5-3 Polynomial Functions

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

1. $11x^6 - 5x^5 + 4x^2$

ANSWER:
degree = 6, leading coefficient = 11

2. $-10x^7 - 5x^3 + 4x - 22$

ANSWER:
degree = 7, leading coefficient = –10

3. $14x^4 - 9x^3 + 3x - 4y$

ANSWER:
not in one variable because there are two variables, x and y

4. $8x^5 - 3x^2 + 4xy - 5$

ANSWER:
not in one variable because there are two variables, x and y

Find $w(5)$ and $w(-4)$ for each function.

5. $w(x) = -2x^3 + 3x - 12$

ANSWER:
$w(5) = -247$; $w(-4) = 104$

6. $w(x) = 2x^4 - 5x^3 + 3x^2 - 2x + 8$

ANSWER:
$w(5) = 698$; $w(-4) = 896$

If $c(x) = 4x^3 - 5x^2 + 2$ and $d(x) = 3x^2 + 6x - 10$, find each value.

7. $c(y^3)$

ANSWER:
$4y^9 - 5y^6 + 2$

8. $-4[d(3z)]$

ANSWER:
$-108z^2 - 72z + 40$

9. $6c(4a) + 2d(3a - 5)$

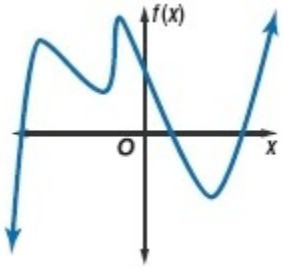
ANSWER:
$1536a^3 - 426a^2 - 144a + 82$

10. $-3c(2b) + 6d(4b - 3)$

ANSWER:
$-96b^3 + 348b^2 - 288b - 12$

For each graph,
a. describe the end behavior,
b. determine whether it represents an odd-degree or an even-degree function, and
c. state the number of real zeros.

11.



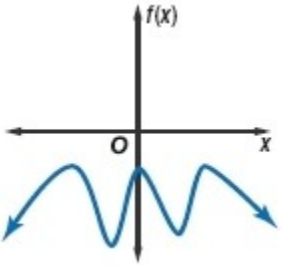
ANSWER:

a. $f(x) \to -\infty \text{ as } x \to -\infty,\ f(x) \to +\infty \text{ as } x \to +\infty$

b. Since the end behavior is in opposite directions, it is an odd-degree function.

c. The graph intersects the x-axis at three points, so there are three real zeros.

12.



ANSWER:

a. $f(x) \to -\infty \text{ as } x \to -\infty,\ f(x) \to -\infty \text{ as } x \to +\infty.$

b. Since the end behavior is in the same direction, it is an even-degree function.

c. The graph intersects the x-axis at zero points, so there are no real zeros.

CCSS PERSEVERANCE State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

13. $-6x^6 - 4x^5 + 13xy$

ANSWER:
not in one variable because there are two variables, x and y.

14. $3a^7 - 4a^4 + \frac{3}{a}$

ANSWER:
not a polynomial because there is a negative exponent

15. $8x^5 - 12x^6 + 14x^3 - 9$

ANSWER:
degree = 6, leading coefficient =–12

16. $-12 - 8x^2 + 5x - 21x^7$

ANSWER:
degree = 7, leading coefficient = –21

17. $15x - 4x^3 + 3x^2 - 5x^4$

ANSWER:
degree = 4, leading coefficient = –5

18. $13b^3 - 9b + 3b^5 - 18$

ANSWER:
degree = 5, leading coefficient = 3

19. $(d + 5)(3d - 4)$

ANSWER:
degree = 2, leading coefficient = 3

20. $(5 - 2y)(4 + 3y)$

ANSWER:
degree = 2, leading coefficient = –6

21. $6x^5 - 5x^4 + 2x^9 - 3x^2$

ANSWER:
degree = 9, leading coefficient = 2

22. $7x^4 + 3x^7 - 2x^8 + 7$

ANSWER:
degree = 8, leading coefficient = –2

Find $p(-6)$ and $p(3)$ for each function.

