6-5 Operations with Radical Expressions

CCSS PRECISION Simplify.

1. $\sqrt{36ab^4c^5}$

ANSWER:
$6b^2c^2\sqrt{ac}$

2. $\sqrt{144x^7y^5}$

ANSWER:
$12x^3y^2\sqrt{xy}$

3. $\dfrac{\sqrt{c^5}}{\sqrt{d^9}}$

ANSWER:
$\dfrac{c^2\sqrt{cd}}{d^5}$

4. $\sqrt[4]{\dfrac{5x}{8y}}$

ANSWER:
$\dfrac{\sqrt[4]{10xy^3}}{2y}$

5. $5\sqrt{2x}\cdot 3\sqrt{8x}$

ANSWER:
$60x$

6. $4\sqrt{5a^5}\cdot\sqrt{125a^3}$

ANSWER:
$100a^4$

7. $3\sqrt[3]{36xy}\cdot 2\sqrt[3]{6x^2y^2}$

ANSWER:
$36xy$

8. $\sqrt[4]{3x^3y^2}\cdot\sqrt[4]{27xy^2}$

ANSWER:
$3x|y|$

9. $5\sqrt{32}+\sqrt{27}+2\sqrt{75}$

ANSWER:
$20\sqrt{2}+13\sqrt{3}$

10. $4\sqrt{40}+3\sqrt{28}-\sqrt{200}$

ANSWER:
$8\sqrt{10}+6\sqrt{7}-10\sqrt{2}$

11. $\left(4+2\sqrt{5}\right)\left(3\sqrt{3}+4\sqrt{5}\right)$

ANSWER:
$12\sqrt{3}+16\sqrt{5}+40+6\sqrt{15}$

12. $\left(8\sqrt{3}-2\sqrt{2}\right)\left(8\sqrt{3}+2\sqrt{2}\right)$

ANSWER:
184

13. $\frac{5}{\sqrt{2}+3}$

ANSWER:
$\frac{15-5\sqrt{2}}{7}$

14. $\frac{8}{\sqrt{6}-5}$

ANSWER:
$\frac{-40-8\sqrt{6}}{19}$

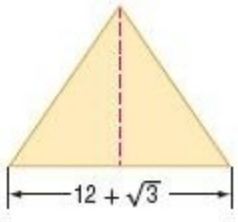
15. $\frac{4+\sqrt{2}}{\sqrt{2}-3}$

ANSWER:
$-2-\sqrt{2}$

16. $\frac{6-\sqrt{3}}{\sqrt{3}+4}$

ANSWER:
$\frac{27-10\sqrt{3}}{13}$

17. **GEOMETRY** Find the altitude of the triangle if the area is $189+4\sqrt{3}$ square centimeters.



ANSWER:
$32-2\sqrt{3}$ cm

Simplify.

18. $\sqrt{72a^8b^5}$

ANSWER:
$6a^4b^2\sqrt{2b}$

19. $\sqrt{9a^{15}b^3}$

ANSWER:
$3a^7b\sqrt{ab}$

20. $\sqrt{24a^{16}b^8c}$

ANSWER:
$2a^8b^4\sqrt{6c}$

21. $\sqrt{18a^6b^3c^5}$

ANSWER:
$3\left|a^3\right|bc^2\sqrt{2bc}$

6-5 Operations with Radical Expressions

22. $\frac{\sqrt{5a^5}}{\sqrt{b^{13}}}$

ANSWER:

$\frac{a^2\sqrt{5ab}}{b^7}$

23. $\sqrt{\frac{7x}{10y^3}}$

ANSWER:

$\frac{\sqrt{70xy}}{10y^2}$

24. $\frac{\sqrt[3]{6x^2}}{\sqrt[3]{5y}}$

ANSWER:

$\frac{\sqrt[3]{150x^2y^2}}{5y}$

25. $\sqrt[4]{\frac{7x^3}{4b^2}}$

ANSWER:

