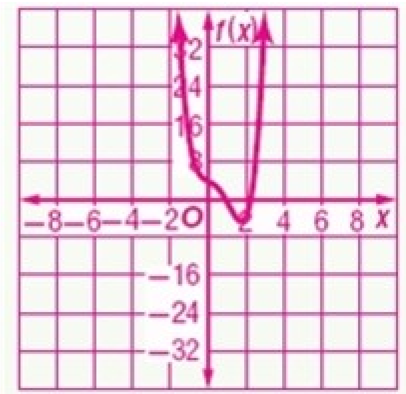
5-4 Analyzing Graphs of Polynomial Functions

Graph each polynomial equation by making a table of values.

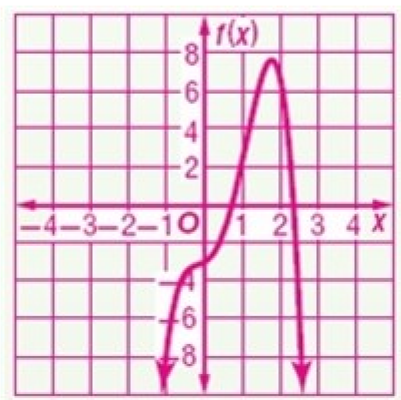
1. $f\ (x) = 2x^4 - 5x^3 + x^2 - 2x + 4$

ANSWER:



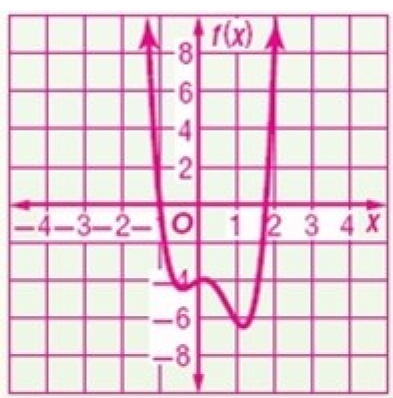
2. $f\ (x) = -2x^4 + 4x^3 + 2x^2 + x - 3$

ANSWER:



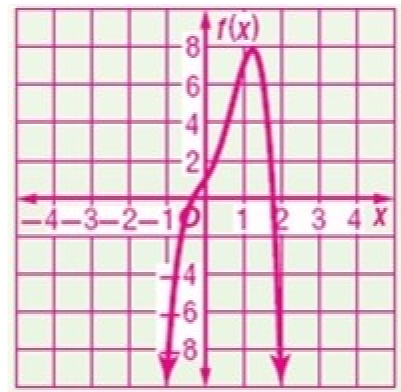
3. $f\ (x) = 3x^4 - 4x^3 - 2x^2 + x - 4$

ANSWER:



4. $f\ (x) = -4x^4 + 5x^3 + 2x^2 + 3x + 1$

ANSWER:

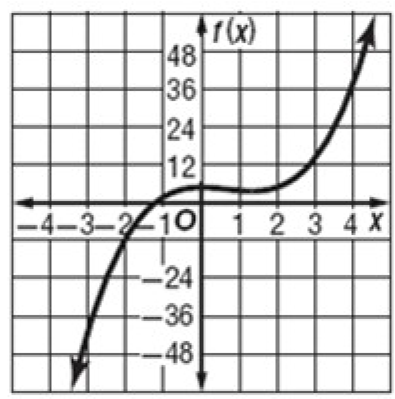


Determine the consecutive integer values of *x* between which each real zero of each function located. Then draw the graph.

5. $f\ (x) = x^3 - 2x^2 + 5$

ANSWER:

between –2 and –1

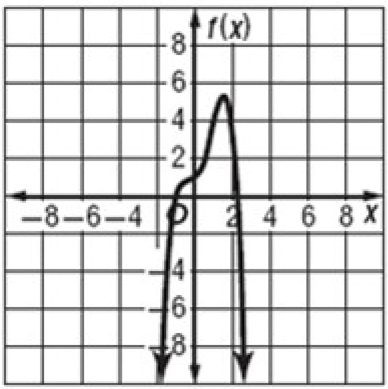


5-4 Analyzing Graphs of Polynomial Functions

6. $f(x) = -x^4 + x^3 + 2x^2 + x + 1$

ANSWER:

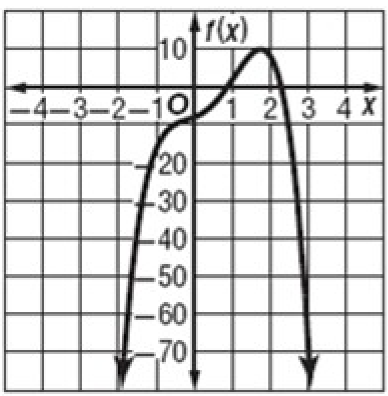
at –1 and between 2 and 3



7. $f(x) = -3x^4 + 5x^3 + 4x^2 + 4x - 8$

ANSWER:

