5-7 Roots and Zeros

Solve each equation. State the number and type of roots.

1. $x^2 - 3x - 10 = 0$

ANSWER:
–2, 5; 2 real

2. $x^3 + 12x^2 + 32x = 0$

ANSWER:
–8, –4, 0; 3 real

3. $16x^4 - 81 = 0$

ANSWER:
$-\frac{3}{2}, \frac{3}{2}, -\frac{3}{2}i, \frac{3}{2}i;$
2 real, 2 imaginary

4. $0 = x^3 - 8$

ANSWER:
$2, -1 + i\sqrt{3}, -1 - i\sqrt{3};$
1 real, 2 imaginary

State the possible number of positive real zeros, negative real zeros, and imaginary zeros of each function.

5. $f(x) = x^3 - 2x^2 + 2x - 6$

ANSWER:
3 or 1; 0; 0 or 2

6. $f(x) = 6x^4 + 4x^3 - x^2 - 5x - 7$

ANSWER:
1; 1 or 3; 0 or 2

7. $f(x) = 3x^5 - 8x^3 + 2x - 4$

ANSWER:
1 or 3; 0 or 2; 0, 2, or 4

8. $f(x) = -2x^4 - 3x^3 - 2x - 5$

ANSWER:
0; 0 or 2; 2 or 4

Find all zeros of each function.

9. $f(x) = x^3 + 9x^2 + 6x - 16$

ANSWER:
–8, –2,1

10. $f(x) = x^3 + 7x^2 + 4x + 28$

ANSWER:
$-7, -2\boldsymbol{i}, 2\boldsymbol{i}$

11. $f(x) = x^4 - 2x^3 - 8x^2 - 32x - 384$

ANSWER:
$-4, 6, -4\boldsymbol{i}, 4\boldsymbol{i}$

12. $f(x) = x^4 - 6x^3 + 9x^2 + 6x - 10$

ANSWER:
$-1, 1, 3 - \boldsymbol{i}, 3 + \boldsymbol{i}$

Write a polynomial function of least degree with integral coefficients that have the given zeros.

13. 4, –1, 6

ANSWER:
$x^3 - 9x^2 + 14x + 24$

14. 3, –1, 1, 2

ANSWER:
$x^4 - 5x^3 + 5x^2 + 5x - 6$

15. $-2, 5, -3\boldsymbol{i}$

ANSWER:
$x^4 - 3x^3 - x^2 - 27x - 90$

16. $-4, 4 + \boldsymbol{i}$

ANSWER:
$x^3 - 4x^2 - 15x + 68$

Solve each equation. State the number and type of roots.

17. $2x^2 + x - 6 = 0$

ANSWER:
$-2, \frac{3}{2}$; 2 real

18. $4x^2 + 1 = 0$

ANSWER:

$-\frac{1}{2}i, \frac{1}{2}i$; 2 imaginary

19. $x^3 + 1 = 0$

ANSWER:

$-1, \frac{1 \pm i\sqrt{3}}{2}$; 1 real, 2 imaginary

20. $2x^2 - 5x + 14 = 0$

ANSWER:

$\frac{5 \pm i\sqrt{87}}{4}$; 2 imaginary

21. $-3x^2 - 5x + 8 = 0$

ANSWER:

$-\frac{8}{3}, 1$; 2 real

22. $8x^3 - 27 = 0$

ANSWER:

$\frac{3}{2}, \frac{-3 \pm 3i\sqrt{3}}{4}$; 1 real, 2 imaginary

23. $16x^4 - 625 = 0$

ANSWER:

$-\frac{5}{2}, \frac{5}{2}, -\frac{5}{2}i, \frac{5}{2}i$; 2 real, 2 imaginary

24. $x^3 - 6x^2 + 7x = 0$

ANSWER:

$0, 3 + \sqrt{2}, 3 - \sqrt{2}$; 3 real

25. $x^5 - 8x^3 + 16x = 0$

ANSWER:

–2, –2, 0, 2, 2; 5 real

26. $x^5 + 2x^3 + x = 0$

ANSWER:

$0, -i, -i, i, i$; 1 real, 4 imaginary

State the possible number of positive real zeros, negative real zeros, and imaginary zeros of each function.

