6-2 Inverse Functions and Relations

Find the inverse of each relation.

1. {(–9, 10), (1, –3), (8, –5)}

ANSWER:
{(10, –9), (–3, 1), (–5, 8)}

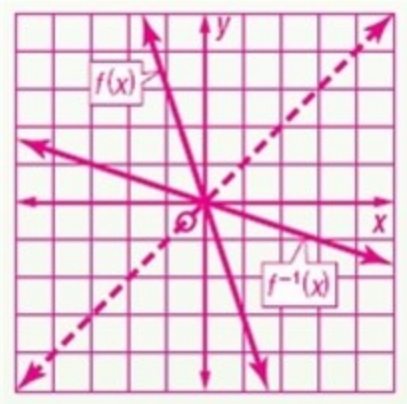
2. {(–2, 9), (4, –1), (–7, 9), (7, 0)}

ANSWER:
{(9, –2), (–1, 4), (9, –7), (0, 7)}

Find the inverse of each function. Then graph the function and its inverse.

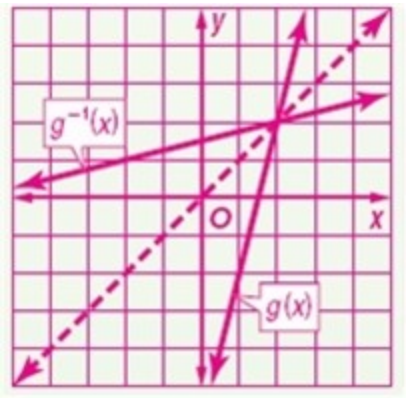
3. $f(x) = -3x$

ANSWER:
$f^{-1}(x) = -\frac{1}{3}x$



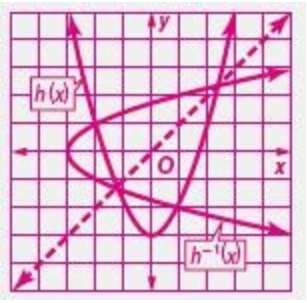
4. $g(x) = 4x - 6$

ANSWER:
$g^{-1}(x) = \frac{x+6}{4}$



5. $h(x) = x^2 - 3$

ANSWER:
$h^{-1} = \pm\sqrt{x+3}$



Determine whether each pair of functions are inverse functions. Write *yes* or *no*.

6. $f(x) = x - 7$
$g(x) = x + 7$

ANSWER:
Yes

6-2 Inverse Functions and Relations

7. $f(x) = \frac{1}{2}x + \frac{3}{4}$
$g(x) = 2x - \frac{4}{3}$

ANSWER:
No

8. $f(x) = 2x^3$
$g(x) = \frac{1}{3}\sqrt{x}$

ANSWER:
No

Find the inverse of each relation.

9. {(–8, 6), (6, –2), (7, –3)}

ANSWER:
{(6, –8), (–2, 6), (–3, 7)}

10. {(7, 7), (4, 9), (3, –7)}

ANSWER:
{(7, 7), (9, 4), (–7, 3)}

11. {(8, –1), (–8, –1), (–2, –8), (2, 8)}

ANSWER:
{(–1, 8), (–1, –8), (–8, –2), (8, 2)}

12. {(4, 3), (–4, –4), (–3, –5), (5, 2)}

ANSWER:
{(3, 4), (–4, –4), (–5, –3), (2, 5)}

13. {(1, –5), (2, 6), (3, –7), (4, 8), (5, –9)}

ANSWER:
{(–5, 1), (6, 2), (–7, 3), (8, 4), (–9, 5)}

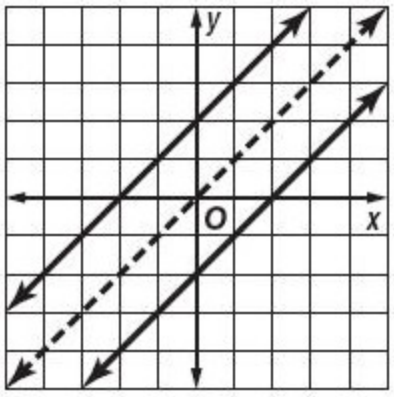
14. {(3, 0), (5, 4), (7, –8), (9, 12), (11, 16)}

ANSWER:
{(0, 3), (4, 5), (–8, 7), (12, 9), (16, 11)}

CCSS SENSE-MAKING Find the inverse of each function. Then graph the function and its inverse.