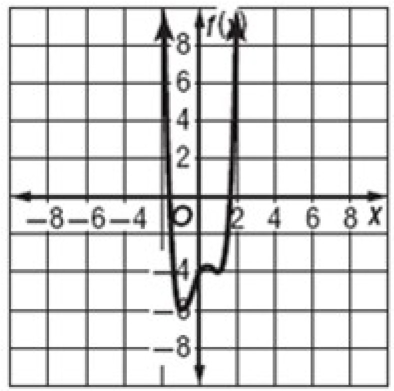
between 0 and 1 and between 2 and 3



8. $f(x) = 2x^4 - x^3 - 3x^2 + 2x - 4$

ANSWER:

between –2 and –1 and between 1 and 2

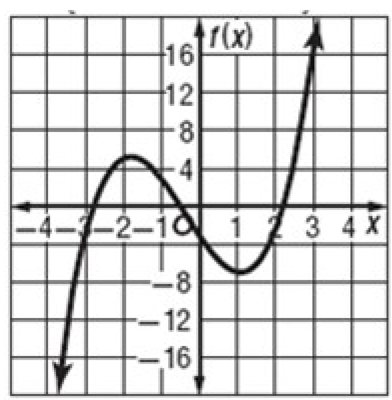


Graph each polynomial function. Estimate the *x*-coordinates at which the relative maxima and relative minima occur. State the domain and range for each function.

9. $f(x) = x^3 + x^2 - 6x - 3$

ANSWER:

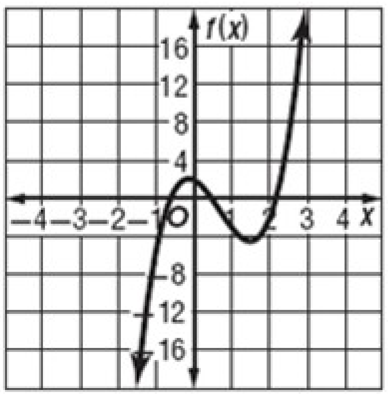
rel. max at $x \approx -1.8$; rel. min at $x \approx 1.1$; D ={all real numbers}, R={all real numbers}



10. $f(x) = 3x^3 - 6x^2 - 2x + 2$

ANSWER:

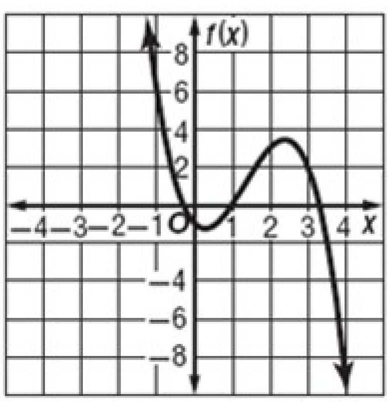
rel. max at $x \approx -0.2$; rel. min at $x \approx 1.5$; D = {all real numbers},R = {all real numbers}



11. $f(x) = -x^3 + 4x^2 - 2x - 1$

ANSWER:

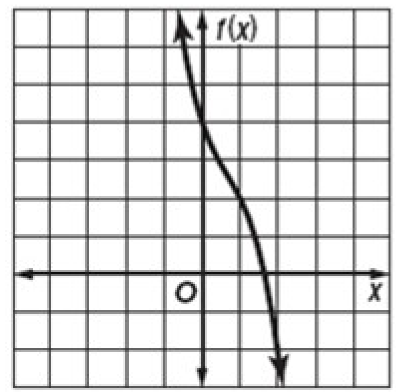
rel. max at $x \approx 2.4$; rel. min at $x \approx 0.3$; D = {all real numbers},R = {all real numbers}



12. $f(x) = -x^3 + 2x^2 - 3x + 4$

ANSWER:

no relative max or min; D = {all real numbers}, R = {all real numbers}



13. **CCSS SENSE-MAKING** Annual compact disc sales can be modeled by the quartic function $f(x) = 0.48x^4 - 9.6x^3 + 53x^2 - 49x + 599$, where x is the number of years after 1995 and $f(x)$ is annual sales in millions.

a. Graph the function for $0 \le x \le 10$

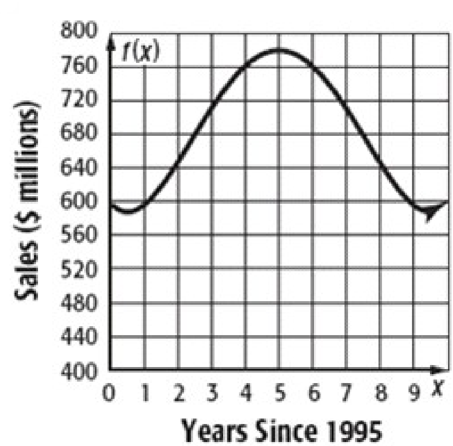
b. Describe the turning points of the graph, its end behavior, and the intervals on which the graph is increasing or decreasing.

c. Continue the graph for $x = 11$ and $x = 12$. What trends in compact disc sales does the graph suggest?

d. Is it reasonable that the trend will continue indefinitely? Explain.

ANSWER:

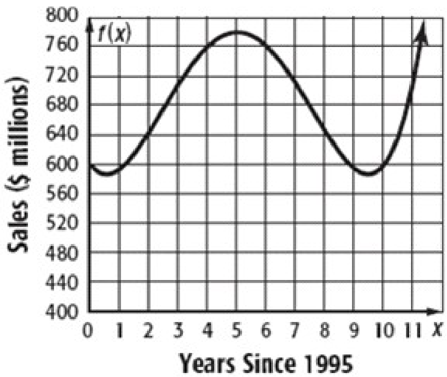
a.



b. Sample answer: Relative maximum at $x = 5$ and relative minimum at $x \approx 9.5$. $f(x) \to \infty$ as $x \to \infty$ and f

$(x) \to \infty$ as $x \to -\infty$. The graph increases when $x < 5$ and $x > 9.5$ and decreases when $5 < x < 9.5$.

c.