23. $p(x) = x^4 - 2x^2 + 3$

ANSWER:
$p(-6) = 1227; p(3) = 66$

24. $p(x) = -3x^3 - 2x^2 + 4x - 6$

ANSWER:
$p(-6) = 546; p(3) = -93$

25. $p(x) = 2x^3 + 6x^2 - 10x$

ANSWER:
$p(-6) = -156; p(3) = 78$

26. $p(x) = x^4 - 4x^3 + 3x^2 - 5x + 24$

ANSWER:
$p(-6) = 2322; p(3) = 9$

27. $p(x) = -x^3 + 3x^2 - 5$

ANSWER:
$p(-6) = 319; p(3) = -5$

28. $p(x) = 2x^4 + x^3 - 4x^2$

ANSWER:
$p(-6) = 2232; p(3) = 153$

If $c(x) = 2x^2 - 4x + 3$ and $d(x) = -x^3 + x + 1$, find each value.

29. $c(3a)$

ANSWER:
$18a^2 - 12a + 3$

30. $5d(2a)$

ANSWER:
$-40a^3 + 10a + 5$

31. $c(b^2)$

ANSWER:

$2b^4 - 4b^2 + 3$

32. $d(4a^2)$

ANSWER:

$-64a^6 + 4a^2 + 1$

33. $d(4y - 3)$

ANSWER:

$-64y^3 + 144y^2 - 104y + 25$

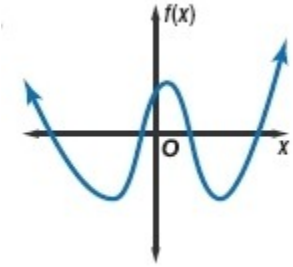
34. $c(y^2 - 1)$

ANSWER:

$2y^4 - 8y^2 + 9$

For each graph,
a. describe the end behavior,
b. determine whether it represents an odd-degree or an even-degree function, and
c. state the number of real zeros.

35.



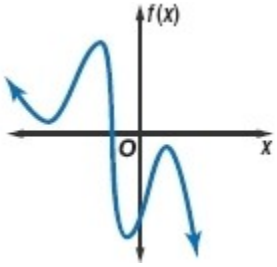
ANSWER:

a. $f(x) \to +\infty$ as $x \to -\infty$. $f(x) \to +\infty$ as $x \to +\infty$.

b. Since the end behavior is in the same direction, it is an even-degree function.

c. The graph intersects the *x*-axis at four points, so there are four real zeros.

36.



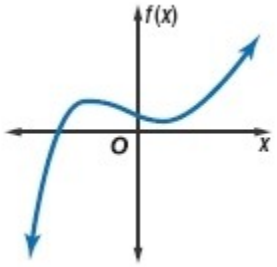
ANSWER:

a. $f(x) \to +\infty$ as $x \to -\infty$. $f(x) \to -\infty$ as $x \to +\infty$.

b. Since the end behavior is in opposite directions, it is an odd-degree function.

c. The graph intersects the *x*-axis at one point, so there is one real zero.

37.



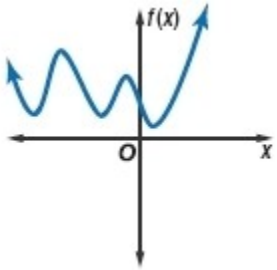
ANSWER:

a. $f(x) \to -\infty$ as $x \to -\infty$. $f(x) \to +\infty$ as $x \to +\infty$.

b. Since the end behavior is in opposite directions, it is an odd-degree function.

c. The graph intersects the *x*-axis at one point, so there is one real zero.

38.

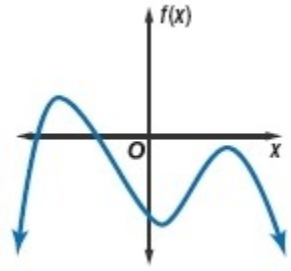


ANSWER:

a. $f(x) \to +\infty$ as $x \to -\infty$. $f(x) \to +\infty$ as $x \to +\infty$.

b. Since the end behavior is in the same direction, it is an even-degree function.

c. The graph intersects the *x*-axis at no points, so there are no real zeros.

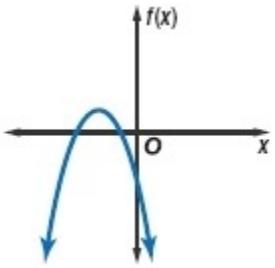
39. 

ANSWER:

a. $f(x) \to -\infty$ as $x \to -\infty$, $f(x) \to -\infty$ as $x \to +\infty$.

b. Since the end behavior is in the same direction, it is an even-degree function.

c. The graph intersects the *x*-axis at two points, so there are two real zeros.

40. 