27. $f(x) = x^4 - 5x^3 + 2x^2 + 5x + 7$

ANSWER:

0 or 2; 0 or 2; 0, 2, or 4

28. $f(x) = 2x^3 - 7x^2 - 2x + 12$

ANSWER:

0 or 2; 1; 0 or 2

29. $f(x) = -3x^5 + 5x^4 + 4x^2 - 8$

ANSWER:

0 or 2; 1; 2 or 4

30. $f(x) = x^4 - 2x^2 - 5x + 19$

ANSWER:

0 or 2; 0 or 2; 0, 2, or 4

31. $f(x) = 4x^6 - 5x^4 - x^2 + 24$

ANSWER:

0 or 2; 0 or 2; 2, 4, or 6

32. $f(x) = -x^5 + 14x^3 + 18x - 36$

ANSWER:

0 or 2; 1; 2 or 4

Find all zeros of each function.

33. $f(x) = x^3 + 7x^2 + 4x - 12$

ANSWER:

–6, –2, 1

34. $f(x) = x^3 + x^2 - 17x + 15$

ANSWER:

–5, 1, 3

35. $f(x) = x^4 - 3x^3 - 3x^2 - 75x - 700$

ANSWER:

$-4, 7, -5\boldsymbol{i}, 5\boldsymbol{i}$

36. $f(x) = x^4 + 6x^3 + 73x^2 + 384x + 576$

ANSWER:

$-3, -3, -8\boldsymbol{i}, 8\boldsymbol{i}$

37. $f(x) = x^4 - 8x^3 + 20x^2 - 32x + 64$

ANSWER:

$4, 4, -2\boldsymbol{i}, 2\boldsymbol{i}$

38. $f(x) = x^5 - 8x^3 - 9x$

ANSWER:

$-3, 0, 3, -\boldsymbol{i}, \boldsymbol{i}$

Write a polynomial function of least degree with integral coefficients that have the given zeros.

39. 5, –2, –1

ANSWER:

$x^3 - 2x^2 - 13x - 10$

40. –4, –3, 5

ANSWER:

$x^3 + 2x^2 - 23x - 60$

41. $-1, -1, 2\boldsymbol{i}$

ANSWER:

$x^4 + 2x^3 + 5x^2 + 8x + 4$

42. $-3, 1, -3\boldsymbol{i}$

ANSWER:

$x^4 + 2x^3 + 6x^2 + 18x - 27$

43. $0, -5, 3 + \boldsymbol{i}$

ANSWER:

$x^4 - x^3 - 20x^2 + 50x$

44. $-2, -3, 4 - 3\boldsymbol{i}$

ANSWER:

$x^4 - 3x^3 - 9x^2 + 77x + 150$

45. **CCSS REASONING** A computer manufacturer determines that for each employee, the profit for producing x computers per day is $P(x) = -0.006x^4 + 0.15x^3 - 0.05x^2 - 1.8x$.

a. How many positive real zeros, negative real zeros, and imaginary zeros exist?

b. What is the meaning of the zeros in this situation?

ANSWER:

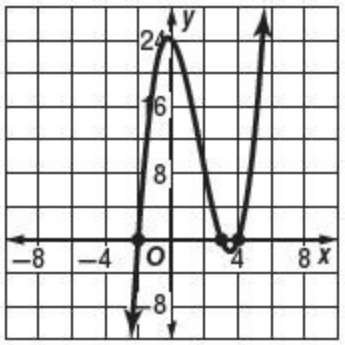
a. 2 or 0; 1; 1 or 3

b. Nonnegative roots represent numbers of computers produced per day which lead to no profit for the manufacturer.

Sketch the graph of each function using its zeros.

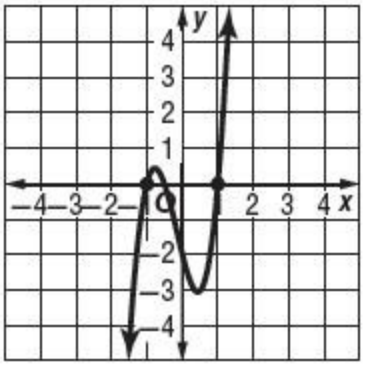
46. $f(x) = x^3 - 5x^2 - 2x + 24$

ANSWER:



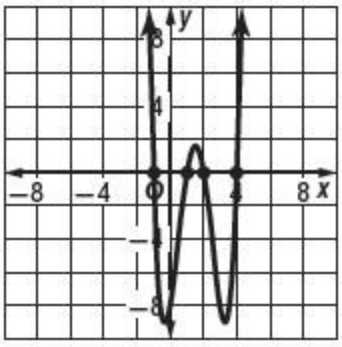
47. $f(x) = 4x^3 + 2x^2 - 4x - 2$

ANSWER:



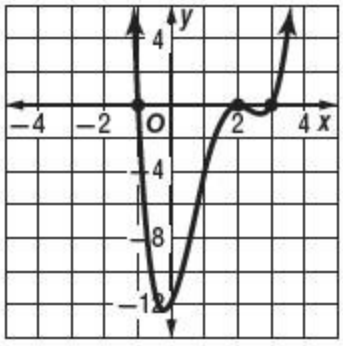
48. $f(x) = x^4 - 6x^3 + 7x^2 + 6x - 8$

ANSWER:

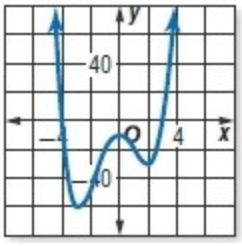


49. $f(x) = x^4 - 6x^3 + 9x^2 + 4x - 12$

ANSWER:

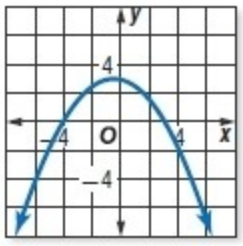


Match each graph to the given zeros.
a. –3, 4, *i*, –*i*
b. –4, 3
c. –4, 3, *i*, –*i*

50. 

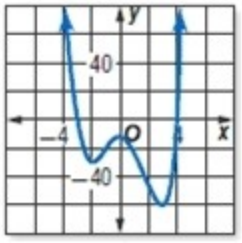
ANSWER:

c

51. 

ANSWER:

b

52. 

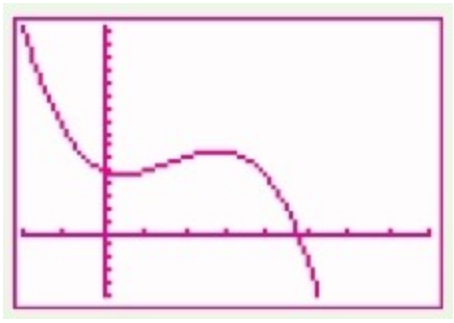
ANSWER:

a

53. **CONCERTS** The amount of money Hoshi's Music Hall took in from 2003 to 2010 can be modeled by $M(x) = -2.03x^3 + 50.1x^2 - 214x + 4020$, where x is the years since 2003.
a. How many positive real zeros, negative real zeros, and imaginary zeros exist?
b. Graph the function using your calculator.
c. Approximate all real zeros to the nearest tenth. What is the significance of each zero in the context of the situation?

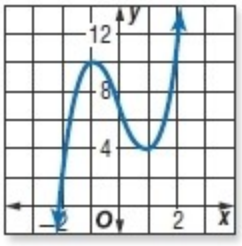
ANSWER:

a. 3 or 1; 0; 2 or 0
b.



[–10, 40] scl: 5 by [–4000, 13,200] scl: 100
c. 23.8; Sample answer: According to the model, the music hall will not earn any money after 2026.

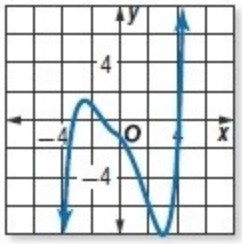
Determine the number of positive real zeros, negative real zeros, and imaginary zeros for each function. Explain your reasoning.

54. 

degree: 3

ANSWER:

0 positive, 1 negative, 2 imaginary;
Sample answer: The graph does not cross the positive *x*-axis, and crosses the negative *x*-axis once. Because the degree of the polynomial is 3, there are 3 – 1 or 2 imaginary zeros.

55. 

degree:5

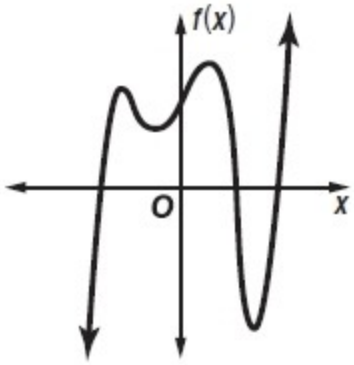
ANSWER:

1 positive, 2 negative, 2 imaginary;
Sample answer: The graph crosses the positive *x*-axis once, and crosses the negative *x*-axis twice. Because the degree of the polynomial is 5, there are 5 – 3 or 2 imaginary zeros.