Sample answer: This suggests a dramatic increase in sales.

d. Sample answer: No; with so many other forms of media on the market today, CD sales will not increase dramatically. In fact, the sales will probably decrease. The function appears to be accurate only until about 2005.

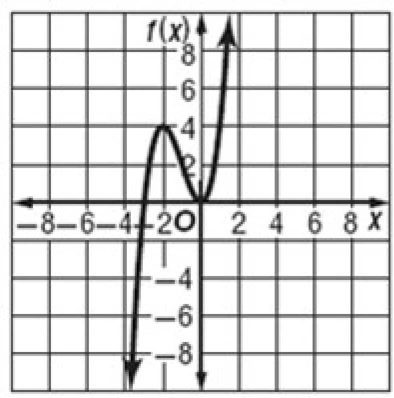
Complete each of the following.
a. Graph each function by making a table of values.
b. Determine the consecutive integer values of *x* between which each real zero is located.
c. Estimate the *x*-coordinates at which the relative maxima and minima occur.

14. $f(x) = x^3 + 3x^2$

ANSWER:

a.

x	$f(x)$
–4	–16
–3	0
–2	4
–1	2
0	0
1	4
2	20
3	54
4	112



b. at 0 and at –3
c. rel. max: $x = -2$; rel. min: $x = 0$

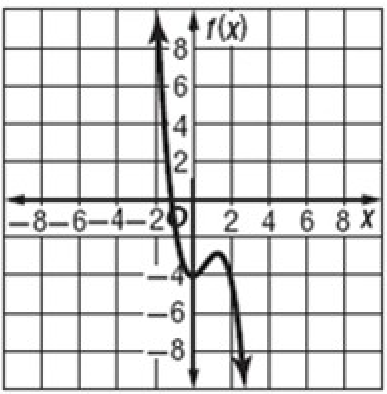
5-4 Analyzing Graphs of Polynomial Functions

15. $f(x) = -x^3 + 2x^2 - 4$

ANSWER:

a.

x	f(x)
–4	92
–3	41
–2	12
–1	–1
0	–4
1	–3
2	–4
3	–13
4	–36



b. between –2 and –1

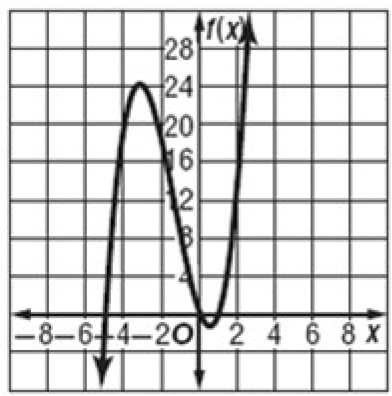
c. rel. min: $x = 0$; rel. max: $x = 1$

16. $f(x) = x^3 + 4x^2 - 5x$

ANSWER:

a.

x	f(x)
–6	–42
–5	0
–4	20
–3	24
–2	18
–1	8
0	0
1	0
2	14
3	48



b. – 5, 0, and 1

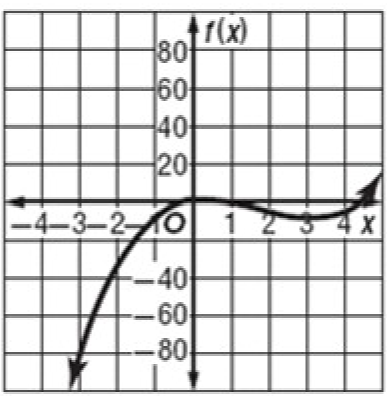
c. rel. max: $x = -3$; rel. min: between $x = 0$ and $x = 1$

17. $f(x) = x^3 - 5x^2 + 3x + 1$

ANSWER:

a.

x	$f(x)$
–4	–155
–3	–80
–2	–33
–1	–8
0	1
1	0
2	–5
3	–8
4	–3
5	16



b. at $x = 1$, between –1 and 0, between $x = 4$ and $x = 5$

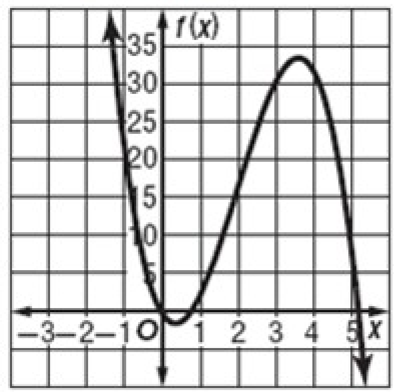
c. rel. max: $x \approx \frac{1}{3}$, rel. min: $x \approx 3$

18. $f(x) = -2x^3 + 12x^2 - 8x$

ANSWER:

a.

x	$f(x)$
–1	22
0	0
1	2
2	16
3	30
4	32
5	10
6	–48
7	–154



b. at 0, between 0 and 1, and between 5 and 6

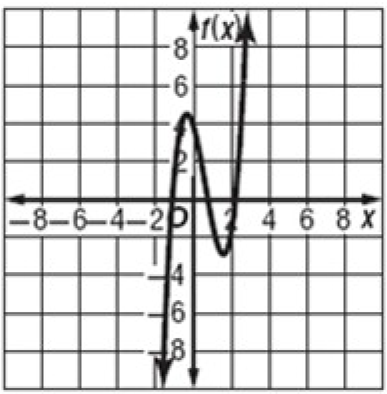
c. rel. min: between $x = 0$ and $x = 1$; rel. max: near $x = 4$