15. $f(x) = x + 2$

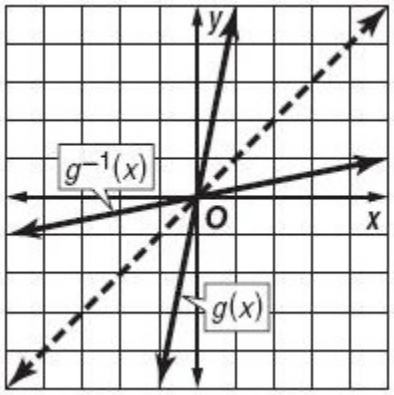
ANSWER:
$f^{-1}(x) = x - 2$



16. $g(x) = 5x$

ANSWER:

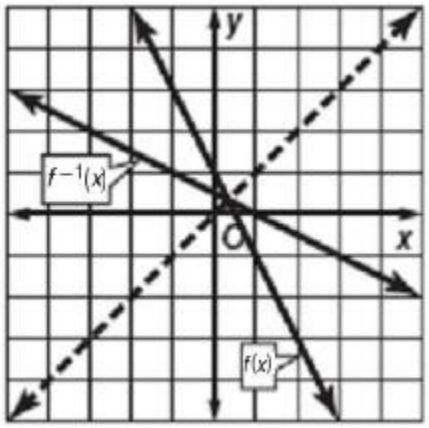
$g^{-1}(x) = \frac{1}{5}x$



17. $f(x) = -2x + 1$

ANSWER:

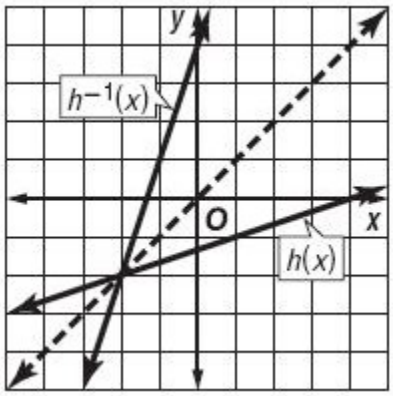
$f^{-1}(x) = -\frac{x}{2} + \frac{1}{2}$



18. $h(x) = \frac{x-4}{3}$

ANSWER:

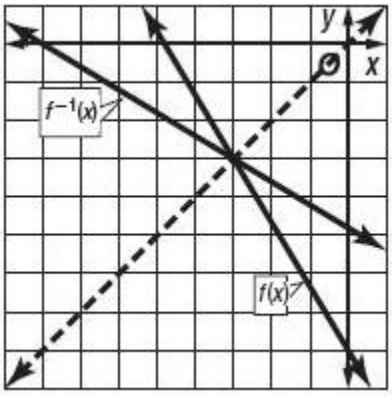
$h^{-1}(x) = 3x + 4$



19. $f(x) = -\frac{5}{3}x - 8$

ANSWER:

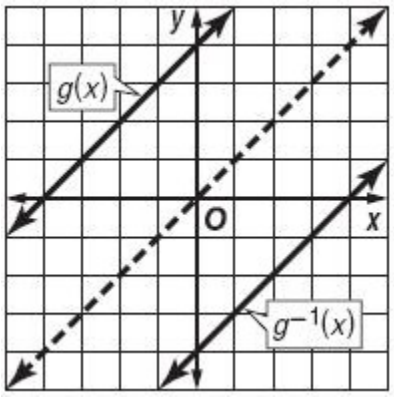
$f^{-1}(x) = -\frac{3}{5}(x + 8)$



20. $g(x) = x + 4$

ANSWER:

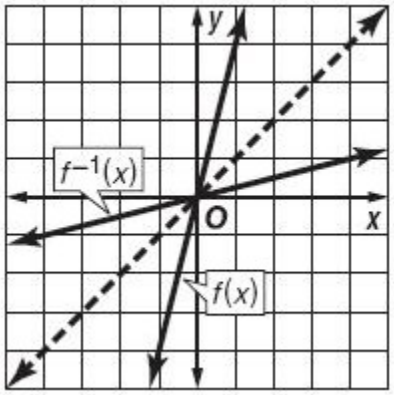
$g^{-1}(x) = x - 4$



21. $f(x) = 4x$

ANSWER:

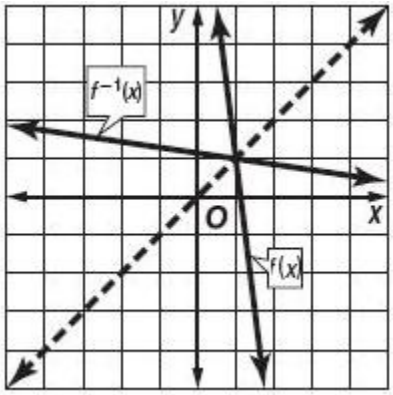
$f^{-1}(x) = \frac{1}{4}x$



22. $f(x) = -8x + 9$

ANSWER:

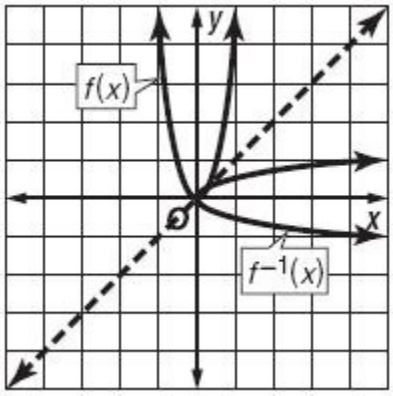
$f^{-1}(x) = -\frac{x}{8} + \frac{9}{8}$



23. $f(x) = 5x^2$

ANSWER:

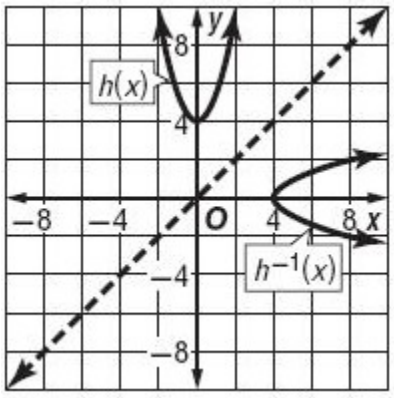
$f^{-1}(x) = \pm\sqrt{\frac{1}{5}x}$



24. $h(x) = x^2 + 4$

ANSWER:

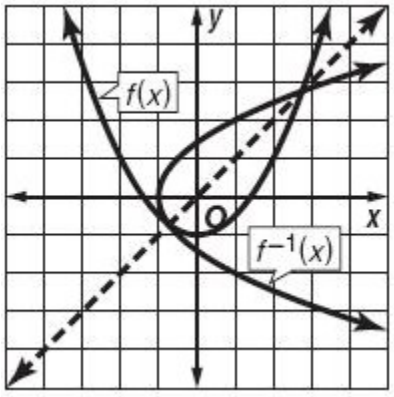
$h^{-1}(x) = \pm\sqrt{x-4}$



25. $f(x) = \frac{1}{2}x^2 - 1$

ANSWER:

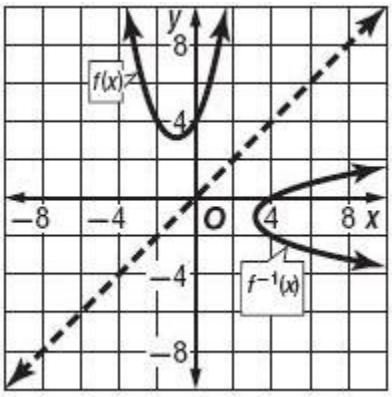
$f^{-1}(x) = \pm\sqrt{2x+2}$



26. $f(x) = (x+1)^2 + 3$

ANSWER:

$f^{-1}(x) = \pm\sqrt{x-3} - 1$



Determine whether each pair of functions are inverse functions. Write *yes* or *no*.

27. $f(x) = 2x + 3$
$g(x) = 2x - 3$

ANSWER:

No

28. $f(x) = 4x + 6$
$g(x) = \frac{x-6}{4}$

ANSWER:

Yes

29. $f(x) = -\frac{1}{3}x + 3$
$g(x) = -3x + 9$

ANSWER:

Yes

30. $f(x) = -6x$
$g(x) = \frac{1}{6}x$

ANSWER:
No

31. $f(x) = \frac{1}{2}x + 5$
$g(x) = 2x - 10$

ANSWER:
Yes

32. $f(x) = \frac{x+10}{8}$
$g(x) = 8x - 10$

ANSWER:
Yes

33. $f(x) = 4x^2$
$g(x) = \frac{1}{2}\sqrt{x}$

ANSWER:
Yes

34. $f(x) = \frac{1}{3}x^2 + 1$
$g(x) = \sqrt{3x - 3}$

ANSWER:
Yes

35. $f(x) = x^2 - 9$
$g(x) = x + 3$

ANSWER:
No

36. $f(x) = \frac{2}{3}x^3$
$g(x) = \sqrt{\frac{2}{3}x}$

ANSWER:
No

37. $f(x) = (x+6)^2$
$g(x) = \sqrt{x} - 6$

ANSWER:
Yes

38. $f(x) = 2\sqrt{x-5}$
$g(x) = \frac{1}{4}x^2 - 5$

ANSWER:
No

39. **FUEL** The average miles traveled for every gallon g of gas consumed by Leroy's car is represented by the function $m(g) = 28g$.

a. Find a function $c(g)$ to represent the cost per gallon of gasoline.

b. Use inverses to determine the function used to represent the cost per mile traveled in Leroy's car.