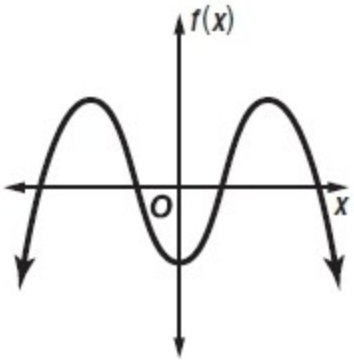
56. **OPEN ENDED** Sketch the graph of a polynomial function with:
a. 3 real, 2 imaginary zeros
b. 4 real zeros
c. 2 imaginary zeros

ANSWER:

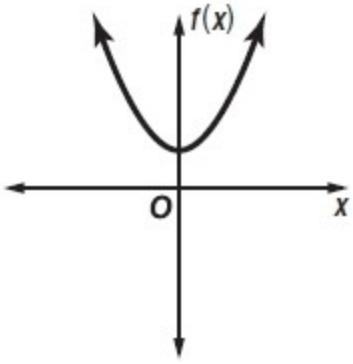
a.



b.



c.



57. **CHALLENGE** Write an equation in factored form of a polynomial function of degree 5 with 2 imaginary zeros, 1 non integral zero, and 2 irrational zeros. Explain.

ANSWER:

Sample answer:
$f(x) = (x + 2i)(x - 2i)(3x + 5)(x + \sqrt{5})(x - \sqrt{5})$ Use conjugates for the imaginary and irrational values.

58. **CCSS ARGUMENTS** Determine which equation is not like the others, Explain

$r^4 + 1 = 0$ $r^3 + 1 = 0$

$r^2 - 1 = 0$ $r^3 - 8 = 0$

ANSWER:

$r^4 + 1 = 0$; Sample answer: The equation has imaginary solutions and all of the others have real solutions.

59. **REASONING** Provide a counter example for each statement.

a. All polynomial functions of degree greater than 2 have at least 1 negative real root.

b. All polynomial functions of degree greater than 2 have at least 1 positive real root.

ANSWER:

a. Sample answer: $f(x) = x^4 + 4x^2 + 4$

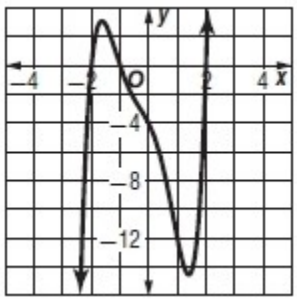
b. Sample answer: $f(x) = x^3 + 6x^2 + 9x$

60. **WRITING IN MATH** Explain to a friend how you would use Descartes' Rule of Signs to determine the number of possible positive real roots and the number of possible negative roots of the polynomial function $f(x) = x^4 - 2x^3 + 6x^2 + 5x - 12$.

ANSWER:

Sample answer: To determine the number of positive real roots, determine how many time the signs change in the polynomial as you move from left to right. In this function there are 3 changes in sign. Therefore, there may be 3 or 1 positive real roots. To determine the number of negative real roots, I would first evaluate the polynomial for $-x$. All of the terms with an odd-degree variable would change signs. Then I would again count the number of sign changes as I move from left to right. There would be only one change. Therefore there may be 1 negative root.

61. Use the graph of the polynomial function below. Which is not a factor of the polynomial $x^5 + x^4 - 3x^3 - 3x^2 - 4x - 4$?



A $x - 2$
B $x + 2$
C $x - 1$
D $x + 1$

ANSWER:

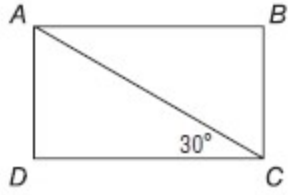
C

62. **SHORT RESPONSE** A window is in the shape of an equilateral triangle. Each side of the triangle is 8 feet long. The window is divided in half by a support from one vertex to the midpoint of the side of the triangle opposite the vertex. Approximately how long is the support?

ANSWER:

6.9 feet

63. **GEOMETRY** In rectangle $ABCD$, $\overline{AD}$ is 8 units long. What is the length of $\overline{AB}$?



F 4 units
G 8 units
H $8\sqrt{3}$ units
J 16 units

ANSWER:
H

64. **SAT/ACT** The total area of a rectangle is $25a^4 - 16b^2$ square units. Which factors could represent the length and width?
A $(5a^2 + 4b)$ units and $(5a^2 + 4b)$ units
B $(5a^2 + 4b)$ units and $(5a^2 - 4b)$ units
C $(5a^2 - 4b)$ units and $(5a^2 - 4b)$ units
D $(5a - 4b)$ units and $(5a - 4b)$ units
E $(5a + 4b)$ units and $(5a - 4b)$ units

ANSWER:
B

Use synthetic substitution to find $f(-8)$ and $f(4)$ for each function.

65. $f(x) = 4x^3 + 6x^2 - 3x + 2$

ANSWER:
$f(-8) = -1638; f(4) = 342$

66. $f(x) = 5x^4 - 2x^3 + 4x^2 - 6x$

ANSWER:
$f(-8) = 21{,}808; f(4) = 1192$

67. $f(x) = 2x^5 - 3x^3 + x^2 - 4$

ANSWER:
$f(-8) = -63{,}940; f(4) = 1868$

Factor completely. If the polynomial is not factorable, write prime.

68. $x^6 - y^6$

ANSWER:
$(x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2)$

69. $a^6 + b^6$

ANSWER:
$(a^2 + b^2)(a^4 - a^2b^2 + b^4)$

70. $4x^2y + 8xy + 16y - 3x^2z - 6xz - 12z$

ANSWER:
$(x^2 + 2x + 4)(4y - 3z)$

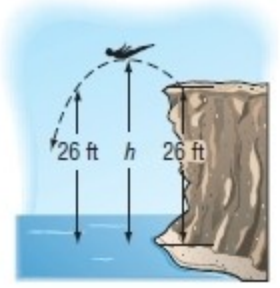
71. $5a^3 - 30a^2 + 40a + 2a^2b - 12ab + 16b$

ANSWER:
$(a - 4)(a - 2)(5a + 2b)$

72. **BUSINESS** A mall owner has determined that the relationship between monthly rent charged for store space r (in dollars per square foot) and monthly profit $P(r)$ (in thousands of dollars) can be approximated by $P(r) = -8.1r^2 + 46.9r - 38.2$. Solve each quadratic equation or inequality. Explain what each answer tells about the relationship between monthly rent and profit for this mall.
a. $-8.1r^2 + 46.9r - 38.2 = 0$
b. $-8.1r^2 + 46.9r - 38.2 > 0$
c. $-8.1r^2 + 46.9r - 38.2 > 10$
d. $-8.1r^2 + 46.9r - 38.2 < 10$

ANSWER:
a. 0.98, 4.81; The owner will break even if he charges \$0.98 or \$4.81 per square foot.
b. $0.98 < r < 4.81$; The owner will make a profit if the rent per square foot is between \$0.98 and \$4.81.
c. $1.34 < r < 4.45$; If rent is set between \$1.34 and \$4.45 per sq ft, the profit will be greater than \$10,000.
d. $r < 1.34$ or $r > 4.45$; If rent is set between \$0 and \$1.34 or above \$4.45 per sq ft, the profit will be less than \$10,000.

73. **DIVING** To avoid hitting any rocks below, a cliff diver jumps up and out. The equation $h = -16t^2 + 4t + 26$ describes her height h in feet t seconds after jumping. Find the time at which she returns to a height of 26 feet.



ANSWER:
0.25 s

Find all of the possible values of $\pm\frac{b}{a}$ for each replacement set.

74. $a = \{1, 2, 4\}$; $b = \{1, 2, 3, 6\}$

ANSWER:

$\pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$

75. $a = \{1, 5\}$; $b = \{1, 2, 4, 8\}$

ANSWER:

$\pm 1, \pm 2, \pm 4, \pm 8, \pm\frac{1}{5}, \pm\frac{2}{5}, \pm\frac{4}{5}, \pm\frac{8}{5}$

76. $a = \{1, 2, 3, 6\}$; $b = \{1, 7\}$

ANSWER:

$\pm 1, \pm 7, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}, \pm\frac{7}{2}, \pm\frac{7}{3}, \pm\frac{7}{6}$