6-4 nth Roots

Simplify.

1. $\pm\sqrt{100y^8}$

ANSWER:
$\pm 10y^4$

2. $-\sqrt{49u^8v^{12}}$

ANSWER:
$-7u^4v^6$

3. $\sqrt{(y-6)^8}$

ANSWER:
$(y-6)^4$

4. $\sqrt[4]{16g^{16}h^{24}}$

ANSWER:
$2g^4h^6$

5. $\sqrt{-16y^4}$

ANSWER:
$\pm 4iy^2$

6. $\sqrt[6]{64(2y+1)^{18}}$

ANSWER:
$2\left|(2y+1)^3\right|$

Use a calculator to approximate each value to three decimal places.

7. $\sqrt{58}$

ANSWER:
7.616

8. $-\sqrt{76}$

ANSWER:
–8.718

9. $\sqrt[5]{-43}$

ANSWER:
–2.122

10. $\sqrt[4]{71}$

ANSWER:
2.903

11. **CCSS PERSEVERANCE** The radius r of the orbit of a television satellite is given by $\sqrt[3]{\frac{GMt^2}{4\pi^2}}$, where G is the universal gravitational constant, M is the mass of Earth, and t is the time it takes the satellite to complete one orbit. Find the radius of the satellite's orbit if G is $6.67\times10^{-11}\ N\cdot m^2/\text{kg}^2$, M is 5.98×10^{24} kg, and t is 2.6×10^6 seconds.

ANSWER:
about 4.088×10^8 m

Simplify.

12. $\pm\sqrt{121x^4y^{16}}$

ANSWER:
$\pm 11x^2y^8$

13. $\pm\sqrt{225a^{16}b^{36}}$

ANSWER:
$\pm 15a^8b^{18}$

14. $\pm\sqrt{49x^4}$

ANSWER:
$\pm 7x^2$

15. $-\sqrt{16c^4d^2}$

ANSWER:
$-4c^2|d|$

16. $-\sqrt{81a^{16}b^{20}c^{12}}$

ANSWER:
$-9a^8b^{10}c^6$

17. $-\sqrt{400x^{32}y^{40}}$

ANSWER:
$-20x^{16}y^{20}$

18. $\sqrt{(x+15)^4}$

ANSWER:
$(x+15)^2$

19. $\sqrt{(x^2+6)^{16}}$

ANSWER:
$(x^2+6)^8$

20. $\sqrt{(a^2+4a)^{12}}$

ANSWER:
$(a^2+4a)^6$

21. $\sqrt[3]{8a^6b^{12}}$

ANSWER:
$2a^2b^4$

22. $\sqrt[6]{d^{24}x^{36}}$

ANSWER:
d^4x^6

23. $\sqrt[3]{27b^{18}c^{12}}$

ANSWER:
$3b^6c^4$

6-4 nth Roots

24. $-\sqrt{(2x+1)^6}$

ANSWER:
$-\left|(2x+1)^3\right|$

25. $\sqrt{-(x+2)^8}$

ANSWER:
$i(x+2)^4$

26. $\sqrt[3]{-(y-9)^9}$

ANSWER:
$-(y-9)^3$

27. $\sqrt[6]{x^{18}}$

ANSWER:
$\left|x^3\right|$

28. $\sqrt[4]{a^{12}}$

ANSWER:
$\left|a^3\right|$

29. $\sqrt[3]{a^{12}}$

ANSWER:
a^4

30. $\sqrt[4]{81(x+4)^4}$

ANSWER:
$3\left|(x+4)\right|$

31. $\sqrt[3]{(4x-7)^{24}}$

ANSWER:
$(4x-7)^8$

32. $\sqrt[3]{(y^3+5)^{18}}$

ANSWER:
$(y^3+5)^6$

33. $\sqrt[4]{256(5x-2)^{12}}$

ANSWER:
$4\left|(5x-2)^3\right|$

34. $\sqrt[8]{x^{16}y^8}$

ANSWER:
$x^2|y|$

35. $\sqrt[5]{32a^{15}b^{10}}$

ANSWER:
$2a^3b^2$

36. **SHIPPING** An online book store wants to increase the size of the boxes it uses to ship orders. The new volume N is equal to the old volume V times the scale factor F cubed, or $N = V \cdot F^3$. What is the scale factor if the old volume was 0.8 cubic feet and the new volume is 21.6 cubic feet?

ANSWER:
3

37. **GEOMETRY** The side length of a cube is determined by $r = \sqrt[3]{V}$, where V is the volume in cubic units. Determine the side length of a cube with a volume of 512 cm^3.

ANSWER:
8 cm

Use a calculator to approximate each value to three decimal places.

38. $\sqrt{92}$

ANSWER:
9.592

39. $-\sqrt{150}$

ANSWER:
–12.247

40. $\sqrt{0.43}$

ANSWER:
0.656

41. $\sqrt{0.62}$

ANSWER:
0.787

42. $\sqrt[3]{168}$

ANSWER:
5.518

43. $\sqrt[5]{-4382}$

ANSWER:
–5.350

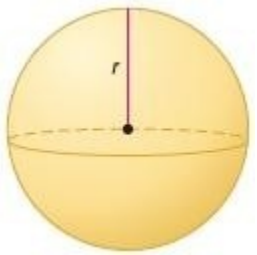
44. $\sqrt[6]{(8912)^2}$

ANSWER:
20.733

45. $\sqrt[5]{(4756)^2}$

ANSWER:
29.573

46. **GEOMETRY** The radius r of a sphere with volume V can be found using the formula $r = \sqrt[3]{\frac{3V}{4\pi}}$.



a. Determine the radius for volumes of 1000 cm^3, 8000 cm^3, and 64,000 cm^3.

b. How does the volume of the sphere change if the radius is doubled? Explain.

ANSWER:

a. (1000, 6.2), (8000, 12.4), (64,000, 24.8)

b. Sample answer: As r doubles, the volume increases by a factor of 2^3 or 8.

Simplify.

47. $\sqrt{196c^6d^4}$

ANSWER:

$14|c^3|d^2$

48. $\sqrt{-64y^8z^6}$

ANSWER:

$8iy^4|z^3|$

49. $\sqrt[3]{-27a^{15}b^9}$

ANSWER:

$-3a^5b^3$

50. $\sqrt[4]{-16x^{16}y^8}$

ANSWER:

$2x^4y^2\sqrt[4]{-1}$

51. $\sqrt{400x^{16}y^6}$

ANSWER:

$20x^8|y^3|$

52. $\sqrt[3]{8c^3d^{12}}$

ANSWER:

$2cd^4$

53. $\sqrt[3]{64(x+y)^6}$

ANSWER:

$4(x+y)^2$

54. $\sqrt[5]{-(y-z)^{15}}$

ANSWER:

$-(y-z)^3$

55. **PHYSICS** Johannes Kepler developed the formula $d = \sqrt[3]{6t^2}$, where d is the distance of a planet from the Sun in millions of miles and t is the number of Earth-days that it takes for the planet to orbit the Sun. If the length of a year on Mars is 687 Earth-days, how far from the Sun is Mars?

ANSWER:

about 141 million mi

56. **CCSS SENSE-MAKING** All matter is composed of atoms. The nucleus of an atom is the center portion of the atom that contains most of the mass of the atom. A theoretical formula for the radius r of the nucleus of an atom is $r = \left(1.3 \times 10^{-15}\right)\sqrt[3]{A}$ meters, where A is the mass number of the nucleus. Find the radius of the nucleus for each atom in the table.

Atom	Mass Number
carbon	6
oxygen	8
sodium	11
aluminum	13
chlorine	17

ANSWER:

carbon: $\approx 2.36 \times 10^{-15}$; oxygen: $= 2.6 \times 10^{-15}$; sodium: $\approx 2.9 \times 10^{-15}$; aluminum: $\approx 3.1 \times 10^{-15}$; chlorine: $\approx 3.34 \times 10^{-15}$

57. **BIOLOGY** Kleiber's Law, $P = 73.3\sqrt[4]{m^3}$, shows the relationship between the mass m in kilograms of an organism and its metabolism P in Calories per day. Determine the metabolism for each of the animals listed at the right.

Animal	Mass (kg)
bald eagle	4.5
golden retriever	30
komodo dragon	72
bottlenose dolphin	156
Asian elephant	2300

ANSWER:

bald eagle: $\approx$ 226.5 Cal/d; golden retriever: $\approx$ 939.6 Cal/d; komodo dragon: $\approx$ 1811.8 Cal/d; bottlenose dolphin: $\approx$ 3235.5 Cal/d; Asian elephant: $\approx$ 24,344.4 Cal/d

58. **MULTIPLE REPRESENTATIONS**

In this problem, you will use $f(x) = x^n$ and $g(x) = \sqrt[n]{x}$ to explore inverses.

a. TABULAR Make tables for $f(x)$ and $g(x)$ using $n = 3$ and $n = 4$

b. GRAPHICAL Graph the equations.

c. ANALYTICAL Which equations are functions? Which functions are one-to-one?

d. ANALYTICAL For what values of n are $g(x)$ and $f(x)$ inverses of each other?

e. VERBAL What conclusions can you make about $g(x) = \sqrt[n]{x}$ and $f(x) = x^n$ for all positive even values of n? for odd values of n?