19. $f(x) = 2x^3 - 4x^2 - 3x + 4$

ANSWER:

a.

x	$f(x)$
–4	–176
–3	–77
–2	–22
–1	1
0	4
1	–1
2	–2
3	13
4	56



b. between $x = -2$ and $x = -1$, between $x = 0$ and $x = 1$, and between $x = 2$ and $x = 3$

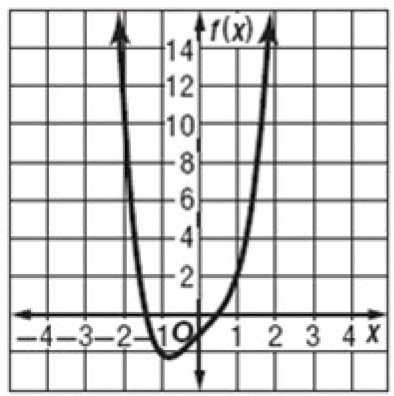
c. rel. max: near $x = -0.3$; rel. min: near $x = 1.6$

20. $f(x) = x^4 + 2x - 1$

ANSWER:

a.

x	$f(x)$
–4	247
–3	74
–2	11
–1	–2
0	–1
1	2
2	19
3	86
4	263



b. between $x = -2$ and $x = -1$ and between $x = 0$ and $x = 1$

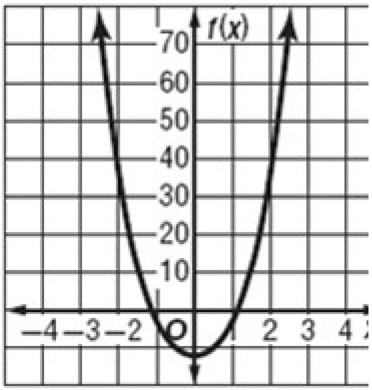
c. min: near $x = -1$; no rel. max

21. $f\ (x) = x^4 + 8x^2 - 12$

ANSWER:

a.

x	$f(x)$
–4	372
–3	141
–2	36
–1	–3
0	–12
1	–3
2	36
3	141
4	372



b. between x = –2 and x = –1 and between x = 1 and = 2

c. min: near x = 0

22. **FINANCIAL LITERACY** The average annual price of gasoline can be modeled by the cubic function $f\,(x) = 0.0007x^3 - 0.014x^2 + 0.08x + 0.96$, where x is the number of years after 1987 and $f\,(x)$ is the price in dollars.

a. Graph the function for $0 \le x \le 30$.

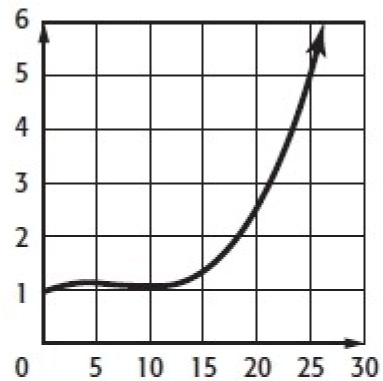
b. Describe the turning points of the graph and its end behavior.

c. What trends in gasoline prices does the graph suggest?

d. Is it reasonable that the trend will continue indefinitely? Explain.

ANSWER:

a.



b. Sample answer: The graph has a relative minimum at x = 10 and then increases as x increases.

c. The graph suggests a fairly steep continuous increase and gas prices at $5 per gallon by 2012, which could be possible.

d. Sample answer: While it is possible for gasoline prices to continue to soar at this rate, it is likely that alternate forms of transportation and fuel will slow down this rapid increase.

Use a graphing calculator to estimate the *x*-coordinates at which the maxima and minima of each function occur. Round to the nearest hundredth.

23. $f\ (x) = x^3 + 3x^2 - 6x - 6$

ANSWER:

rel. max: x = –2.73; rel. min: x = 0.73

24. $f\ (x) = -2x^3 + 4x^2 - 5x + 8$

ANSWER:

no relative max or min

25. $f(x) = -2x^4 + 5x^3 - 4x^2 + 3x - 7$

ANSWER:
rel. max: $x = 1.34$; no rel. min

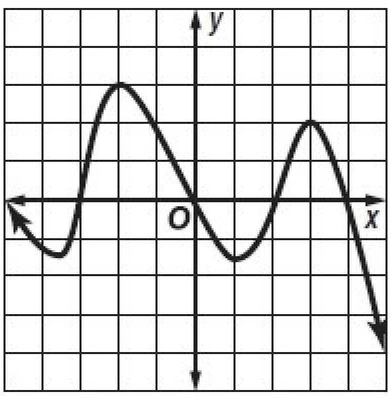
26. $f(x) = x^5 - 4x^3 + 3x^2 - 8x - 6$

ANSWER:
rel. max: $x = -1.87$; rel. min: $x = 1.52$

Sketch the graph of polynomial functions with the following characteristics.