$\frac{\sqrt[4]{28b^2x^3}}{2|b|}$

26. $3\sqrt{5y}\cdot 8\sqrt{10yz}$

ANSWER:

$120y\sqrt{2z}$

27. $2\sqrt{32a^3b^5}\cdot\sqrt{8a^7b^2}$

ANSWER:

28. $6\sqrt{3ab}\cdot 4\sqrt{24ab^3}$

ANSWER:

$144|a|b^2\sqrt{2}$

29. $5\sqrt{x^8y^3}\cdot 5\sqrt{2x^5y^4}$

ANSWER:

$25x^6y^3\sqrt{2xy}$

30. $3\sqrt{90}+4\sqrt{20}+\sqrt{162}$

ANSWER:

$9\sqrt{10}+8\sqrt{5}+9\sqrt{2}$

31. $9\sqrt{12}+5\sqrt{32}-\sqrt{72}$

ANSWER:

$18\sqrt{3}+14\sqrt{2}$

32. $4\sqrt{28}-8\sqrt{810}+\sqrt{44}$

ANSWER:

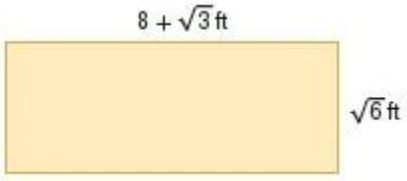
$8\sqrt{7}-72\sqrt{10}+2\sqrt{11}$

33. $3\sqrt{54}+6\sqrt{288}-\sqrt{147}$

ANSWER:

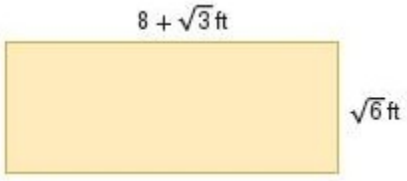
$9\sqrt{6}+72\sqrt{2}-7\sqrt{3}$

34. **GEOMETRY** Find the perimeter of the rectangle.



ANSWER:
$16+2\sqrt{3}+2\sqrt{6}$ ft

35. **GEOMETRY** Find the area of the rectangle.



ANSWER:
$8\sqrt{6}+3\sqrt{2}$ ft^2

36. **GEOMETRY** Find the exact surface area of a sphere with radius of $4+\sqrt{5}$ inches.

ANSWER:
$\left(84+32\sqrt{5}\right)\pi$ in^2

Simplify.

37. $\left(7\sqrt{2}-3\sqrt{3}\right)\left(4\sqrt{6}+3\sqrt{12}\right)$

ANSWER:
$56\sqrt{3}+42\sqrt{6}-36\sqrt{2}-54$

38. $\left(8\sqrt{5}-6\sqrt{3}\right)\left(8\sqrt{5}+6\sqrt{3}\right)$

ANSWER:
212

39. $\left(12\sqrt{10}-6\sqrt{5}\right)\left(12\sqrt{10}+6\sqrt{5}\right)$

ANSWER:
1260

40. $\left(6\sqrt{3}+5\sqrt{2}\right)\left(2\sqrt{6}+3\sqrt{8}\right)$

ANSWER:
$36\sqrt{2}+36\sqrt{6}+20\sqrt{3}+60$

41. $\dfrac{6}{\sqrt{3}-\sqrt{2}}$

ANSWER:
$6\sqrt{3}+6\sqrt{2}$

42. $\dfrac{\sqrt{2}}{\sqrt{5}-\sqrt{3}}$

ANSWER:
$\dfrac{\sqrt{10}+\sqrt{6}}{2}$

43. $\dfrac{9-2\sqrt{3}}{\sqrt{3}+6}$

ANSWER:
$\dfrac{20-7\sqrt{3}}{11}$

44. $\frac{2\sqrt{2}+2\sqrt{5}}{\sqrt{5}+\sqrt{2}}$

ANSWER:
2

Simplify.