ANSWER:

a. $n = 3$

x	–5	–4	–3	–2	–1	0
$f(x)$	–125	–64	–27	–8	–1	0
x	1	2	3	4	5	
$f(x)$	1	8	27	64	125	

$n = 4$

x	−5	−4	−3	−2	−1	0
$f(x)$	625	256	81	16	1	0
x	1	2	3	4	5	
$f(x)$	1	16	81	256	625	

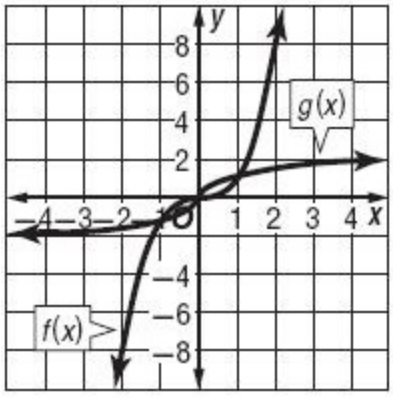
$n = 3$

x	−125	−64	−27	−8	−1	0
$g(x)$	−5	−4	−3	−2	−1	0
x	1	8	27	64	125	
$g(x)$	1	2	3	4	5	

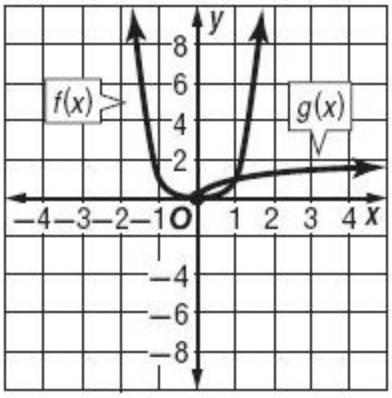
$n = 4$

x	625	256	81	16	1	0
$g(x)$	imaginary	imaginary	imaginary	imaginary	imaginary	0
x	1	16	18	256	625	
$g(x)$	1	2	3	4	5	

b. $n = 3$



$n = 4$

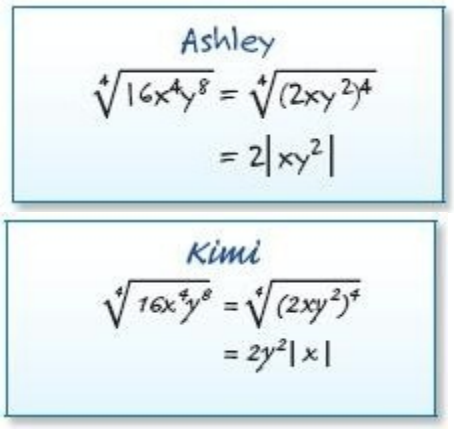


c. $f(x) = x^3; f(x) = x^4; g(x) = \sqrt[3]{x};$
$f(x) = x^3$ *and* $g(x) = \sqrt[3]{x}$

d. positive odd values

e. Sample answer: For all positive odd values of n, $f(x)$ and $g(x)$ are inverse functions. For all positive even values of n, $f(x)$ and $g(x)$ are inverse functions only if the range of $f(x)$ and the domain of $g(x)$ are restricted to positive values.

59. **CCSS CRITIQUE** Ashley and Kimi are simplifying $\sqrt[4]{16x^4y^8}$. Is either of them correct? Explain your reasoning.



ANSWER:

Kimi; Ashley's error was keeping the y^2 inside the absolute value symbol.

60. **CHALLENGE** Under what conditions is $\sqrt{x^2 + y^2} = x + y$ true?

ANSWER:

Case #1: $x = 0$ and $y \geq 0$

Case #2: $y = 0$ and $x \geq 0$

61. **REASONING** Determine whether the statement $\sqrt[4]{(-x)^4} = x$ is *sometimes*, *always*, or *never* true.

ANSWER:

Sample answer: Sometimes; when $x = -3, \sqrt[4]{(-x)^4} = |(-x)|$ or 3.

When $x = 3, \sqrt[4]{(-x)^4} = |(3)|$ or 3.

62. **CHALLENGE** For what real values of x is $\sqrt[3]{x} > x$?

ANSWER:

$0 < x < 1, x < -1$

63. **OPEN ENDED** Write a number for which the principal square root and cube root are both integers.

ANSWER:

Sample answers: 1, 64

64. **WRITING IN MATH** Explain when and why absolute value symbols are needed when taking an nth root.

ANSWER:

Sample answer: They are needed to ensure that the answer is not a negative number. When we take any odd root of a number, we find that there is just one answer. If the number is positive, the root is positive. If the number is negative, the root is negative. Every positive real number has two nth roots when n is even; one of these roots is positive and one is negative. Negative real numbers do not have nth roots when n is even. Absolute value signs are never needed when finding odd roots. When finding even nth roots, absolute value signs are sometimes necessary, as with square roots.

65. **CHALLENGE** Write an equivalent expression in for $\sqrt[3]{2x} \cdot \sqrt[3]{8y}$. Simplify the radical.

ANSWER:

$2\sqrt[3]{2xy}$

CHALLENGE Simplify each expression.

66. $\sqrt[4]{0.0016}$

ANSWER:

0.2

67. $\sqrt[7]{-0.0000001}$

ANSWER:

–0.1

68. $\dfrac{\sqrt[5]{-0.00032}}{\sqrt[3]{-0.027}}$

ANSWER:

$\dfrac{2}{3}$

69. **CHALLENGE** Solve $-\dfrac{5}{\sqrt{a}} = -125$ for a.

ANSWER:

$\dfrac{1}{625}$

70. What is the value of *w* in the equation $\frac{1}{2}(4w+36)=3(4w-3)$?

A 2

B 2.7

C 27

D 36

ANSWER:
B

71. What is the product of the complex numbers (5 + *i*) and (5 – *i*)?

F 24

G 26

H 25 – *i*

J 26 – 10*i*

ANSWER:
G

72. **EXTENDED RESPONSE** A cylindrical cooler has a diameter of 9 inches and a height of 11 inches. Tate plans to use it for soda cans that have a diameter of 2.5 inches and a height of 4.75 inches.

a. Tate plans to place two layers consisting of 9 cans each into the cooler. What is the volume of the space that will not be filled with the cans?

b. Find the ratio of the volume of the cooler to the volume of the cans in part a.

ANSWER:
a. 280.1 in^3

b. about 1.7

73. **SAT/ACT** Which of the following is closest to $\sqrt[3]{7.32}$?

A 1.8

B 1.9

C 2.0

D 2.1

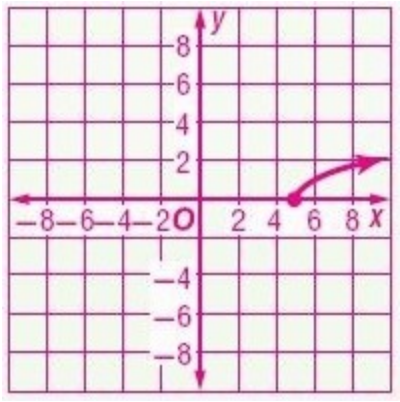
E 2.2

ANSWER:
B

Graph each function.

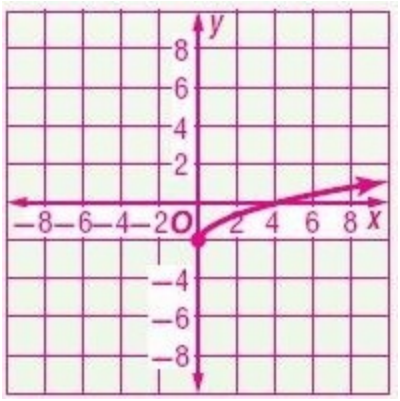
74. $y = \sqrt{x-5}$

ANSWER:



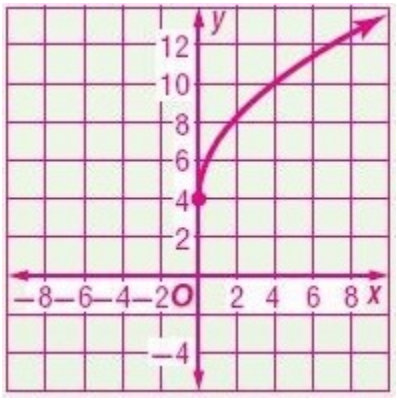
75. $y = \sqrt{x} - 2$

ANSWER:



76. $y = 3\sqrt{x} + 4$

ANSWER:



77. **HEALTH** The average weight of a baby born at a certain hospital is $7\frac{1}{2}$ pounds and the average length is 19.5 inches. One kilogram is about 2.2 pounds and 1 centimeter is about 0.3937 inches. Find the average weight in kilograms and the length in centimeters.

ANSWER:

3.41 kg and 49.53 cm

Simplify.

78. $(4c - 5) - (c + 11) + (-6c + 17)$

ANSWER:

$-3c + 1$

79. $(11x^2 + 13x - 15) - (7x^2 - 9x + 19)$

ANSWER:

$4x^2 + 22x - 34$

80. $(d - 5)(d + 3)$

ANSWER:

$d^2 - 2d - 15$

81. $\left(2a^2 + 6\right)^2$

ANSWER:

$4a^4 + 24a^2 + 36$

82. **GAS MILEAGE** The gas mileage y in miles per gallon for a certain vehicle is given by the equation $y = 10 + 0.9x - 0.01x^2$, where x is the speed of the vehicle between 10 and 75 miles per hour. Find the range of speeds that would give a gas mileage of at least 25 miles per gallon.

ANSWER:

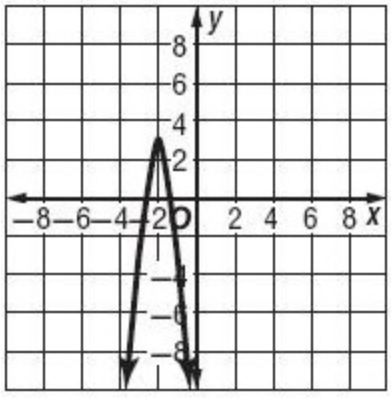
$22.087 \le x \le 67.91$ mph

Write each equation in vertex form, if not already in that form. Identify the vertex, axis of symmetry, and direction of opening. Then graph the function.

83. $y = -6\left(x + 2\right)^2 + 3$

ANSWER:

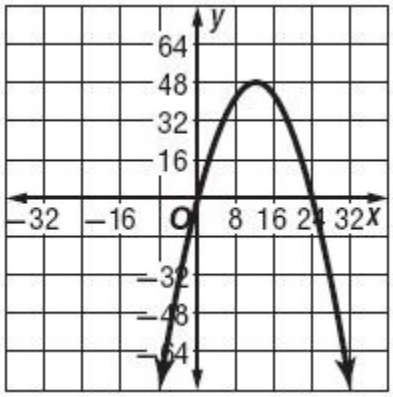
(–2, 3); $x = -2$; down



84. $y = -\frac{1}{3}x^2 + 8x$

ANSWER:

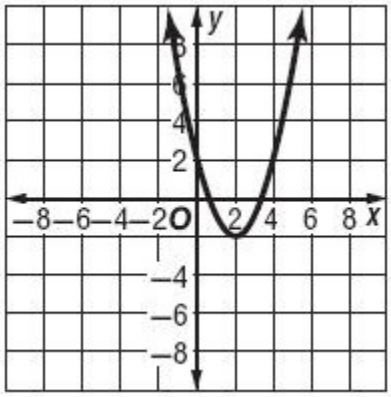
$y = -\frac{1}{3}\left(x - 12\right)^2 + 48; (12, 48); x = 12$; down



85. $y = \left(x - 2\right)^2 - 2$

ANSWER:

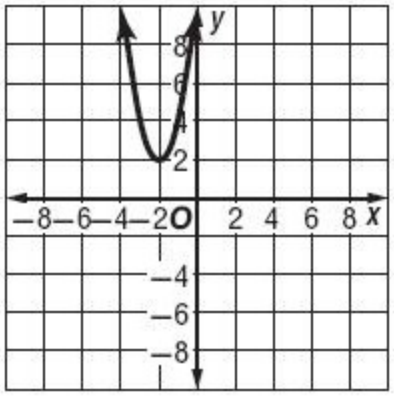
(2, –2); $x = 2$; up



86. $y = 2x^2 + 8x + 10$

ANSWER:

$y = 2(x+2)^2 + 2; (-2, 2); x = -2$; up



Find each product.

87. $(x+4)(x+5)$

ANSWER:

$x^2 + 9x + 20$

88. $(y-3)(y+4)$

ANSWER:

$y^2 + y - 12$

89. $(a+2)(a-9)$

ANSWER:

$a^2 - 7a - 18$

90. $(a-b)(a-3b)$

ANSWER:

$a^2 - 4ab + 3b^2$

91. $(x+2y)(x-y)$

ANSWER:

$x^2 + xy - 2y^2$

92. $2(w+z)(w-4z)$

ANSWER:

$2w^2 - 6wz - 8z^2$