ANSWER:

a. $f(x) \to -\infty as x \to -\infty$. $f(x) \to -\infty as x \to +\infty$.

b. Since the end behavior is in the same direction, it is an even-degree function.

c. The graph intersects the *x*-axis at two points, so there are two real zeros.

41. **PHYSICS** For a moving object with mass *m* in kilograms, the kinetic energy *KE* in joules is given by the function $KE(v) = 0.5mv^2$, where *v* represents the speed of the object in meters per second. Find the kinetic energy of an all-terrain vehicle with a mass of 171 kilograms moving at a speed of 11 meters/second.

ANSWER:

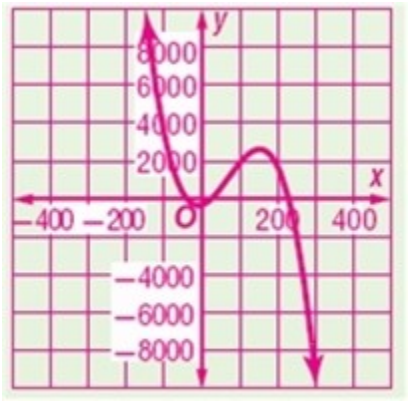
10,345.5 joules

42. **CCSS MODELING** A microwave manufacturing firm has determined that their profit function is $P(x) = -0.0014x^3 + 0.3x^2 + 6x - 355$ where *x* is the number of microwaves sold annually.

a. Graph the profit function using a calculator.
b. Determine a reasonable viewing window for the function.
c. Approximate all of the zeros of the function using the **CALC** menu.
d. What must be the range of microwaves sold in order for the firm to have a profit?

ANSWER:

a.



b. Sample answer: [–500, 500] scl: 50 by [–10,000, 10,000] scl: 1000

c. – 41.0, 27.1, 228.2

d. 28 to 228 microwaves

Find *p* (–2) and *p* (8) for each function.

43. $p(x) = \frac{1}{4}x^4 + \frac{1}{2}x^3 - 4x^2$

ANSWER:

$p(-2) = -16; p(8) = 1024$

44. $p(x) = \frac{1}{8}x^4 - \frac{3}{2}x^3 + 12x - 18$

ANSWER:

$p(-2) = -28; p(8) = -178$

45. $p(x) = \frac{3}{4}x^4 - \frac{1}{8}x^2 + 6x$

ANSWER:

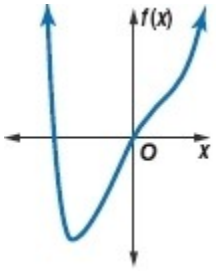
$p(-2) = -0.5; p(8) = 3112$

46. $p(x) = \frac{5}{8}x^3 - \frac{1}{2}x^2 + \frac{3}{4}x + 10$

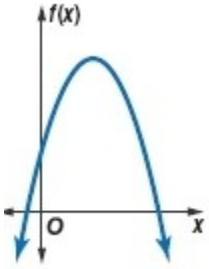
ANSWER:
$p(-2) = 1.5; p(8) = 304$

Use the degree and end behavior to match each polynomial to its graph.

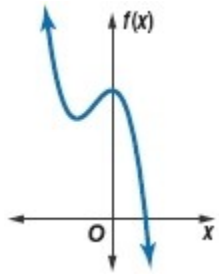
A.



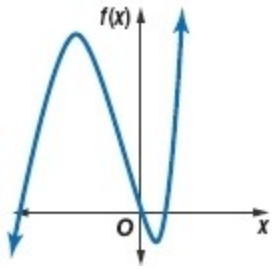
B.



C.



D.



47. $f(x) = x^3 + 3x^2 - 4x$

ANSWER:
D

48. $f(x) = -2x^2 + 8x + 5$

ANSWER:
B

49. $f(x) = x^4 - 3x^2 + 6x$

ANSWER:
A

50. $f(x) = -4x^3 - 4x^2 + 8$

ANSWER:
C

If $c(x) = x^3 - 2x$ and $d(x) = 4x^2 - 6x + 8$, find each value.

51. $3c(a - 4) + 3d(a + 5)$

ANSWER:
$3a^3 - 24a^2 + 240a + 66$

52. $-2d(2a + 3) - 4c(a^2 + 1)$

ANSWER:
$-4a^6 - 12a^4 - 36a^2 - 72a - 48$

53. $5c(a^2) - 8d(6 - 3a)$

ANSWER:
$5a^6 - 298a^2 + 1008a - 928$

54. $-7d(a^3) + 6c(a^4 + 1)$

ANSWER:
$6a^{12} + 18a^8 - 28a^6 + 6a^4 + 42a^3 - 62$

55. **BUSINESS** A clothing manufacturer's profitability can be modeled by $p(x) = -x^4 + 40x^2 - 144$, where x is the number of items sold in thousands and $p(x)$ is the company's profit in thousands of dollars.

a. Use a table of values to sketch the function.

b. Determine the zeros of the function.

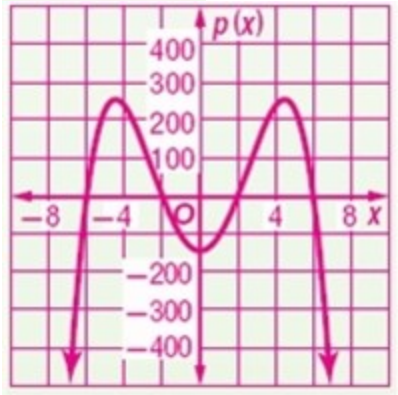
c. Between what two values should the company sell in order to be profitable?

d. Explain why only two of the zeros are considered in part c.

ANSWER:

a.

x	$p(x)$
–7	–585
–6	0
–4	240
–3	135
–2	0
0	–144
1	–105
2	0
4	240
6	0
7	–585



b. –6, –2, 2, 6

c. 2000 and 6000 items

d. Sample answer: The negative values should not be considered because the company will not produce negative items.

56. **MULTIPLE REPRESENTATIONS** Consider $g(x) = (x - 2)(x + 1)(x - 3)(x + 4)$

a. ANALYTICAL Determine the x- and y-intercepts, roots, degree, and end behavior of $g(x)$.

b. ALGEBRAIC Write the function in standard form.

c. TABULAR Make a table of values for the function.

d. GRAPHICAL Sketch a graph of the function by plotting points and connecting them with a smooth curve.

ANSWER:

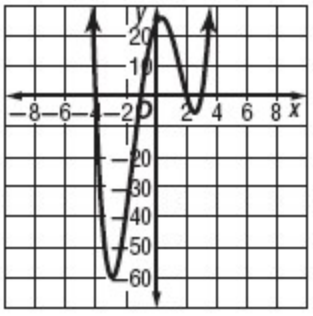
a. degree: 4; x-intercepts: 2, –1, 3, –4; y-intercept: 24; roots: 2, –1, 3, –4, end behavior: as $x \to -\infty, g(x) \to +\infty$; as $x \to +\infty, g(x) \to +\infty$

b. $g(x) = x^4 - 5x^2 + 10x + 24$

c.

x	$g(x)$
–5	224
–4	0
–3	–60
–2	–40
–1	0
0	24
1	20
2	0
3	0
4	80
5	324

d.



Describe the end behavior of the graph of each function.

57. $f(x) = -5x^4 + 3x^2$

ANSWER:

$f(x) \to -\infty$ as $x \to -\infty$; $f(x) \to -\infty$ as $x \to +\infty$.

58. $g(x) = 2x^5 + 6x^4$

ANSWER:

$g(x) \to -\infty as x \to -\infty, g(x) \to +\infty as x \to +\infty$.

59. $h(x) = -4x^7 + 8x^6 - 4x$

ANSWER:

$h(x) \to +\infty as x \to -\infty, h(x) \to -\infty as x \to +\infty$.

60. $f(x) = 6x - 7x^2$

ANSWER:

$f(x) \to -\infty \text{ as } x \to -\infty; f(x) \to -\infty \text{ as } x \to +\infty$.

61. $g(x) = 8x^4 + 5x^5$

ANSWER:

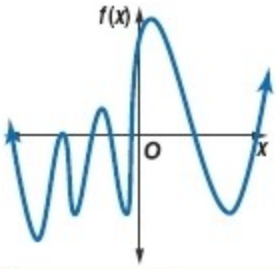
$g(x) \to -\infty as x \to -\infty, g(x) \to +\infty as x \to +\infty$.

62. $h(x) = 9x^6 - 5x^7 + 3x^2$

ANSWER:

$h(x) \to +\infty as x \to -\infty, h(x) \to -\infty as x \to +\infty$.

63. **CCSS CRITIQUE** Shenequa and Virginia are determining the number of real zeros of the graph. Is either of them correct? Explain your reasoning.



Shenequa

There are 7 real zeros because the graph intersects the *x*-axis 7 times.

Virginia

There are 8 real zeros because the graph intersects the *x*-axis 7 times, and there is a double zero.

ANSWER:

Sample answer: Shenequa is correct; the number of real zeros is equal to exactly the number of times the graph intersects the *x*-axis.

64. **CHALLENGE** Of $f(x)$ and $g(x)$, which function has more potential real zeros? What is the degree of that function?

x	−24	−18	−12	−6	0	6	12	18	24
$f(x)$	−8	−1	3	−2	4	7	−1	−8	5

$g(x) = x^4 + x^3 - 13x^2 + x + 4$

ANSWER:

$f(x)$; $f(x)$ has potential for 5 or more real zeros and a degree of 5 or more. $g(x)$ has potential for 4 real zeros and a degree of 4.

65. **CHALLENGE** If $f(x)$ has a degree of 5 and a positive leading coefficient and $g(x)$ has a degree of 3 and a positive leading coefficient, determine the end behavior of $\frac{f(x)}{g(x)}$. Explain your reasoning.

ANSWER:

Sample answer:

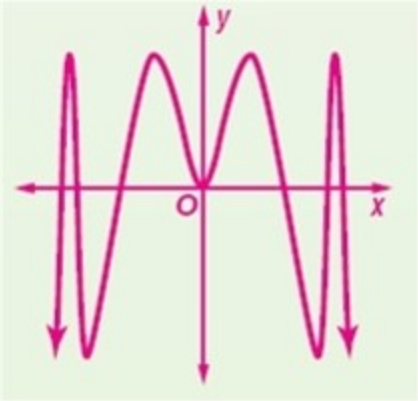
$f(x) \to +\infty$ as $x \to -\infty$. $f(x) \to +\infty$ as $x \to +\infty$.

$\frac{f(x)}{g(x)}$ will become a 2nd-degree function with a positive leading coefficient.

66. **OPEN ENDED** Sketch the graph of an even-degree polynomial with 7 real zeros, one of them a double zero.

ANSWER:

Sample answer:



67. **REASONING** Determine whether the following statement is *always*, *sometimes*, or *never* true. Explain.

A polynomial function that has four real zeros is a fourth-degree polynomial.

ANSWER:

Sometimes; a polynomial function with four real zeros may be a sixth-degree polynomial function with two imaginary zeros. A polynomial function that has four real zeros is at least a fourth-degree polynomial.

68. **WRITING IN MATH** Describe what the end behavior of a polynomial function is and how to determine it.

ANSWER:

Sample answer: The end behavior of a polynomial function is what the graph does as the input value approaches negative and positive infinity. It can be determined by the leading coefficient and the degree of the polynomial.

69. **SHORT RESPONSE** Four students solved the same math problem. Each student's work is shown below. Who is correct?

Student A

$$x^2x^{-5} = \frac{x^2}{x^5} = \frac{1}{x^3}, x \neq 0$$

Student B

$$x^2x^{-5} = \frac{x^2}{x^{-5}} = x^{-7}, x \neq 0$$

Student C

$$x^2x^{-5} = \frac{x^2}{x^{-5}} = x^7, x \neq 0$$

Student D

$$x^2x^{-5} = \frac{x^2}{x^5} = x^3, x \neq 0$$

ANSWER:

Student A

70. **SAT/ACT** What is the remainder when $x^3 - 7x + 5$ is divided by $x + 3$?

A –11
B –1
C 1
D 11
E 35

ANSWER:

B

71. **EXTENDED RESPONSE** A company manufactures tables and chairs. It costs \$40 to make each table and \$25 to make each chair. There is \$1440 available to spend on manufacturing each week. Let t = the number of tables produced and c = the number of chairs produced.

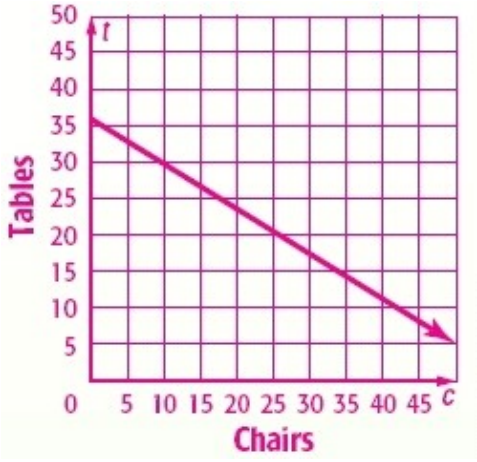
a. The manufacturing equation is $40t + 25c = 1500$. Construct a graph of this equation.

b. The company always produces two chairs with

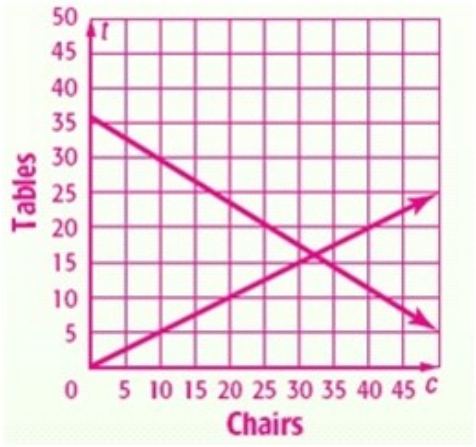
each table. Write and graph an equation to represent this situation on the same graph as the one in part **a**.
c. Determine the number of tables and chairs that the company can produce each week.
d. Explain how to determine this answer using the graph.

ANSWER:

a.



b. $t = 0.5c$;



c. 16 tables and 32 chairs
d. Sample answer: This can be determined by the intersection of he graphs. This point of intersection is the optimal amount of tables and chairs manufactured.

72. If $i = \sqrt{-1}$ then $5i(7i) =$
F 70
H –35
G 35
J –70

ANSWER:
H

Simplify.

73. $\dfrac{16x^4y^3 + 32x^6y^5z^2}{8x^2y}$

ANSWER:
$2x^2y^2 + 4x^4y^4z^2$

74. $\dfrac{18ab^4c^5 - 30a^4b^3c^2 + 12a^5bc^3}{6abc^2}$

ANSWER:
$3b^3c^3 - 5a^3b^2 + 2a^4c$

75. $\dfrac{18c^5d^2 - 3c^2d^2 + 12a^5c^3d^4}{3c^2d^2}$

ANSWER:
$6c^3 - 1 + 4a^5cd^2$

Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

76. $8x^2 + 5xy^3 - 6x + 4$

ANSWER:
yes; 4

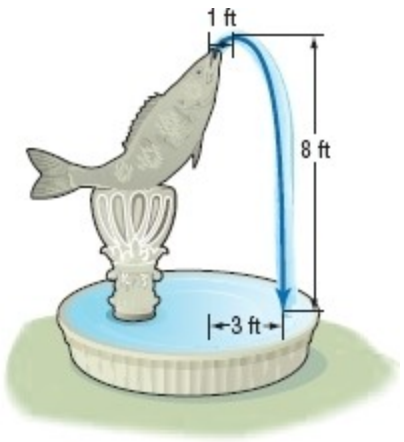
77. $9x^4 + 12x^6 - 16$

ANSWER:
yes; 6

78. $3x^4 + 2x^2 - x^{-1}$

ANSWER:
not a polynomial

79. **FOUNTAINS** The height of a fountain's water stream can be modeled by a quadratic function. Suppose the water from a jet reaches a maximum height of 8 feet at a distance 1 foot away from the jet.
a. If the water lands 3 feet away from the jet, find a quadratic function that models the height $h(d)$ of the water at any given distance d feet from the jet. Then compare the graph of the function to the parent function.
b. Suppose a worker increases the water pressure so that the stream reaches a maximum height of 12.5 feet at a distance of 15 inches from the jet. The water now lands 3.75 feet from the jet. Write a new quadratic function for $h(d)$. How do the changes in h and k affect the shape of the graph?



ANSWER:

a. $h(d) = -2d^2 + 4d + 6$;
The graph opens downward and is narrower than the parent graph, and the vertex is at (1, 8).
b. $h(d) = -2(d - 1.25)^2 + 12.5$;
It shifted the graph up 4.5 ft and to the right 3 in.

Solve each inequality.

80. $|2x + 4| \le 8$

ANSWER:

$-6 \le x \le 2$

81. $|-3x + 2| \ge 4$

ANSWER:

$x \le -\frac{2}{3}$ or $x \ge 2$

82. $|2x - 8| - 4 \le -6$

ANSWER:

no solution

Determine whether each function has a maximum or minimum value, and find that value.

83. $f(x) = 3x^2 - 8x + 4$

ANSWER:

minimum; $-\frac{4}{3}$

84. $f(x) = -4x^2 + 2x - 10$

ANSWER:

maximum; −9.75

85. $f(x) = -0.25x^2 + 4x - 5$

ANSWER:

maximum;11