5-6 The Remainder and Factor Theorems

Use synthetic substitution to find $f(4)$ and $f(-2)$ for each function.

1. $f(x) = 2x^3 - 5x^2 - x + 14$

ANSWER:
58; –20

2. $f(x) = x^4 + 8x^3 + x^2 - 4x - 10$

ANSWER:
758; –46

3. **NATURE** The approximate number of bald eagle nesting pairs in the United States can be modeled by the function $P(x) = -0.16x^3 + 15.83x^2 - 154.15x + 1147.97$, where x is the number of years since 1970. About how many nesting pairs of bald eagles can be expected in 2018?

ANSWER:
12,526

Given a polynomial and one of its factors, find the remaining factors of the polynomial.

4. $x^3 - 6x^2 + 11x - 6$; $x - 1$

ANSWER:
$x - 2$, $x - 3$

5. $x^3 + x^2 - 16x - 16$; $x + 1$

ANSWER:
$x + 4$, $x - 4$

6. $3x^3 + 10x^2 - x - 12$; $x - 1$

ANSWER:
$x + 3$, $3x + 4$

7. $2x^3 - 5x^2 - 28x + 15$; $x + 3$

ANSWER:
$x - 5$, $2x - 1$

Use synthetic substitution to find $f(-5)$ and $f(2)$ for each function.

8. $f(x) = x^3 + 2x^2 - 3x + 1$

ANSWER:
–59; 11

9. $f(x) = x^2 - 8x + 6$

ANSWER:
71; –6

10. $f(x) = 3x^4 + x^3 - 2x^2 + x + 12$

ANSWER:
1707; 62

11. $f(x) = 2x^3 - 8x^2 - 2x + 5$

ANSWER:
–435; –15

12. $f(x) = x^3 - 5x + 2$

ANSWER:
–98; 0

13. $f(x) = x^5 + 8x^3 + 2x - 15$

ANSWER:
–4150; 85

14. $f(x) = x^6 - 4x^4 + 3x^2 - 10$

ANSWER:
13,190; 2

15. $f(x) = x^4 - 6x - 8$

ANSWER:
647; –4

16. **FINANCIAL LITERACY** A specific car's fuel economy in miles per gallon can be approximated by $f(x) = 0.00000056x^4 - 0.000018x^3 - 0.016x^2 + 1.38x - 0.38$, where x represents the car's speed in miles per hour. Determine the fuel economy when the car is travelling 40, 50 and 60 miles per hour.

ANSWER:
29.5 mpg; 29.87 mpg; 28.19 mpg

Given a polynomial and one of its factors, find the remaining factors of the polynomial.

17. $x^3 - 3x + 2$; $x + 2$

ANSWER:
$(x-1)^2$

18. $x^4 + 2x^3 - 8x - 16$; $x + 2$

ANSWER:

$x - 2$, $x^2 + 2x + 4$

19. $x^3 - x^2 - 10x - 8$; $x + 2$

ANSWER:

$x - 4$, $x + 1$

20. $x^3 - x^2 - 5x - 3$; $x - 3$

ANSWER:

$(x+1)^2$

21. $2x^3 + 17x^2 + 23x - 42$; $x - 1$

ANSWER:

$x + 6$, $2x + 7$

22. $2x^3 + 7x^2 - 53x - 28$; $x - 4$

ANSWER:

$x + 7$, $2x + 1$

23. $x^4 + 2x^3 + 2x^2 - 2x - 3$; $x - 1$

ANSWER:

$x + 1$, $x^2 + 2x + 3$

24. $x^3 + 2x^2 - x - 2$; $x + 2$

ANSWER:

$x - 1$, $x + 1$

25. $6x^3 - 25x^2 + 2x + 8$; $2x + 1$

ANSWER:

$x - 4$, $3x - 2$

26. $16x^5 - 32x^4 - 81x + 162$; $2x - 3$

ANSWER:

$x - 2$, $2x + 3$, $4x^2 + 9$

27. **BOATING** A motor boat travelling against waves accelerates from a resting position. Suppose the speed of the boat in feet per second is given by the function $f(t) = -0.04t^4 + 0.8t^3 + 0.5t^2 - t$, where t is the time in seconds.

a. Find the speed of the boat at 1, 2, and 3 seconds.

b. It takes 6 seconds for the boat to travel between two buoys while it is accelerating. Use synthetic substitution to find $f(6)$ and explain what this means.

ANSWER:

a. 0.26 ft/s, 5.76 ft/s, 19.86 ft/s

b. 132.96 ft/s; This means the boat is travelling at 132.96 ft/s when it passes the second buoy.