45. $\sqrt[3]{16y^4z^{12}}$

ANSWER:
$2yz^4\sqrt[3]{2y}$

46. $\sqrt[3]{-54x^6y^{11}}$

ANSWER:
$-3x^2y^3\sqrt[3]{2y^2}$

47. $\sqrt[4]{162a^6b^{13}c}$

ANSWER:
$3|a|b^3\sqrt[4]{2a^2bc}$

48. $\sqrt[4]{48a^9b^3c^{16}}$

ANSWER:
$2a^2c^4\sqrt[4]{3ab^3}$

49. $\sqrt[4]{\frac{12x^3y^2}{5a^2b}}$

ANSWER:
$\frac{\sqrt[4]{1500a^2b^3x^3y^2}}{5|a|b}$

50. $\frac{\sqrt[3]{36xy^2}}{\sqrt[3]{10xz}}$

ANSWER:
$\frac{\sqrt[3]{450y^2z^2}}{5z}$

51. $x+1/\sqrt{x}-1$

ANSWER:
$\frac{(x+1)(\sqrt{x}+1)}{x-1}$ or $\frac{x\sqrt{x}+\sqrt{x}+x+1}{x-1}$

52. $\frac{x-2}{\sqrt{x^2-4}}$

ANSWER:
$\frac{\sqrt{x^2-4}}{x+2}$

53. $\frac{\sqrt{x}}{\sqrt{x^2-1}}$

ANSWER:
$\frac{\sqrt{x^3-x}}{x^2-1}$

54. **APPLES** The diameter of an apple is related to its weight and can be modeled by the formula $d=\sqrt[3]{3w}$, where d is the diameter in inches and w is the weight in ounces. Find the diameter of an apple that weighs 6.47 ounces.

ANSWER:
2.69 in.

Simplify each expression if *b* is an even number.

55. $\sqrt[b]{a^b}$

ANSWER:

$|a|$

56. $\sqrt[b]{a^{4b}}$

ANSWER:

a^4

57. $\sqrt[b]{a^{2b}}$

ANSWER:

a^2

58. $\sqrt[b]{a^{3b}}$

ANSWER:

$|a^3|$

59. **MULTIPLE REPRESENTATIONS** In this problem, you will explore operations with like radicals.

a. NUMERICAL Copy the diagram at the right on dot paper. Use the Pythagorean Theorem to prove that the length of the red segment is $\sqrt{2}$ units.

b. GRAPHICAL Extend the segment to represent $\sqrt{2}+\sqrt{2}$.

c. ANALYTICAL Use your drawing to show that $\sqrt{2}+\sqrt{2} \neq \sqrt{2+2}$ or 2.

d. GRAPHICAL Use the dot paper to draw a square with side lengths $\sqrt{2}$ units.

e. NUMERICAL Prove that the area of the square is $\sqrt{2} \cdot \sqrt{2} = 2$ square units.

ANSWER:

a. $a^2 + b^2 = c^2$

$1^2 + 1^2 = c^2$

$2 = c^2$

$c = \sqrt{2}$

b.

c. $\sqrt{2}+\sqrt{2}$ units is the length of the hypotenuse of an isosceles right triangle with legs of length 2 units. Therefore, $\sqrt{2}+\sqrt{2} > 2$.

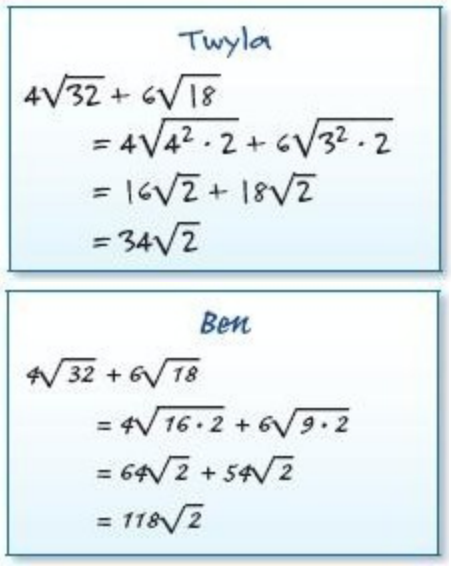
d.

e. The square creates 4 triangles with a base of 1 and a height of 1.