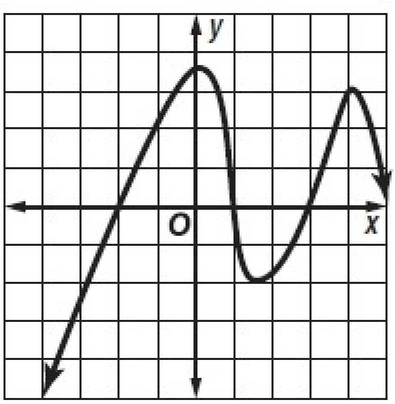
27. an odd function with zeros at –5, –3, 0, 2 and 4

ANSWER:



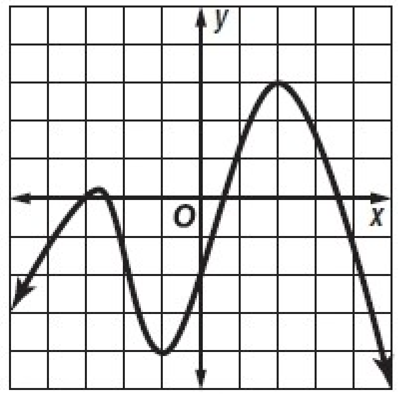
28. an even function with zeros at –2, 1, 3, and 5

ANSWER:



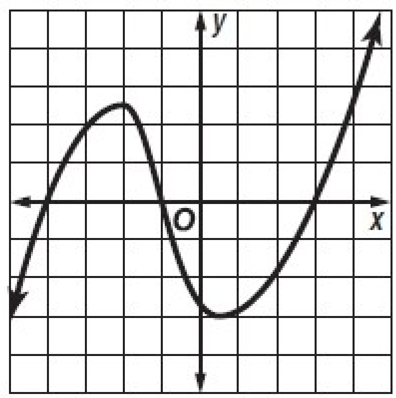
29. a 4-degree function with a zero at –3, maximum at $x = 2$, and minimum at $x = -1$

ANSWER:



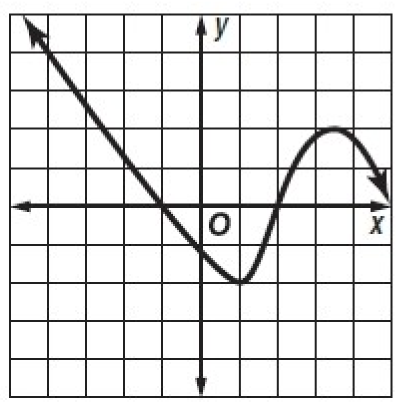
30. a 5-degree function with zeros at –4, –1, and 3, maximum at $x = -2$

ANSWER:



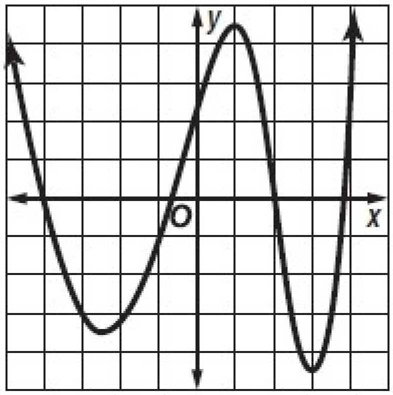
31. an odd function with zeros at –1, 2, and 5 and a negative leading coefficient

ANSWER:

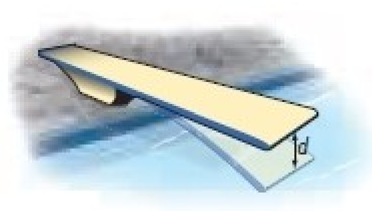


32. an even function with a minimum at $x = 3$ and a positive leading coefficient

ANSWER:



33. **DIVING** The deflection d of a 10-foot-long d diving board can be calculated using the function $d(x) = 0.015x^2 - 0.0005x^3$, where x is the distance between the diver and the stationary end of the board in feet.



a. Make a table of values of the function for $0 \le x \le 10$.

b. Graph the function.

c. What does the end behavior of the graph suggest as x increases?

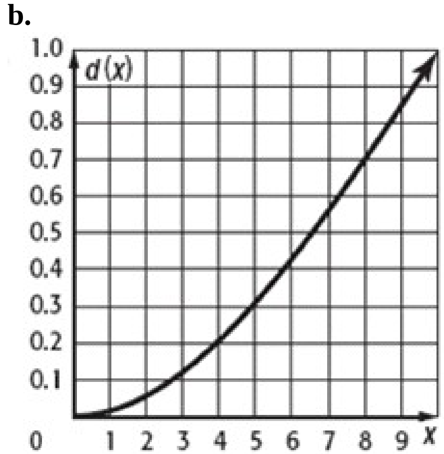
d. Will this trend continue indefinitely? Explain your reasoning.

ANSWER:

a.

x	$d(x)$
0	0
1	0.0145
2	0.056
3	0.1215
4	0.208
5	0.3125
6	0.432
7	0.5635
8	0.704
9	0.8505
10	1

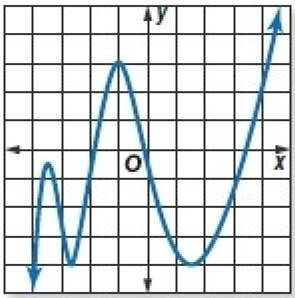
b.



c. $d(x)$ increases.

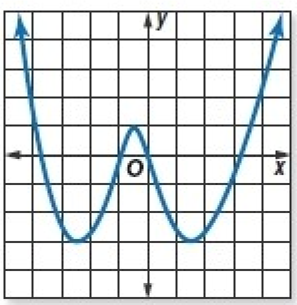
d. No, because x cannot be greater than 10.