ANSWER:

a. $c(g) = 2.95g$

b. $c(m) \approx 0.105m$

40. **SHOES** The shoe size for the average U.S. teen or adult male can be determined using the formula $M(x) = 3x - 22$, where x is length of a foot in measured inches. The shoe size for the average U.S. teen or adult female can be found by using the formula $F(x) = 3x - 21$.

a. Find the inverse of each function.

b. If Lucy wears a size $7\frac{1}{2}$ shoe, how long are her feet?

ANSWER:

a. $M^{-1}(x) = \frac{x+22}{3}; F^{-1}(x) = \frac{x+21}{3}$

b. $9\frac{1}{2}$ in.

41. **GEOMETRY** The formula for the area of a circle is $A = \pi r^2$.

a. Find the inverse of the function.

b. Use the inverse to find the radius of a circle with an area of 36 square centimeters.

ANSWER:

a. $r = \sqrt{\frac{A}{\pi}}$

b. ≈ 3.39 cm

Use the horizontal line test to determine whether the inverse of each function is also a function.

42. $f(x) = 2x^2$

ANSWER:

No

43. $f(x) = x^3 - 8$

ANSWER:

Yes

44. $g(x) = x^4 - 6x^2 + 1$

ANSWER:

No

45. $h(x) = -2x^4 - x - 2$

ANSWER:

No

46. $g(x) = x^5 + x^2 - 4x$

ANSWER:
No

47. $h(x) = x^3 + x^2 - 6x + 12$

ANSWER:
No

48. **SHOPPING** Felipe bought a used car. The sales tax rate was 7.25% of the selling price, and he paid $350 in processing and registration fees. Find the selling price if Felipe paid a total of $8395.75.

ANSWER:
$7501.86

49. **TEMPERATURE** A formula for converting degrees Celsius to Fahrenheit is $F(x) = \frac{9}{5}x + 32$.

a. Find the inverse $F^{-1}(x)$. Show that $F(x)$ and $F^{-1}(x)$ are inverses.

b. Explain what purpose $F^{-1}(x)$ serves.

ANSWER:
a.

$$F^{-1}(x) = \frac{5}{9}(x - 32);$$

$$F\left[F^{-1}(x)\right] = \frac{9}{5}\left[\frac{5}{9}(x - 32)\right] + 32$$
$$= -32 + 32$$
$$= x$$

$$F^{-1}\left[F(x)\right] = \frac{5}{9}\left(\frac{9}{5}x + 32 - 32\right)$$
$$= \frac{5}{9}\left(\frac{9}{5}x + 0\right)$$
$$= x$$

b. It can be used to convert Fahrenheit to Celsius.

50. **MEASUREMENT** There are approximately 1.852 kilometers in a nautical mile.

a. Write a function that converts nautical miles to kilometers.

b. Find the inverse of the function that converts kilometers back to nautical miles.

c. Using composition of functions, verify that these two functions are inverses.

ANSWER:

a. $K(m) = 1.852\ m$

b. $K^{-1}(m) = \frac{1}{1.852}m$

c. $K\left[K^{-1}(m)\right] = m$ and $K^{-1}\left[K(m)\right] = m$, so the two functions are inverses of each other.

51. **MULTIPLE REPRESENTATIONS** Consider the functions $y = x^n$ for $n = 0, 1, 2, \ldots$

.

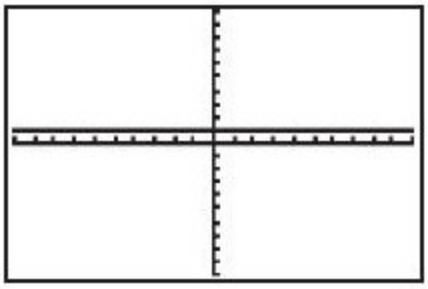
a. GRAPHING Use a graphing calculator to graph $y = x^n$ for $n = 0, 1, 2, 3,$ and 4.

b. TABULAR For which values of n is the inverse a function? Record your results in a table.

