250,190.26 years

b. 760 seconds or about 12.67 min

Simplify.

52. $\frac{1}{4}g^2(8g+12h-16gh^2)$

ANSWER:

$$2g^3 + 3g^2h - 4g^3h^2$$

53. $\frac{1}{3}n^3(6n-9p+18np^4)$

ANSWER:

$$2n^4 - 3n^3p + 6n^4p^4$$

54. $x^{-2}(x^4 - 3x^3 + x^{-1})$

ANSWER:

$$x^2 - 3x + \frac{1}{x^3}$$

55. $a^{-3}b^2(ba^3 + b^{-1}a^2 + b^{-2}a)$

ANSWER:

$$b^3 + \frac{b}{a} + \frac{1}{a^2}$$

56. $(g^3 - h)(g^3 + h)$

ANSWER:

$$g^6 - h^2$$

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57. $(n^2 - 7)(2n^3 + 4)$

ANSWER:

$$2n^5 - 14n^3 + 4n^2 - 28$$

58. $(2x - 2y)^3$

ANSWER:

$$8x^3 - 24x^2y + 24xy^2 - 8y^3$$

59. $(4n - 5)^3$

ANSWER:

$$64n^3 - 240n^2 + 300n - 125$$

60. $(3z - 2)^3$

ANSWER:

$$27z^3 - 54z^2 + 36z - 8$$

61. **CCSS MODELING** The polynomials $0.108x^2 - 0.876x + 474.1$ and $0.047x^2 + 9.694x + 361.7$ approximate the number of bachelor's degrees, in thousands, earned by males and females, respectively, where x is the number of years after 1971.

a. Find the polynomial that represents the total number of bachelor's degrees (in thousands) earned by both men and women.

b. Find the polynomial that represents the difference between bachelor's degrees earned by men and by women.

ANSWER:

a. $0.155x^2 + 8.818x + 835.8$

b. $0.061x^2 - 10.57x + 112.4$

62. If $5^{k+7} = 5^{2k-3}$, what is the value of k ?

ANSWER:

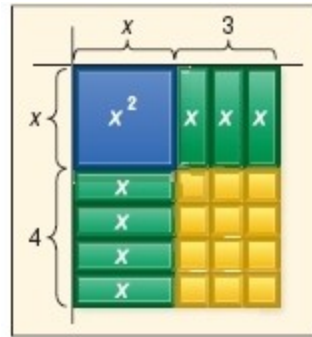
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63. What value of k makes $q^{41} = q^{4k} \cdot q^5$ true?

ANSWER:

9

64. **MULTIPLE REPRESENTATIONS** Use the model that represents the product of $x + 3$ and $x + 4$.
- a. **GEOMETRIC** The area of the each rectangle is the product of its length and width. Use the model to find the product of $x + 3$ and $x + 4$.
- b. **ALGEBRAIC** Use FOIL to find the product of $x + 3$ and $x + 4$.
- c. **VERBAL** Explain how each term of the product is represented in the model.



ANSWER:

a. $x^2 + 7x + 12$

b. $x^2 + 7x + 12$

c. Each term is represented by one or more rectangles with an area that represents the variable and the power in the term.

65. **PROOF** Show how the property of negative exponents can be proven using the Quotient of Powers Property and the Zero Power Property.

ANSWER:

$$\frac{1}{a^n} = \frac{a^0}{a^n} = a^{0-n} = a^{-n}$$

66. **CHALLENGE** What happens to the quantity of x^{-y} as y increases, for $y > 0$ and $x \geq 1$?

ANSWER:

Sample answer: It approaches 0.

67. **REASONING** Explain why the expression 0^{-2} is undefined.

ANSWER:

Sample answer: We would have a 0 in the denominator, which makes the expression undefined.

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68. **OPEN ENDED** Write three different expressions that are equivalent to x^{12} .

ANSWER:

Sample answer:

$$x^9 \cdot x^3, \frac{x^{14}}{x^2}, (x^6)^2$$

69. **WRITING IN MATH** Explain why properties of exponents are useful in astronomy. Include an explanation of how to find the amount of time it takes for light from a source to reach a planet.

ANSWER:

Sample answer: Astronomy deals with very large numbers that are sometimes difficult to work with because they contain so many digits. Properties of exponents make very large or very small numbers more manageable. As long as you know how far away a planet is from a light source you can divide that distance by the speed of light to obtain how long it will take light to reach that planet.

70. **Simplify** $\frac{(2x^2)^3}{12x^4}$.

ANSWER:

$$\frac{2x^2}{3}$$

71. **STATISTICS** For the numbers a , b , and c , the average (arithmetic mean) is twice the median. If $a = 0$ and $a < b < c$, what is the value of $\frac{c}{b}$?

- A 2
- B 3
- C 4
- D 5

ANSWER:

D

72. Which is not a factor of $x^3 - x^2 - 2x$?

- F x
- G $x + 1$
- H $x - 1$
- J $x - 2$

ANSWER:

H

73. **SAT/ACT** The expression $(-6 + i)^2$ is equivalent to which of the following expressions?

- A 35
- B $-12i$
- C $-12 + i$
- D $35 - 12i$
- E $37 - 12i$

ANSWER:

D

Solve each inequality algebraically.

74. $x^2 - 6x \leq 16$

ANSWER:

$$-2 \leq x \leq 8$$

75. $x^2 + 3x > 40$

ANSWER:

$$x > 5 \text{ or } x < -8$$

76. $2x^2 - 12 \leq -5x$

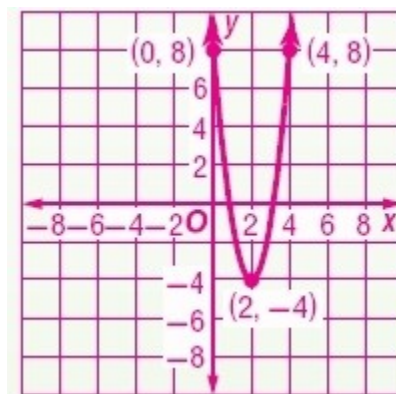
ANSWER:

$$-4 \leq x \leq 1.5$$

Graph each function.

77. $y = 3(x - 2)^2 - 4$

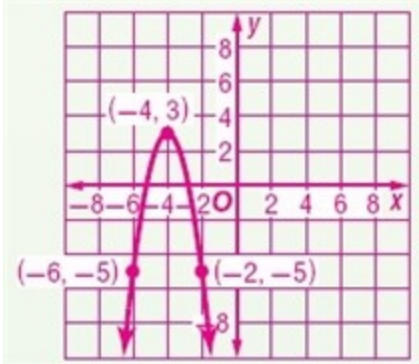
ANSWER:



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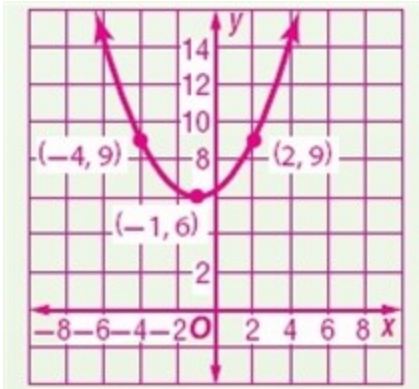
78. $y = -2(x + 4)^2 + 3$

ANSWER:

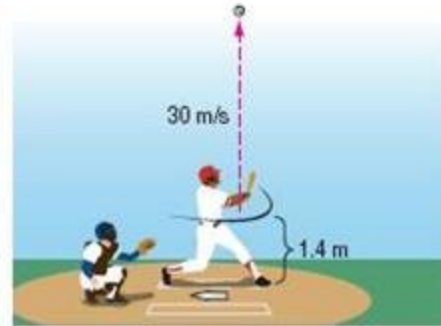


79. $y = \frac{1}{3}(x + 1)^2 + 6$

ANSWER:



80. **BASEBALL** A baseball player hits a high pop-up with an initial upward velocity of 30 meters per second, 1.4 meters above the ground. The height $h(t)$ of the ball in meters t seconds after being hit is modeled by $h(t) = -4.9t^2 + 30t + 1.4$. How long does an opposing player have to get under the ball if he catches it 1.7 meters above the ground? Does your answer seem reasonable? Explain.



ANSWER:

Sample answer: About 6.1 seconds; this answer seems reasonable. The equation has two solutions. The first solution, 0.01 second, is the time required for the ball to rise from 1.4 m to 1.7 m, and 6.1 seconds is the time required for the ball to come back down to 1.7 m.

Evaluate each determinant.

81. $\begin{vmatrix} 3 & 0 & -2 \\ -1 & 4 & 3 \\ 5 & -2 & -1 \end{vmatrix}$

ANSWER:

42

82. $\begin{vmatrix} -2 & -4 & -6 \\ 0 & 6 & -5 \\ -1 & 3 & -1 \end{vmatrix}$

ANSWER:

-74

83. $\begin{vmatrix} -3 & -1 & -2 \\ -2 & 3 & 4 \\ 6 & 1 & 0 \end{vmatrix}$

ANSWER:

28

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84. **FINANCIAL LITERACY** A couple is planning to invest \$15,000 in certificates of deposit (CDs). For tax purposes, they want their total interest the first year to be \$800. They want to put \$1000 more in a 2-year CD than in a 1-year CD and then invest the rest in a 3-year CD. How much should they invest in each type of CD?

Years	1	2	3
Rate	3.4%	5.0%	6.0%

ANSWER:

\$2500 in the 1-year; \$3500 in the 2-year; \$9000 in the 3-year

Find the slope of the line that passes through each pair of points.

85. (6, -2) and (-2, -9)

ANSWER:

$$\frac{7}{8}$$

86. (-4, -1) and (3, 8)

ANSWER:

$$\frac{9}{7}$$

87. (3, 0) and (-7, -5)

ANSWER:

$$\frac{1}{2}$$

88. $\left(\frac{1}{2}, \frac{2}{3}\right)$ and $\left(\frac{1}{4}, \frac{1}{3}\right)$

ANSWER:

$$\frac{4}{3}$$

89. $\left(\frac{2}{5}, \frac{1}{4}\right)$ and $\left(\frac{1}{10}, \frac{1}{12}\right)$

ANSWER:

$$\frac{5}{9}$$

90. (-4.5, 2.5) and (-3, -1)

ANSWER:

$$-\frac{7}{3}$$

Factor each polynomial.

91. $12ax^3 + 20bx^2 + 32cx$

ANSWER:

$$4x(3ax^2 + 5bx + 8c)$$

92. $x^2 + 2x + 6 + 3x$

ANSWER:

$$(x + 3)(x + 2)$$

93. $12y^2 + 9y + 8y + 6$

ANSWER:

$$(3y + 2)(4y + 3)$$

94. $2my + 7x + 7m + 2xy$

ANSWER:

$$(m + x)(2y + 7)$$

95. $8ax - 6x - 12a + 9$

ANSWER:

$$(2x - 3)(4a - 3)$$

96. $10x^2 - 14xy - 15x + 21y$

ANSWER:

$$(2x - 3)(5x - 7y)$$