28. **CCSS REASONING** A company's sales, in millions of dollars, of consumer electronics can be modeled by $S(x) = -1.2x^3 + 18x^2 + 26.4x + 678$, where x is the number of years since 2005.

a. Use synthetic substitution to estimate the sales for 2017 and 2020.

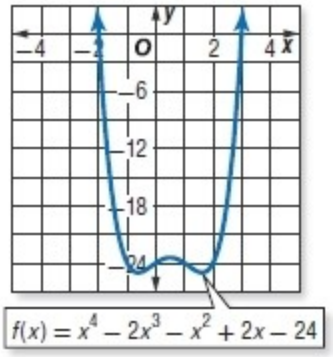
b. Do you think this model is useful in estimating future sales? Explain.

ANSWER:

a. \$1513.2 million; \$1074 million

b. Sample answer: The graph of the function has a relative maximum at about $x = 11$ or the year 2016 and then the values rapidly decrease. This model could be useful for the next 15 years. After that, is is unlikely that the sales would decrease as rapidly as indicated.

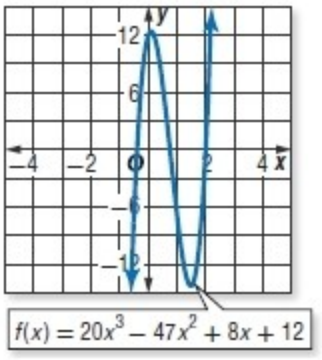
Use the graphs to find all of the factors for each polynomial function.

29. 

ANSWER:

$x + 2$, $x - 3$, $x^2 - x + 4$

30.



ANSWER:

$x - 2, 4x - 3, 5x + 2$

31. **MULTIPLE REPRESENTATIONS** In this problem, you will consider the function $f(x) = -9x^5 + 104x^4 - 249x^3 - 456x^2 + 828x + 432$.

a. ALGEBRAIC If $x - 6$ is a factor of the function, find the depressed polynomial.

b. TABULAR Make a table of values for $-5 \le x \le 6$ for the depressed polynomial.

c. ANALYTICAL What conclusions can you make about the locations of the other zeros based on the table? Explain your reasoning.

d. GRAPHICAL Graph the original function to confirm your conclusions.

ANSWER:

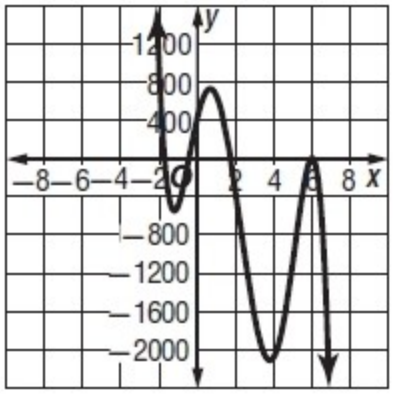
a. $g(x) = -9x^4 + 50x^3 + 51x^2 - 150x - 72$

b.

x	g(x)
–5	–9922
–4	–4160
–3	–1242
–2	–112
–1	70
0	–72
1	–130
2	88
3	558
4	1040
5	1078
6	0

c. There is a zero between $x = -2$ and $x = -1$ because $g(x)$ changes sign between the two values. There are also zeros between $x = -1$ and 0 and between and $x = 1$ and $x = 2$ because $g(x)$ changes sign between the two values. There is also a zero at $x = 6$.

d.



CCSS Find values of *k* so that each remainder is 3.

32. $(x^2 - x + k) \div (x - 1)$

ANSWER:

3

33. $(x^2 + kx - 17) \div (x - 2)$

ANSWER:

8

34. $(x^2 + 5x + 7) \div (x + k)$

ANSWER:

1,4

35. $(x^3 + 4x^2 + x + k) \div (x + 2)$

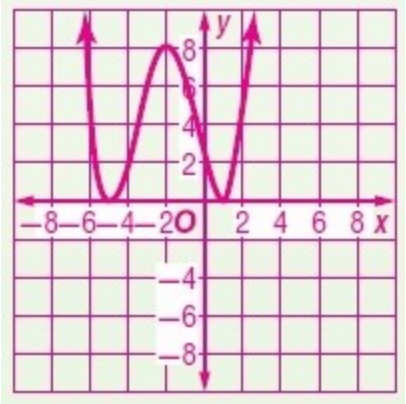
ANSWER:

–3

36. **OPEN ENDED** Write a polynomial function that has a double zero of 1 and a double zero of –5. Graph the function.

ANSWER:

Sample answer: $f(x) = 0.1(x - 1)^2(x + 5)^2$



CHALLENGE Find the solutions of each polynomial equation.

37. $(x^2 - 4)^2 - (x^2 - 4) - 2 = 0$

ANSWER:

$\pm\sqrt{6}, \pm\sqrt{3}$

38. $(x^2 + 3)^2 - 7(x^2 + 3) + 12 = 0$

ANSWER:

–1, 0, 1

39. **REASONING** Polynomial $f(x)$ is divided by $x - c$. What can you conclude if:

a. the remainder is 0?

b. the remainder is 1?

c. the quotient is 1, and the remainder is 0?

ANSWER:

a. $x - c$ is a factor of $f(x)$.

b. $x - c$ is not a factor of $f(x)$.

c. $f(x) = x - c$

40. **CHALLENGE** Review the definition for the Factor Theorem. Provide a proof of the theorem.

ANSWER:

If $x - a$ is a factor of $f(x)$, then $f(a)$ has a factor of $(a - a)$ or 0. Since a factor of $f(a)$ is 0, $f(a) = 0$. Now assume that $f(a) = 0$. If $f(a) = 0$, then the Remainder Theorem states that the remainder is 0 when $f(x)$ is divided by $x - a$. This means that $x - a$ is a factor of $f(x)$. This proves the Factor Theorem.

41. **OPEN ENDED** Write a cubic function that has a remainder of 8 for $f(2)$ and a remainder of –5 for $f(3)$.

ANSWER:

Sample answer: $f(x) = -x^3 + x^2 + x + 10$

42. **CHALLENGE** Show that the quadratic function $ax^4 + bx^3 + cx^2 + dx + e = 0$ will always have a rational root when the numbers 1, –2, 3, 4, and –6 are randomly assigned to replace a through f, and all of the numbers are used.

ANSWER:

Sample answer: When $x = 1$, $f(1)$ is the sum of all of the coefficients and constants in $f(x)$, in this case, a, b, c, d, and e. The sum of a, b, c, d, and e is 0, so however the coefficients are arranged, $f(1)$ will always equal 0, and $f(x)$ will have a rational root.

43. **WRITING IN MATH** Explain how the zeros of a function can be located by using the Remainder Theorem and making a table of values for different input values and then comparing the remainders.

ANSWER:

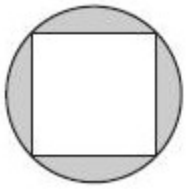
Sample answer: A zero can be located using the Remainder Theorem and a table of values by determining when the output, or remainder, is equal to zero. For instance, if $f(6)$ leaves a remainder of 2 and $f(7)$ leaves a remainder of –1, then you know that there is a zero between $x = 6$ and $x = 7$.

44. $27x^3 + y^3 =$

A $(3x + y)(3x + y)(3x + y)$
B $(3x + y)(9x^2 - 3xy + y^2)$
C $(3x - y)(9x^2 + 3xy + y^2)$
D $(3x - y)(9x^2 + 9xy + y^2)$

ANSWER:
B

45. **GRIDDED RESPONSE** In the figure, a square with side length $2\sqrt{2}$ is inscribed in a circle. The area of the circle is $k\pi$. What is the exact value of k?



ANSWER:
4

46. What is the product of the complex numbers $(4 + i)(4 - i)$?

F 15
G $16 - i$
H 17
J $17 - 8i$

ANSWER:
H

47. **SAT/ACT** The measure of the largest angle of a triangle is 14 less than twice the measure of the smallest angle. The third angle measure is 2 more than the measure of the smallest angle. What is the measure of the smallest angle?

A 46
B 48
C 50
D 52
E 82

ANSWER:
B

Solve each equation.

48. $x^4 - 4x^2 - 21 = 0$

ANSWER:
$\pm\sqrt{7}, \pm i\sqrt{3}$

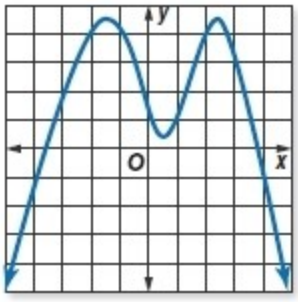
49. $x^4 - 6x^2 = 27$

ANSWER:
$\pm 3, \pm i\sqrt{3}$

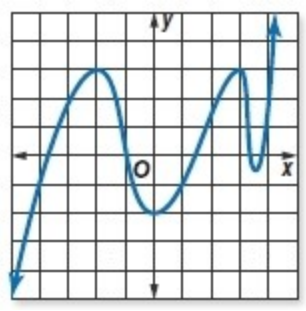
50. $4x^4 - 8x^2 - 96 = 0$

ANSWER:
$\pm\sqrt{6}, \pm 2i$

Complete each of the following.
a. Estimate the *x* coordinate of every turning point and determine if those coordinates are relative maxima or relative minima.
b. Estimate the *x*-coordinate of every zero.
c. Determine the smallest possible degree of the function.
d. Determine the domain and range of the function.

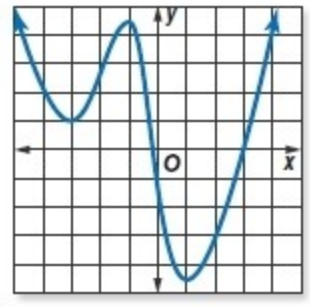
51. 

ANSWER:
a. –1.5(max), 0.5(min), 2.5(max)
b. –3.5, 3.75
c. 4
d. D: {all real numbers}; R: $\{y \mid y \leq 4.5\}$

52. 

ANSWER:

a. –2(max), 0(min), 3(max), 3.5(min)
b. –3.75, –1, 1.5, 3.25, 3.75
c. 5
d. D: {all real numbers}; R: {all real numbers}

53. 

ANSWER:

a. –3(min), –1(max), 1(min)
b. –0.25, 3
c. 4
d. D: {all real numbers}; R: $\{y \mid y \geq 4.5\}$

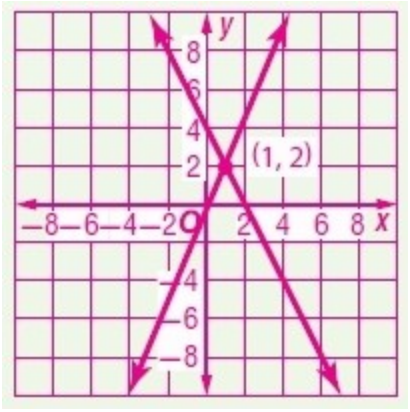
54. **HIGHWAY SAFETY** Engineers can use the formula $d = 0.05v^2 + 1.1v$ to estimate the minimum stopping distance d in feet for a vehicle traveling v miles per hour. If a car is able to stop after 125 feet, what is the fastest it could have been traveling when the driver first applied the brakes?

ANSWER:
about 40.2 mph

Solve by graphing.

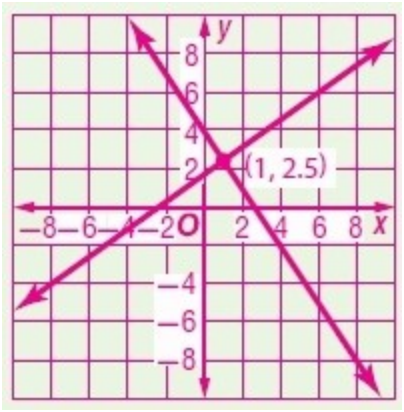
55. $y = 3x - 1$
$y = -2x + 4$

ANSWER:



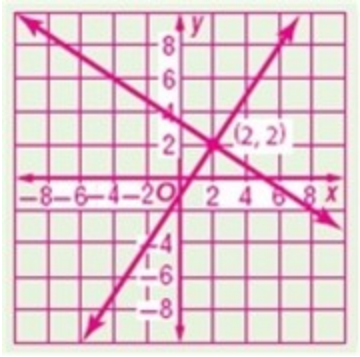
56. $3x + 2y = 8$
$-4x + 6y = 11$

ANSWER:



57. $5x - 2y = 6$
$3x - 2y = 2$

ANSWER:



If $c(x) = x^2 - 2x$ and $d(x) = 3x^2 - 6x + 4$, find each value.

58. $c(a + 2) - d(a - 4)$

ANSWER:

$-2a^2 + 32a - 76$

59. $c(a - 3) + d(a + 1)$

ANSWER:

$4a^2 - 8a + 16$

60. $c(-3a) + d(a + 4)$

ANSWER:

$12a^2 + 24a + 28$

61. $3d(3a) - 2c(-a)$

ANSWER:

$79a^2 - 58a + 12$

62. $c(a) + 5d(2a)$

ANSWER:

$61a^2 - 62a + 20$

63. $-2d(2a + 3) - 4c(a^2 + 1)$

ANSWER:

$-4a^4 - 24a^2 - 48a - 22$