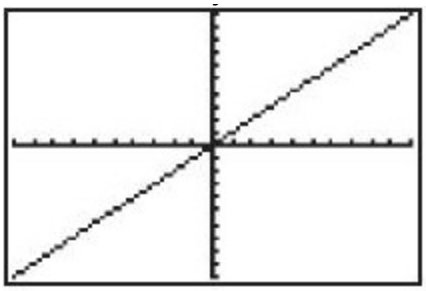
c. ANALYTICAL Make a conjecture about the values of n for which the inverse of $f(x) = x^n$ is a function. Assume that n is a whole number.

ANSWER:

a.



[–10, 10] SCI: 1 by [–10, 10] SCI: 1



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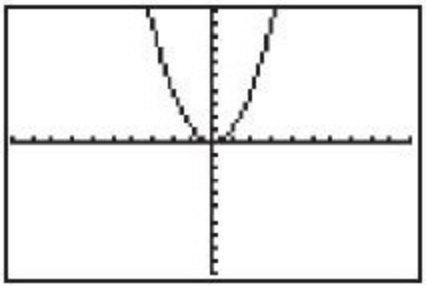

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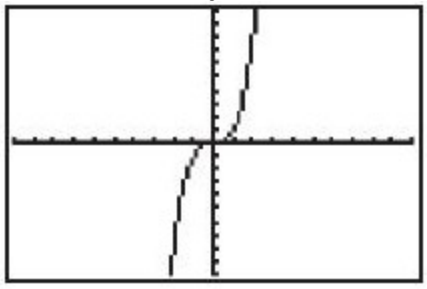
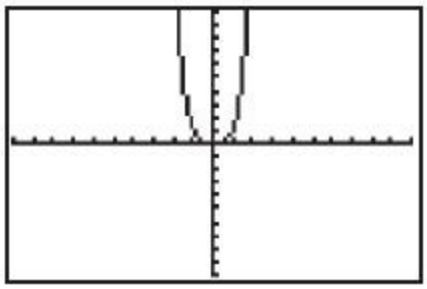


[–10, 10] SCI: 1 by [–10, 10] SCI: 1



[–10, 10] SCI: 1 by [–10, 10] SCI: 1

b.

Function	Inverse a function?
$y = x^0$ or $y = 1$	no
$y = x^1$ or $y = x$	yes
$y = x^2$	no
$y = x^3$	yes
$y = x^4$	no

c. n is odd.

52. **REASONING** If a relation is not a function, then its inverse is sometimes, always, or never a function. Explain your reasoning.

ANSWER:

Sample answer: Sometimes; $y = \pm\sqrt{x}$ is an example of a relation that is not a function, with an inverse being a function. A circle is an example of a relation that is not a function with an inverse not being a function.

53. **OPEN ENDED** Give an example of a function and its inverse. Verify that the two functions are inverses.

ANSWER:

Sample answer: $f(x) = 2x, f^{-1}(x) = 0.5x$;

$f[f^{-1}(x)] = f^{-1}[f(x)] = x$

54. **CHALLENGE** Give an example of a function that is its own inverse.

ANSWER:

Sample answer: $f(x) = x$ and $f^{-1}(x) = x$ or $f(x) = -x$ and $f^{-1}(x) = -x$

55. **CCSS ARGUMENTS** Show that the inverse of a linear function $y = mx + b$, where $m \neq 0$ and $x \neq b$, is also a linear function.

ANSWER:

The inverse function is $y = \frac{1}{m}x - \frac{b}{m}$.

56. **WRITING IN MATH** Suppose you have a composition of two functions that are inverses. When you put in a value of 5 for x, why is the result always 5?

ANSWER:

Sample answer: One of the functions carries out an operation on 5. Then the second function that is an inverse of the first function reverse the operation on 5. Thus, the result is 5.

57. **SHORT RESPONSE** If the length of a rectangular television screen is 24 inches and its height is 18 inches, what is the length of its diagonal in inches?

ANSWER:

30 in.

58. **GEOMETRY** If the base of a triangle is represented by $2x + 5$ and the height is represented by $4x$, which expression represents the area of the triangle?

A $(2x + 5) + (4x)$

B $(2x + 5)(4x)$

C $\frac{1}{2}(2x + 5) + (4x)$

D $\frac{1}{2}(2x + 5)(4x)$

ANSWER:

D

59. Which expression represents $f\ [g(x)]$ if $f(x) = x^2 + 3$ and $g(x) = -x + 1$?

F $x^2 - x + 2$

G $-x^2 - 2$

H $-x^3 + x^2 - 3x + 3$

J $x^2 - 2x + 4$

ANSWER:
J

60. **SAT/ACT** Which of the following is the inverse of $f(x) = \frac{3x-5}{2}$?