Complete each of the following.
a. Estimate the *x*-coordinate of every turning point and determine if those coordinates are relative maxima or relative minima.
b. Estimate the *x*-coordinate of every zero.
c. Determine the smallest possible degree of the function.
d. Determine the domain and range of the function.

34. 

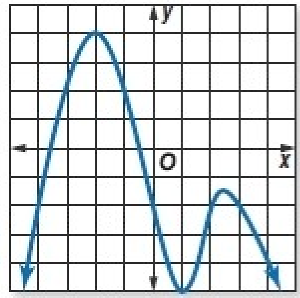
ANSWER:

a. –3.5(max), –2.5(min), –1(max), 2(min)
b. –1.75, –0.25, 3.5
c. 5
d. D: {all real numbers}; R: {all real numbers}

35. 

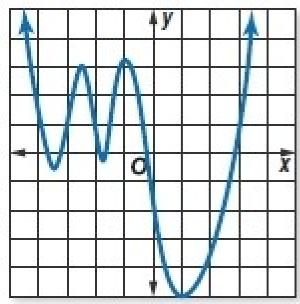
ANSWER:

a. –2.5(min), –0.5(max), 1.5(min)
b. –3.5, –1, 0, 3
c. 4
d. D: {all real numbers}; R: {y | $y \geq -3.1$}

36. 

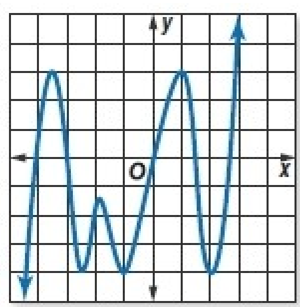
ANSWER:

a. –2(max), 1(min), 2.5(max)
b. –3.5, –0.5
c. 4
d. D: {all real numbers}; R: {y | $y \leq 4.1$}

37. 

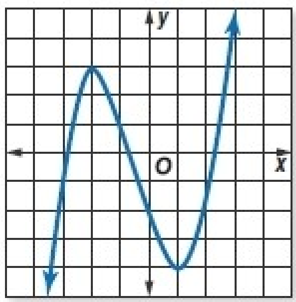
ANSWER:

a. –3.5(min), –2.5(max), –2(min), –1(max), 1(min)
b. –3.75, –3.25, –2, –1.75, –0.25, 2.9
c. 6
d. D: {all real numbers}; R: {y | $y \geq -5$}

38. 

ANSWER:

a. –3.5(max), –2.5(min), –1.75(max), –1(min), 1 (max), 2(min)
b. –4, –3, 0, 1.5, 2.75
c. 7
d. D: {all real numbers}; R: {all real numbers}

39. 

ANSWER:

a. –2(max), 1(min)
b. –3, –0.5, 2
c. 3
d. D: {all real numbers}; R: {all real numbers}

40. **CCSS REASONING** The number of subscribers using pagers in the United States can be modeled by

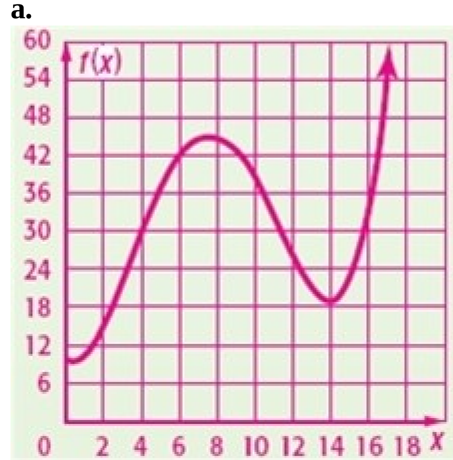
$$f(x) = 0.015x^4 - 0.44x^3 + 3.46x^2 - 2.7x + 9.68$$

where x is the number of years after 1990 and $f(x)$ is the number of subscribers in millions.

a. Graph the function.
b. Describe the end behavior of the graph.
c. What does the end behavior suggest about the number of pager subscribers?
d. Will this trend continue indefinitely? Explain your reasoning.

ANSWER:

a.



b. As x increases, $f(x)$ increases.
c. Sample answer: The graph suggests that the number of pager subscribers will increase dramatically and continue to increase.
d. Sample answer: The graph is unreasonable for $x \geq 15$ since pager use is currently decreasing rapidly and pagers have been replaced by more efficient products.

41. **PRICING** Jin's vending machines currently sell an average of 3500 beverages per week at a rate of $0.75 per can. She is considering increasing the price. Her weekly earnings can be represented by

$f(x) = -5x^2 + 100x + 2625$

where x is the number of $0.05 increases. Graph the function and determine the most profitable price for Jin.

ANSWER:

$1.25

For each function,
a. determine the zeros, x- and y-intercepts, and turning points,
b. determine the axis of symmetry, and
c. determine the intervals for which it is increasing, decreasing, or constant.

42. $f(x) = x^4 - 8x^2 + 16$

ANSWER:

a. zeros: $x = \pm 2$; x-intercepts: ± 2; y-intercept: 16; turning points: $x = -2, 0, 2$
b. $x = 0$
c. decreasing: $x < -2$ and $0 < x < 2$; increasing: $-2 < x < 0$ and $2 < x$

43. $f(x) = x^5 - 3x^3 + 2x - 4$

ANSWER:

a. zeros: $x \approx 1.75$; x-intercept: ≈ 1.75; y-intercept: –4; turning points: $x \approx -1.25, -0.5, 0.5, 1.25$
b. no axis of symmetry
c. decreasing: $-1.25 \le x \le -0.5$ and $0.5 \le x \le 1.25$;
increasing: $x \le -1.25$, $-0.5 \le x \le 0.5$, and $x \ge 1.25$

44. $f(x) = -2x^4 + 4x^3 - 5x$

ANSWER:

a. zeros: $x \approx -1$ and 0; x-intercept: ≈ -1 and 0; y-intercept: 0; turning point: $x \approx -0.5$
b. no axis of symmetry
c. decreasing: $x \ge -0.5$; increasing: $x \le -0.5$

45. $f(x) = \begin{cases} x^2 \text{ if } x \le -4 \\ 5 \text{ if } -4 < x \le 0 \\ x^3 \text{ if } x > 0 \end{cases}$

ANSWER:

a. no zeros, no x-intercepts, y-intercept: 5; no turning points
b. no axis of symmetry
c. decreasing: $x \le -4$; constant: $-4 < x \le 0$; increasing $x > 0$

46. **MULTIPLE REPRESENTATIONS** Consider the following function.

$f(x) = x^4 - 8.65x^3 + 27.34x^2 - 37.2285x + 18.27$

a. ANALYTICAL What are the degree, leading coefficient, and end behavior?
b. TABULAR Make a table of integer values $f(x)$ if $-4 \le x \le 4$ How many zeros does the function appear to have from the table?
c. GRAPHICAL Graph the function by using a graphing calculator.
d. GRAPHICAL Change the viewing window to [0, 4] scl: 1 by [–0.4, 0.4] scl: 0.2. What conclusions can you make from this new view of the graph?

ANSWER:

a. degree: 4; leading coefficient: 1; end behavior: as

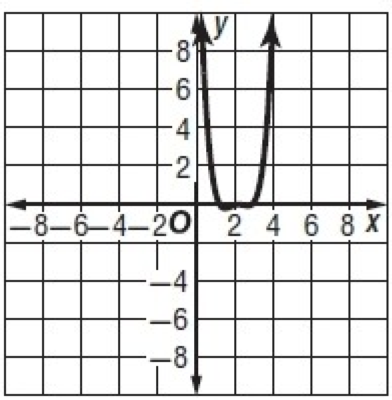
$x \to -\infty,\ f(x) \to +\infty,\ as\ x \to +\infty,\ f(x) \to +\infty,$

b.

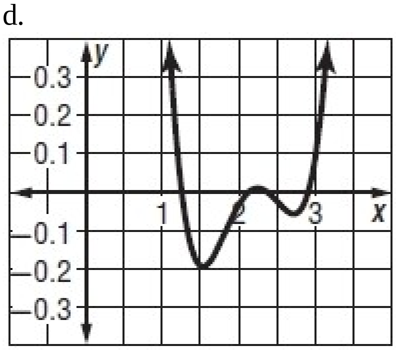
x	f(x)
–4	1414
–3	690
–2	287
–1	92
0	18
1	0.7
2	–0.03
3	0.09
4	9.2

2 zeros

c.



d.



Sample answer: Sometimes it is necessary to have a more accurate viewing window or to change the interval values of the table function in order to assess the graph more accurately.

47. **REASONING** Explain why the leading coefficient and the degree are the only determining factors in the end behavior of a polynomial function.

ANSWER:

As the *x*-values approach large positive or negative numbers, the term with the largest degree becomes more and more dominant in determining the value of *f*(*x*).

48. **REASONING** The table below shows the values of *g*(*x*), a cubic function. Could there be a zero between *x* = 2 and *x* = 3? Explain your reasoning.

x	–2	–1	0	1	2	3
g(x)	4	–2	–1	1	–2	–2

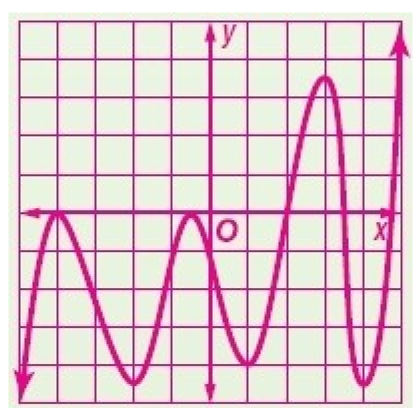
ANSWER:

Sample answer: No; the cubic function is of degree 3 and cannot have any more than three zeros. Those zeros are located between –2 and –1, 0 and 1, and 1 and 2.

49. **OPEN ENDED** Sketch the graph of an odd polynomial function with 6 turning points and 2 double roots.

ANSWER:

Sample answer:



50. **CCSS ARGUMENTS** Determine whether the following statement is sometimes, always, or never true. Explain your reasoning.
For any continuous polynomial function, the y-coordinate of a turning point is also either a relative maximum or relative minimum.

ANSWER:
Sample answer: Always; the definition of a turning point of a graph is a point in which the graph stops increasing and begins to decrease, causing a maximum, or stops decreasing and begins to increase, causing a minimum.

51. **REASONING** A function is said to be even if for every x in the domain of f, $f(x) = f(-x)$. Is every even-degree polynomial function also an even function? Explain.

ANSWER:
Sample answer: No; $f(x) = x^2 + x$ is an even degree, but $f(1) \neq f(-1)$.

52. **REASONING** A function is said to be odd if for every x in the domain, $-f(x) = f(-x)$. Is every odd-degree polynomial function also an odd function? Explain.

ANSWER:
Sample answer: No; $f(x) = x^3 + 2x^2$ is an odd degree, but $-f(1) \neq f(-1)$.

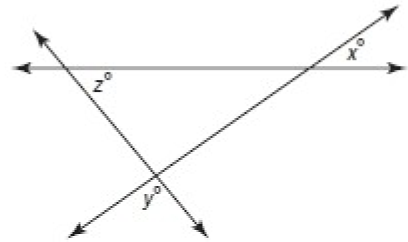
53. **WRITING IN MATH** How can you use the characteristics of a polynomial function to sketch its graph?

ANSWER:
Sample answer: From the degree, you can determine whether the graph is even or odd and the maximum number of zeros and turning points for the graph. You can create a table of values to help you find the approximate locations of turning points and zeros. The leading coefficient can be used to determine the end behavior of the graph, and, along with the degree, build the shape of the graph.

54. Which of the following is the factorization of $2x - 15 + x^2$?
A. $(x - 3)(x - 5)$
B. $(x - 3)(x + 5)$
C. $(x + 3)(x - 5)$
D. $(x + 3)(x + 5)$

ANSWER:
B

55. **SHORT RESPONSE** In the figure below, if $x = 35$ and $z = 50$, what is the value of y?

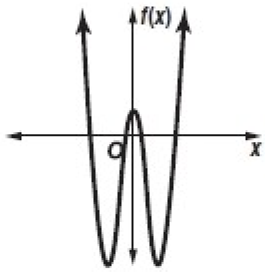


ANSWER:
95

56. Which polynomial represents $(4x^2 + 5x - 3)(2x - 7)$?
F $8x^3 - 18x^2 - 41x - 21$
G $8x^3 + 18x^2 + 29x - 21$
H $8x^3 - 18x^2 - 41x + 21$
J $8x^3 + 18x^2 - 29x + 21$

ANSWER:
H

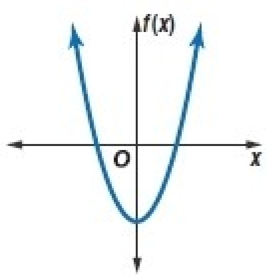
57. **SAT/ACT** The figure shows the graph of a polynomial function $f(x)$. Which of the following could be the degree of $f(x)$?



A 2
B 3
C 4
D 5
E 6

ANSWER:
C

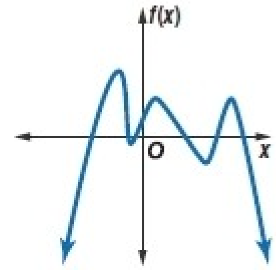
For each graph,
a. describe the end behavior,
b. determine whether it represents an odd-degree or an even-degree function, and
c. state the number of real zeros.



58.

ANSWER:

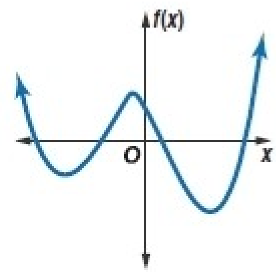
$f(x) \to +\infty$ as $x \to -\infty$. $f(x) \to +\infty$ as $x \to +\infty$.
Since the end behavior is in the same direction, it is an even-degree function. The graph intersects the x-axis at two points, so there are two real zeros.



59.

ANSWER:

$f(x) \to -\infty$ as $x \to -\infty$. $f(x) \to -\infty$ as $x \to +\infty$.
Since the end behavior is in the same direction, it is an even-degree function. The graph intersects the x-axis at six points, so there are six real zeros.



60.

ANSWER:

$f(x) \to +\infty$ as $x \to -\infty$. $f(x) \to +\infty$ as $x \to +\infty$.
Since the end behavior is in the same direction, it is an even-degree function. The graph intersects the x-axis at four points, so there are four real zeros.

Simplify.

61. $(x^3 + 2x^2 - 5x - 6) \div (x + 1)$

ANSWER:
$(x - 2)(x + 3)$

62. $(4y^3 + 18y^2 + 5y - 12) \div (y + 4)$

ANSWER:
$4y^2 + 2y - 3$

63. $(2a^3 - a^2 - 4a) \div (a - 1)$

ANSWER:
$2a^2 + a - 3 - \frac{3}{a-1}$

64. **CHEMISTRY** Tanisha needs 200 milliliters of a 48% concentration acid solution. She has 60% and 40% concentration solutions in her lab. How many milliliters of 40% acid solution should be mixed with 60% acid solution to make the required amount of 48% acid solution?

ANSWER:

80 mL of the 60% solution and 120 mL of the 40% solution

Factor.

65. $x^2 + 6x + 3x + 18$

ANSWER:

$(x + 6)(x + 3)$

66. $y^2 – 5y – 8y + 40$

ANSWER:

$(y – 5)(y – 8)$

67. $a^2 + 6a – 16$

ANSWER:

$(a + 8)(a – 2)$

68. $b^2 – 4b – 21$

ANSWER:

$(b – 7)(b + 3)$

69. $6x^2 – 5x – 4$

ANSWER:

$(3x – 4)(2x + 1)$

70. $4x^2 – 7x – 15$

ANSWER:

$(4x + 5)(x – 3)$