5-8 Rational Zero Theorem

List all of the possible rational zeros of each function.

1. $f(x) = x^3 - 6x^2 - 8x + 24$

ANSWER:

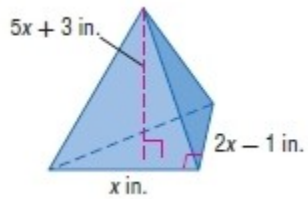
$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

2. $f(x) = 2x^4 + 3x^2 - x + 15$

ANSWER:

$\pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{15}{2}$

3. **CCSS REASONING** The volume of the triangular pyramid is 210 cubic inches. Find the dimensions of the solid.



ANSWER:

5 in. × 9 in. × 28 in.

Find all of the rational zeros of each function.

4. $f(x) = x^3 - 6x^2 - 13x + 42$

ANSWER:

$-3, 2, 7$

5. $f(x) = 2x^4 + 11x^3 + 26x^2 + 29x + 12$

ANSWER:

$-\frac{3}{2}, -1$

Find all of the zeros of each function.

6. $f(x) = 3x^3 - 2x^2 - 8x + 5$

ANSWER:

$\frac{5}{3}, \frac{-1 \pm \sqrt{5}}{2}$

7. $f(x) = 8x^3 + 14x^2 + 11x + 3$

ANSWER:

$-\frac{1}{2}, \frac{-5 \pm i\sqrt{23}}{8}$

8. $f(x) = 4x^4 + 13x^3 - 8x^2 + 13x - 12$

ANSWER:

$-4, \frac{3}{4}, -i, i$

9. $f(x) = 4x^4 - 12x^3 + 25x^2 - 14x - 15$

ANSWER:

$-\frac{1}{2}, \frac{3}{2}, 1 + 2i, 1 - 2i$

List all of the possible rational zeros of each function.

10. $f(x) = x^4 + 8x - 32$

ANSWER:

$\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32$

11. $f(x) = x^3 + x^2 - x - 56$

ANSWER:

$\pm 1, \pm 2, \pm 4, \pm 7, \pm 8, \pm 14, \pm 28, \pm 56$

12. $f(x) = 2x^3 + 5x^2 - 8x - 10$

ANSWER:

$\pm 1, \pm 2, \pm 5, \pm 10, \pm \frac{1}{2}, \pm \frac{5}{2}$

13. $f(x) = 3x^6 - 4x^4 - x^2 - 35$

ANSWER:

$\pm 1, \pm 5, \pm 7, \pm 35, \pm \frac{1}{3}, \pm \frac{5}{3}, \pm \frac{7}{3}, \pm \frac{35}{3}$

14. $f(x) = 6x^5 - x^4 + 2x^3 - 3x^2 + 2x - 18$

ANSWER:

$\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{9}{2}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{1}{6}$

15. $f(x) = 8x^4 - 4x^3 - 4x^2 + x + 42$

ANSWER:

$\pm 1, \pm 2, \pm 3, \pm 6, \pm 7, \pm 14, \pm 21, \pm 42, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{7}{2}, \pm \frac{21}{2}, \pm \frac{1}{4}, \pm \frac{3}{4}, \pm \frac{7}{4}, \pm \frac{21}{4}, \pm \frac{1}{8}, \pm \frac{3}{8}, \pm \frac{7}{8}, \pm \frac{21}{8}$

16. $f(x) = 15x^3 + 6x^2 + x + 90$

ANSWER:

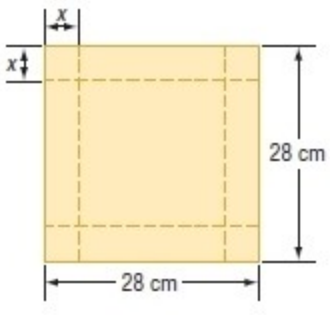
$\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 9, \pm 10, \pm 15, \pm 18, \pm 30, \pm 45, \pm 90, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{5}{3}, \pm \frac{10}{3}, \pm \frac{1}{5}, \pm \frac{2}{5}, \pm \frac{3}{5}, \pm \frac{6}{5}, \pm \frac{9}{5}, \pm \frac{18}{5}, \pm \frac{1}{15}, \pm \frac{2}{15}$

17. $f(x) = 16x^4 - 5x^2 + 128$

ANSWER:

$\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32, \pm 64, \pm 128, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm \frac{1}{8}, \pm \frac{1}{16}$

18. **MANUFACTURING** A box is to be constructed by cutting out equal squares from the corners of a square piece of cardboard and turning up the sides.



a. Write a function $V(x)$ for the volume of the box.
b. For what value of x will the volume of the box equal 1152 cubic centimetres?
c. What will be the volume of the box if $x = 6$ centimetres?

ANSWER:

a. $V(x) = (28 - 2x)(28 - 2x)x = 4x^3 - 112x^2 + 784x$
b. 2 or 8
c. 1536 cm^6

Find all of the rational zeros of each function.

19. $f(x) = x^3 + 10x^2 + 31x + 30$

ANSWER:

–5, –3, –2

20. $f(x) = x^3 - 2x^2 - 56x + 192$

ANSWER:

–8, 4, 6

21. $f(x) = 4x^3 - 3x^2 - 100x + 75$

ANSWER:

$-5, \frac{3}{4}, 5$

22. $f(x) = 4x^4 + 12x^3 - 5x^2 - 21x + 10$

ANSWER:

$-\frac{5}{2}, -2, \frac{1}{2}, 1$

23. $f(x) = x^4 + x^3 - 8x - 8$

ANSWER:

–1, 2

24. $f(x) = 2x^4 - 3x^3 - 24x^2 + 4x + 48$

ANSWER:

$-2, 4, \frac{3}{2}$

25. $f(x) = 4x^3 + x^2 + 16x + 4$

ANSWER:

$-\frac{1}{4}$

26. $f(x) = 81x^4 - 256$

ANSWER:

$-\frac{4}{3}, \frac{4}{3}$

Find all of the zeros of each function.

27. $f(x) = x^3 + 3x^2 - 25x + 21$

ANSWER:

$-7, 1, 3$

28. $f(x) = 6x^3 + 5x^2 - 9x + 2$

ANSWER:

$\frac{2}{3}, \frac{-3 \pm \sqrt{17}}{4}$

29. $f(x) = x^4 - x^3 - x^2 - x - 2$

ANSWER:

$2, -1, \boldsymbol{i}, -\boldsymbol{i}$

30. $f(x) = 10x^3 - 17x^2 - 7x + 2$

ANSWER:

$-\frac{1}{2}, \frac{1}{5}, 2$

31. $f(x) = x^4 - 3x^3 + x^2 - 3x$

ANSWER:

$0, 3, -\boldsymbol{i}, \boldsymbol{i}$

32. $f(x) = 6x^3 + 11x^2 - 3x - 2$

ANSWER:

$\frac{1}{2}, -\frac{1}{3}, -2$

33. $f(x) = 6x^4 + 22x^3 + 11x^2 - 38x - 40$

ANSWER:

$-2, \frac{4}{3}, \frac{-3 \pm i}{2}$

34. $f(x) = 2x^3 - 7x^2 - 8x + 28$

ANSWER:

$-2, 2, \frac{7}{2}$

35. $f(x) = 9x^5 - 94x^3 + 27x^2 + 40x - 12$

ANSWER:

$3, \frac{2}{3}, -\frac{2}{3}, \frac{-3 \pm \sqrt{13}}{2}$

36. $f(x) = x^5 - 2x^4 - 12x^3 - 12x^2 - 13x - 10$

ANSWER:

$-1, -2, 5, \boldsymbol{i}, -\boldsymbol{i}$

37. $f(x) = 48x^4 - 52x^3 + 13x - 3$

ANSWER:

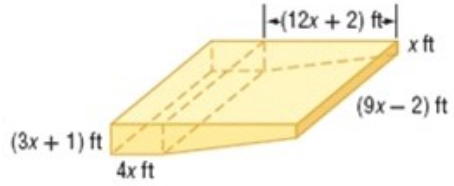
$-\frac{1}{2}, \frac{1}{3}, \frac{1}{2}, \frac{3}{4}$

38. $f(x) = 5x^4 - 29x^3 + 55x^2 - 28x$

ANSWER:

$\frac{4}{5}, 0, \frac{5 \pm i\sqrt{3}}{2}$

39. **SWIMMING POOLS** A diagram of the swimming pool at the Midtown Community Center is shown below. The pool can hold 9175 cubic feet of water.



a. Write a polynomial function that represents the volume of the swimming pool.

b. What are the possible values of x? Which of these values are reasonable?

ANSWER:

a. $V(x) = 324x^3 + 54x^2 - 19x - 2$

b. $1.05i, -4.22i, 3$; 3 is the only reasonable value for x. The other two values are imaginary.

40. **CCSS MODELING** A portion of the path of a certain roller coaster can be modeled by $f(t) = t^4 - 31t^3 + 308t^2 - 1100t + 1200$ where t represents the time in seconds and $f(t)$ represents when the height of the roller coaster is at a relative maximum. Use the Rational Zero Theorem to determine the four times at which the roller coaster is at ground level.

ANSWER:

2 s, 4 s, 10 s, and 15 s

41. **FOOD** A restaurant orders spaghetti sauce in cylindrical metal cans. The volume of each can is about 160π cubic inches, and the height of the can is 6 inches more than the radius.

a. Write a polynomial equation that represents the volume of a can. Use the formula for the volume of a cylinder, $v = \pi r^2 h$

b. What are the possible values of r? Which of these values are reasonable for this situation?

c. Find the dimensions of the can.

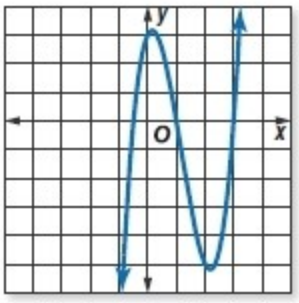
ANSWER:

a. $v = \pi r^3 + 6\pi r^2$

b. $4, -5 \pm i\sqrt{15}; 4$

c. $r = 4$ in., $h = 10$ in.

42. Refer to the graph.



a. Find all of the zeros of $f(x) = 2x^3 + 7x^2 + 2x - 3$ and $g(x) = 2x^3 - 7x^2 + 2x + 3$.

b. Determine which function, f or g, is shown in the graph at the right.

ANSWER:

a. $-1, \frac{1}{2}, -3; -\frac{1}{2}, 1, 3$

b. g

43. **MUSIC SALES** Refer to the beginning of the lesson.

a. Write a polynomial equation that could be used to determine the year in which music sales would be about $9,000,000,000.

b. List the possible whole number solutions for your equation in part **a**.

c. Determine the approximate year in which music sales will reach $9,000,000,000.

d. Does the model represent a realistic estimate for all future music sales? Explain your reasoning.

ANSWER:

a. $30t^3 - 478t^2 + 1758t + 1092 = 0$

b. 1, 2, 3, 4, 6, 7, 12, 13, 14, 21, 26, 28, 39, 42, 52, 78, 84, 91, 156, 182, 273, 364, 546, 1092

c. 2013

d. No; Sample answer: Music sales fluctuate from 2005 to 2015, then increase indefinitely. It is not reasonable to expect sales to increase forever.

Find all of the zeros of each function.

44. $f(x) = x^5 + 3x^4 - 19x^3 - 43x^2 + 18x + 40$

ANSWER:

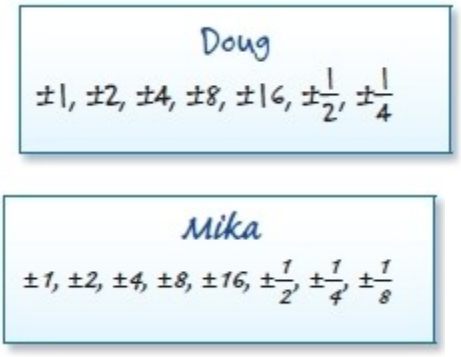
1, –1, –2, 4, –5

45. $f(x) = x^5 - x^4 - 23x^3 + 33x^2 + 126x - 216$

ANSWER:

2, 3, 3, –3, –4

46. **CCSS CRITIQUE** Doug and Mika are listing all of the possible rational zeros for $f(x) = 4x^4 + 8x^5 + 10x^2 + 3x + 16$. Is either of them correct? Explain your reasoning.



ANSWER:

Sample answer: Doug; the value of q is the leading coefficient, which is 4, not 8.

47. **CHALLENGE** Give a polynomial function that has zeros at and $5 + 2\boldsymbol{i}$.

ANSWER:

Sample answer: $f(x) = x^4 - 12x^3 + 47x^2 - 38x - 58$

48. **REASONING** Determine if the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning. *If all of the possible zeros of a polynomial function are integers, then the leading coefficient of the function is* 1 or -1.

ANSWER:

Sample answer: Always; in order for the possible zeros of a polynomial function to be integers, the value of q must be 1 or -1. Otherwise, the possible zeros could be a fraction. In order for q to be 1 or -1, the leading coefficient of the polynomial must also be 1 or -1.

49. **OPEN ENDED** Write a function that has possible zeros of $\pm 18, \pm 9, \pm 6, \pm 3, \pm 2, \pm 1, \pm\frac{9}{4}, \pm\frac{9}{2}, \pm\frac{3}{2}, \pm\frac{3}{4}, \pm\frac{1}{2}$ *and* $\pm\frac{1}{4}$.

ANSWER:

Sample answer: $f(x) = 4x^5 + 3x^3 + 8x + 18$

50. **CHALLENGE** The roots of $x^2 + bx + c = 0$ are M and N. If $|M - N| = 1$, express c in terms of b.

ANSWER:

$$c = \frac{b^2 - 1}{4}$$

51. **WRITING IN MATH** Explain the process of using the Rational Zero Theorem to determine the number of possible rational zeros of a function.

ANSWER:

Sample answer: For any polynomial function, the constant term represents p and the leading coefficient represents q. The possible zeros of the function can be found with $\pm\frac{p}{q}$ where the fraction is every combination of factors of p and q. For example, if p is 4 and q is 3, then $\pm 4, \pm 2, \pm 1, \pm\frac{4}{3}, \pm\frac{2}{3}, \pm\frac{1}{3}$, and $\pm\frac{1}{3}$ are all possible zeros.

52. **ALGEBRA** Which of the following is a zero of the function $f(x) = 12x^5 - 5x^3 + 2x - 9$?

A –6

B $-\frac{2}{3}$

C $\frac{3}{8}$

D 1

ANSWER:

D

53. **SAT/ACT** How many negative real zeros does $f(x) = x^5 - 2x^4 - 4x^3 + 4x^2 - 5x + 6$ have?

F 5

G 3

H 2

J 1

K 0

ANSWER:

J

54. **ALGEBRA** For all nonnegative numbers n, let $\boxed{n} = \frac{\sqrt{n}}{2}$. if $\boxed{n} = 4$, what is the value of n?

A 2
B 4
C 16
D 64

ANSWER:
D

55. **GRIDDED RESPONSE** What is the y-intercept of a line that contains the point (–1, 4) and has the same x-intercept as $x + 2y = –3$?

ANSWER:
6

Write a polynomial function of least degree with integral coefficients that has the given zeros.

56. 6, –3, $\sqrt{2}$

ANSWER:
$f(x) = x^4 – 3x^3 – 20x^2 + 6x + 36$

57. 5, –1, $4i$

ANSWER:
$f(x) = x^4 – 4x^3 + 11x^2 – 64x – 80$

58. –4, –2, $i\sqrt{2}$

ANSWER:
$f(x) = x^4 + 6x^3 + 10x^2 + 12x + 16$

Given a polynomial and one of its factors, find the remaining factors of the polynomial.

59. $x^4 + 5x^3 + 5x^2 – 5x – 6$; $x + 3$

ANSWER:
$(x – 1)(x + 2)(x + 1)$

60. $a^4 – 2a^3 – 17a^2 + 18a + 72$; $a – 3$

ANSWER:
$(a + 3)(a – 4)(a + 2)$

61. $x^4 + x^3 – 11x^2 + x – 12$; $x + i$

ANSWER:
$(x – 3)(x + 4)(x – i)$

62. **BRIDGES** The supporting cables of the Golden Gate Bridge approximate the shape of a parabola. The parabola can be modeled by the quadratic function $y = 0.00012x^2 + 6$, where x represents the distance from the axis of symmetry and y represents the height of the cables. The related quadratic equation is $0.00012x^2 + 6 = 0$.
a. Calculate the value of the discriminate.
b. What does the discriminate tell you about the supporting cables of the Golden Gate Bridge?

ANSWER:
a. –0.00288
b. Sample answer: This means that the cables do not touch the floor of the bridge, since the graph does not intersect the x-axis and the roots are imaginary.

63. **RIDES** An amusement park ride carries riders to the top of a 225-foot tower. The riders then free-fall in their seats until they reach 30 feet above the ground.
a. Use the formula $h(t) = –16t^2 + h_0$, where the time t is in seconds and the initial height h_0 is in feet, to find how long the riders are in free-fall.
b. Suppose the designer of the ride wants the riders to experience free-fall for 5 seconds before stopping 30 feet above the ground. What should be the height of the tower?

ANSWER:
a. about 3.5 s
b. 430 ft

Simplify.

64. $(x – 4)(x + 3)$

ANSWER:
$x^2 – x – 12$

65. $3x(x^2 + 4)$

ANSWER:
$3x^3 + 12x$

66. $x^2(x - 2)(x + 1)$

ANSWER:

$x^4 - x^3 - 2x^2$

Find each value if $f(x) = 6x + 2$ and $g(x) = -4x^2$.

67. $f(5)$

ANSWER:

32

68. $g(-3)$

ANSWER:

-36

69. $f(3c)$

ANSWER:

$18c + 2$