A $g(x) = \frac{2x+5}{3}$

B $g(x) = \frac{2x-5}{3}$

C $g(x) = \frac{3x+5}{2}$

D $g(x) = 2x + 5$

E $g(x) = \frac{3x-5}{2}$

ANSWER:
A

If $f(x) = 3x + 5$, $g(x) = x - 2$, and $h(x) = x^2 - 1$, find each value.

61. $g[f(3)]$

ANSWER:
12

62. $f\ [h(-2)]$

ANSWER:
14

63. $h[g(1)]$

ANSWER:
0

64. **CONSTRUCTION** A picnic area has the shape of a trapezoid. The longer base is 8 more than 3 times the length of the shorter base, and the height is 1 more than 3 times the shorter base. What are the dimensions if the area is 4104 square feet?

ANSWER:
$b_1 = 25$ ft, $b_2 = 83$ ft, $h = 76$ ft

Find the value of c that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

65. $x^2 + 34x + c$

ANSWER:
289; $(x + 17)^2$

66. $x^2 - 11x + c$

ANSWER:
$\frac{121}{4}; \left(x - \frac{11}{2}\right)^2$

Simplify.

67. $(3 + 4i)(5 - 2i)$

ANSWER:
$23 + 14i$

68. $\left(\sqrt{6}+i\right)\left(\sqrt{6}-i\right)$

ANSWER:
7

69. $\frac{1+i}{1-i}$

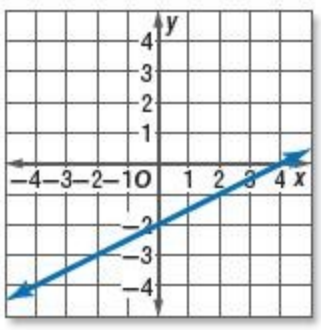
ANSWER:
i

70. $\frac{4-3i}{1+2i}$

ANSWER:
$-\frac{2}{5}-\frac{11}{5}i$

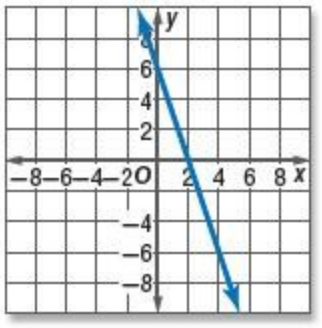
Determine the rate of change of each graph.

71.



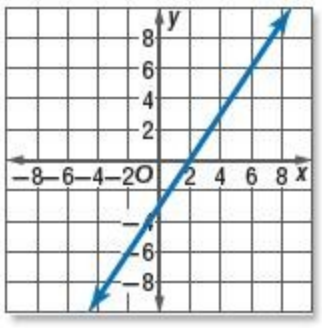
ANSWER:
$\frac{1}{2}$

72.



ANSWER:
-3

73.

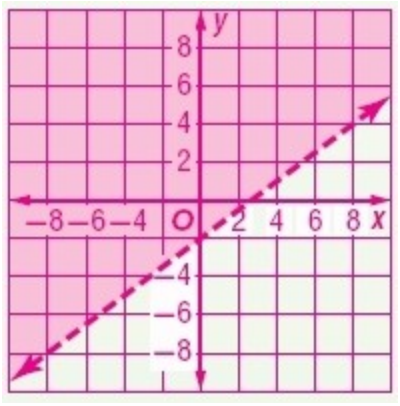


ANSWER:
$\frac{3}{2}$

Graph each inequality.

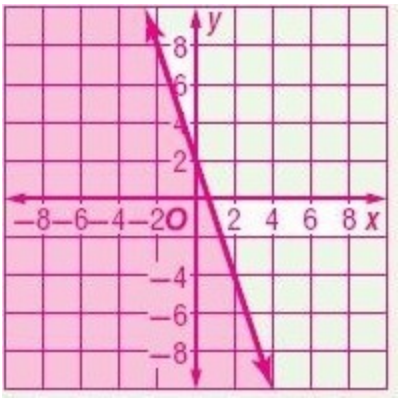
74. $y > \frac{3}{4}x - 2$

ANSWER:



75. $y \leq -3x + 2$

ANSWER:



76. $y < -x - 4$

ANSWER: