5-2 Dividing Polynomials

Simplify.

1. $\frac{4xy^2 - 2xy + 2x^2y}{xy}$

ANSWER:
$4y + 2x - 2$

2. $(3a^2b - 6ab + 5ab^2)(ab)^{-1}$

ANSWER:
$3a + 5b - 6$

3. $(x^2 - 6x - 20) \div (x + 2)$

ANSWER:
$x - 8 - \frac{4}{x+2}$

4. $(2a^2 - 4a - 8) \div (a + 1)$

ANSWER:
$2a - 6 - \frac{2}{a+1}$

5. $(3z^4 - 6z^3 - 9z^2 + 3z - 6) \div (z + 3)$

ANSWER:
$3z^3 - 15z^2 + 36z - 105 + \frac{309}{z+3}$

6. $(y^5 - 3y^2 - 20) \div (y - 2)$

ANSWER:
$y^4 + 2y^3 + 4y^2 + 5y + 10$

7. **MULTIPLE CHOICE** Which expression is equal to $(x^2 + 3x - 9)(4 - x)^{-1}$?

A. $-x - 7 + \frac{19}{4-x}$

B. $-x - 7$

C. $x + 7 - \frac{19}{4-x}$

D. $-x - 7 - \frac{19}{4-x}$

ANSWER:
A

Simplify.

8. $(10x^2 + 15x + 20) \div (5x + 5)$

ANSWER:
$2x + 1 + \frac{3}{x+1}$

9. $(18a^2 + 6a + 9) \div (3a - 2)$

ANSWER:
$6a + 6 + \frac{21}{3a-2}$

10. $\frac{12b^2 + 23b + 15}{3b + 8}$

ANSWER:
$4b - 3 + \frac{39}{3b+8}$

11. $\frac{27y^2 + 27y - 30}{9y - 6}$

ANSWER:
$3y + 5$

Simplify

12. $\frac{24a^3b^2 - 16a^2b^3}{8ab}$

ANSWER:
$3a^2b - 2ab^2$

13. $\frac{5x^2y - 10xy + 15xy^2}{5xy}$

ANSWER:
$x + 3y - 2$

14. $\frac{7g^3h^2 + 3g^2h - 2gh^3}{gh}$

ANSWER:
$7g^2h + 3g - 2h^2$

15. $\frac{4a^3b - 6ab + 2ab^2}{2ab}$

ANSWER:

$2a^2 + b - 3$

16. $\frac{16c^4d^4 - 24c^2d^2}{4c^2d^2}$

ANSWER:

$4c^2d^2 - 6$

17. $\frac{9n^3p^3 - 18n^2p^2 + 21n^2p^3}{3n^2p^2}$

ANSWER:

$3np - 6 + 7p$

18. **ENERGY** Compact fluorescent light (CFL) bulbs reduce energy waste. The amount of energy waste that is reduced each day in a certain community can be estimated by $-b^2 + 8b$, where b is the number of bulbs. Divide by b to find the average amount of energy saved per CFL bulb.

ANSWER:

$-b + 8$

19. **BAKING** The number of cookies produced in a factory each day can be estimated by $-w^2 + 16w + 1000$, where w is the number of workers. Divide by w to find the average number of cookies produced per worker.

ANSWER:

$-w + 16 + \frac{1000}{w}$

Simplify.

20. $(a^2 - 8a - 26) \div (a + 2)$

ANSWER:

$a - 10 - \frac{6}{a+2}$

21. $(b^3 - 4b^2 + b - 2) \div (b + 1)$

ANSWER:

$b^2 - 5b + 6 - \frac{8}{b+1}$

22. $(z^4 - 3z^3 + 2z^2 - 4z + 4)(z - 1)^{-1}$

ANSWER:

$z^3 - 2z^2 - 4$

23. $(x^5 - 4x^3 + 4x^2) \div (x - 4)$

ANSWER:

$x^4 + 4x^3 + 12x^2 + 52x + 208 + \frac{832}{x-4}$

24. $\frac{y^3 + 11y^2 - 10y + 6}{y + 2}$

ANSWER:

$y^2 + 9y - 28 + \frac{62}{y+2}$

25. $(g^4 - 3g^2 - 18) \div (g - 2)$

ANSWER:

$g^3 + 2g^2 + g + 2 - \frac{14}{g-2}$

26. $(6a^2 - 3a + 9) \div (3a - 2)$

ANSWER:

$2a + \frac{1}{3} + \frac{29}{9a-6}$

27. $\frac{6x^5 + 5x^4 + x^3 - 3x^2 + x}{3x + 1}$

ANSWER:

$2x^4 + x^3 - x + \frac{2}{3} - \frac{2}{9x+3}$

28. $\frac{4g^4 - 6g^3 + 3g^2 - g + 12}{4g - 4}$

ANSWER:

$g^3 - \frac{1}{2}g^2 + \frac{1}{4}g + \frac{3}{g-1}$

29. $(2b^3 - 6b^2 + 8b) \div (2b + 2)$

ANSWER:

$b^2 - 4b + 8 - \frac{8}{b+1}$

30. $(6z^6 + 3z^4 - 9z^2)(3z - 6)^{-1}$

ANSWER:

$2z^5 + 4z^4 + 9z^3 + 18z^2 + 33z + 66 + \frac{132}{z-2}$

31. $(10y^6 + 5y^5 + 10y^3 - 20y - 15)(5y + 5)^{-1}$

ANSWER:

$2y^5 - y^4 + y^3 + y^2 - y - 3$

32. **CCSS REASONING** A rectangular box for a new product is designed in such a way that the three dimensions always have a particular relationship defined by the variable x. The volume of the box can be written as $6x^3 + 31x^2 + 53x + 30$, and the height is always $x + 2$. What are the width and length of the box?

ANSWER:

$2x + 3$, $3x + 5$

33. **PHYSICS** The voltage V is related to current I and power P by the equation $V = \frac{P}{I}$. The power of a generator is modeled by $P(t) = t^3 + 9t^2 + 26t + 24$. If the current of the generator is $I = t + 4$, write an expression that represents the voltage.

ANSWER:

$V(t) = t^2 + 5t + 6$

34. **ENTERTAINMENT** A magician gives these instructions to a volunteer.
• Choose a number and multiply it by 4.
• Then add the sum of your number and 15 to the product you found.
• Now divide by the sum of your number and 3.
a. What number will the volunteer always have at the end?
b. Explain the process you used to discover the answer.

ANSWER:

a. 5
b. Sample answer: Let x be the number. Multiply the x by 4 to get $4x$. Then add $x + 15$ to the product to get $5x + 15$. Divide the polynomial by $x + 3$. The quotient is 5.

35. **BUSINESS** The number of magazine subscriptions sold can be estimated by $n = \frac{3500a^2}{a^2 + 100}$, where a is the amount of money the company spent on advertising in hundreds of dollars and n is the number of subscriptions sold.

a. Perform the division indicated by $\frac{3500a^2}{a^2 + 100}$.

b. About how many subscriptions will be sold if $1500 is spent on advertising?

ANSWER:

a. $3500 - \frac{350,000}{a^2 + 100}$
b. about 2423 subscriptions

Simplify.

36. $(x^4 - y^4) \div (x - y)$

ANSWER:

$(x^2 + y^2)(x + y)$

37. $(28c^3d^2 - 21cd^2) \div (14cd)$

ANSWER:

$\frac{4c^2d - 3d}{2}$

38. $(a^3b^2 - a^2b + 2b)(-ab)^{-1}$

ANSWER:

$-a^2b + a - \frac{2}{a}$

39. $\frac{n^3 + 3n^2 - 5n - 4}{n + 4}$

ANSWER:

$n^2 - n - 1$

40. $\frac{p^3 + 2p^2 - 7p - 21}{p + 3}$

ANSWER:

$p^2 - p - 4 - \frac{9}{p + 3}$

41. $\frac{3z^5 + 5z^4 + z + 5}{z + 2}$

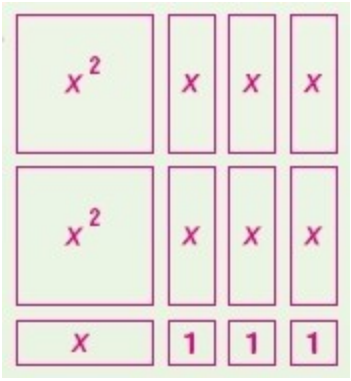
ANSWER:

$3z^4 - z^3 + 2z^2 - 4z + 9 - \frac{13}{z + 2}$

42. **MULTIPLE REPRESENTATIONS** Consider a rectangle with area $2x^2 + 7x + 3$ and length $2x + 1$.
a. **CONCRETE** Use algebra tiles to represent this situation. Use the model to find the width.
b. **SYMBOLIC** Write an expression to represent the model.
c. **NUMERICAL** Solve this problem algebraically using synthetic or long division. Does your concrete model check with your algebraic model?

ANSWER:

a.



The width is $x + 3$.

b. $2x^2 + 7x + 3 \div (2x + 1)$

c.

$-\frac{1}{2}$			
	1	$\frac{7}{2}$	$\frac{3}{2}$
		$-\frac{1}{2}$	$-\frac{3}{2}$
	1	3	0

yes

43. **ERROR ANALYSIS** Sharon and Jamal are dividing $2x^3 - 4x^2 + 3x - 1$ by $x - 3$. Sharon claims that the remainder is 26. Jamal argues that the remainder is –100. Is either of them correct? Explain your reasoning.

ANSWER:

Sample answer: Sharon; Jamal actually divided by $x + 3$.

44. **CHALLENGE** If a polynomial is divided by a binomial and the remainder is 0, what does this tell you about the relationship between the binomial and the polynomial?

ANSWER:

The binomial is a factor of the polynomial.

45. **REASONING** Review any of the division problems in this lesson. What is the relationship between the degrees of the dividend, the divisor, and the quotient?

ANSWER:

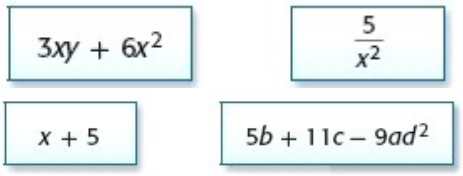
Sample answer: The degree of the quotient plus the degree of the divisor equals the degree of the dividend.

46. **OPEN ENDED** Write a quotient of two polynomials for which the remainder is 3.

ANSWER:

Sample answer: $\frac{x^2+5x+9}{x+2}$

47. **CCSS ARGUMENTS** Identify the expression that does not belong with the other three. Explain your reasoning.



ANSWER:

$\frac{5}{x^2}$ does not belong with the other three. The other three expressions are polynomials. Since the denominator of $\frac{5}{x^2}$ contains a variable, it is not a polynomial.

48. **WRITING IN MATH** Use the information at the beginning of the lesson to write assembly instruction using the division of polynomials to make a paper cover for your textbook.

ANSWER:

Sample answer: By dividing $140x^2 + 60x$ by $10x$, the quotient of $14x + 6$ provides the length of the book jacket. Then, subtracting $14x$, we are left with 6 inches. Half of this length is the width of each flap.

49. An office employs x women and 3 men. What is the ratio of the total number of employees to the number of women?

A $\frac{x+3}{x}$

B $\frac{x}{x+3}$

C $\frac{3}{x}$

D $\frac{x}{3}$

ANSWER:

A

50. **SAT/ACT** Which polynomial has degree 3?

A $x^3 + x^2 - 2x^4$

B $-2x^2 - 3x + 4$

C $3x - 3$

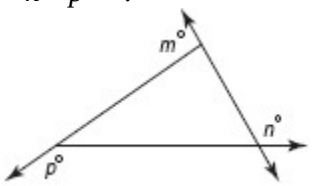
D $x^2 + x + 12^3$

E $1 + x + x^3$

ANSWER:

E

51. **GRIDDED RESPONSE** In the figure below, $m + n + p = ?$



ANSWER:

360

52. $(-4x^2 + 2x + 3) - 3(2x^2 - 5x + 1) =$

F $2x^2$

H $-10x^2 + 17x$

G $-10x^2$

J $2x^2 + 17x$

ANSWER:

H

Simplify.

53. $(5x^3 + 2x^2 - 3x + 4) - (2x^3 - 4x)$

ANSWER:

$3x^3 + 2x^2 + x + 4$

54. $(2y^3 - 3y + 8) + (3y^2 - 6y)$

ANSWER:

$2y^3 + 3y^2 - 9y + 8$

55. $4a(2a - 3) + 3a(5a - 4)$

ANSWER:

$23a^2 - 24a$

56. $(c + d)(c - d)(2c - 3d)$

ANSWER:

$2c^3 - 3c^2d - 2cd^2 + 3d^3$

57. $(xy)^2(2xy^2z)^3$

ANSWER:

$8x^5y^8z^3$

58. $(3ab^2)^{-2}(2a^2b)^2$

ANSWER:

$\frac{4a^2}{9b^2}$

59. **LANDSCAPING** Amado wants to plant a garden and surround it with decorative stones. He has enough stones to enclose a rectangular garden with a perimeter of 68 feet, but he wants the garden to cover no more than 240 square feet. What could the width of his garden be?

ANSWER:

0 to 10 ft or 24 to 34 ft

Solve each equation by completing the square.

60. $x^2 + 6x + 2 = 0$

ANSWER:

$-3 \pm \sqrt{7}$

61. $x^2 - 8x - 3 = 0$

ANSWER:

$4 \pm \sqrt{19}$

62. $2x^2 + 6x + 5 = 0$

ANSWER:

$-1.5 \pm \frac{i}{2}$

State the consecutive integers between which the zeros of each quadratic function are located.

63.

x	−7	−6	−5	−4	−3	−2	−1	0
$f(x)$	4	1	−3	−8	−1	2	8	16

ANSWER:

between –6 and –5; between –3 and –2

64.

x	−2	−1	0	1	2	3	4	5
$f(x)$	−16	−7	−4	3	3	−4	−7	−16

ANSWER:

between 0 and 1; between 2 and 3

65.

x	−2	−1	0	1	2	3	4	5
$f(x)$	6	1	−3	−5	−3	1	6	14

ANSWER:

between –1 and 0; between 2 and 3

66. **BUSINESS** A landscaper can mow a lawn in 30 minutes and perform a small landscape job in 90 minutes. He works at most 10 hours per day, 5 days per week. He earns $35 per lawn and $125 per landscape job. He cannot do more than 3 landscape jobs per day. Find the combination of lawns mowed and completed landscape jobs per week that will maximize income. Then find the maximum income.

ANSWER:

15 landscape jobs and 55 lawns; $3800

Find each value if $f(x) = 4x + 3$, $g(x) = -x^2$, and $h(x) = -2x^2 - 2x + 4$.

67. $f(-6)$

ANSWER:

–21

68. $g(-8)$

ANSWER:
-64

69. $h(3)$

ANSWER:
-20

70. $f(c)$

ANSWER:
$4c + 3$

71. $g(3d)$

ANSWER:
$-9d^2$

72. $h(2b + 1)$

ANSWER:
$-8b^2 - 12b$