Therefore the area of each triangle is $\frac{1}{2}bh = \frac{1}{2}(1)(1)$.

The area of the square is 2, so $\sqrt{2} \cdot \sqrt{2} = 2$.

60. **ERROR ANALYSIS** Twyla and Ben are simplifying $4\sqrt{32}+6\sqrt{18}$. Is either of them correct? Explain your reasoning.



ANSWER:

Twyla; Ben's mistakes were multiplying the 4 by 16 instead of 4 and multiplying the 6 by 9 instead of 3.

61. **CHALLENGE** Show that $\frac{-1-i\sqrt{3}}{2}$ is a cube root of 1.

ANSWER:

$$\left(\frac{-1-i\sqrt{3}}{2}\right)^3$$
$$=\left(\frac{-1-i\sqrt{3}}{2}\right)\cdot\left(\frac{-1-i\sqrt{3}}{2}\right)\cdot\left(\frac{-1-i\sqrt{3}}{2}\right)$$
$$=\frac{\left(-1-i\sqrt{3}\right)\left(-1-i\sqrt{3}\right)\left(-1-i\sqrt{3}\right)}{8}$$
$$=\frac{\left(1+i\sqrt{3}+i\sqrt{3}+3i^2\right)\left(-1-i\sqrt{3}\right)}{8}$$
$$=\frac{\left(2i\sqrt{3}-2\right)\left(-1-i\sqrt{3}\right)}{8}$$
$$=\frac{-2i\sqrt{3}-6i^2+2+2i\sqrt{3}}{8}$$
$$=\frac{-6i^2+2}{8}=\frac{8}{8}\text{ or }1$$

62. **CCSS ARGUMENTS** For what values of a is $\sqrt{a}\cdot\sqrt{-a}$ a real number? Explain.

ANSWER:

0 is the only possible value for a since $\sqrt{a}$ is defined for $a\geq 0$, and $\sqrt{-a}$ is defined for $a\leq 0$.

63. **CHALLENGE** Find four combinations of whole numbers that satisfy $\sqrt[a]{256}=b$.

ANSWER:

$a = 1$, $b = 256$; $a = 2$, $b = 16$; $a = 4$, $b = 4$; $a = 8$, $b = 2$

64. **OPEN ENDED** Find a number other than 1 that has a positive whole number for a square root, cube root, and fourth root.

ANSWER:
Sample answer: 4096

65. **WRITING IN MATH** Explain why absolute values may be unnecessary when an *n*th root of an even power results in an odd power.

ANSWER:
Sample answer: It is only necessary to use absolute values when it is possible that *n* could be odd or even and still be defined. It is when the radicand must be nonnegative in order for the root to be defined that the absolute values are not necessary.

66. **PROBABILITY** A six-sided number cube has faces with the numbers 1 though 6 marked on it. What is the probability that a number less than 4 will occur on one toss of the number cube?

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. $\frac{1}{5}$

ANSWER:
A

67. When the number of a year is divisible by 4, the year is a leap year. However, when the year is divisible by 100, the year is not a leap year, unless the year is divisible by 400. Which is not a leap year?

F 1884

G 1900

H 1904

J 1940

ANSWER:
G

68. **SHORT RESPONSE** Which property is illustrated by $4x + 0 = 4x$?

ANSWER:
Additive Identity Property

69. **SAT/ACT** The expression $\sqrt{180a^2b^8}$ is equivalent to which of the following?

A $3\sqrt{10}|a|b^4$

B $5\sqrt{6}|a|b^4$

C $6\sqrt{5}|a|b^4$

D $18\sqrt{10}|a|b^4$

E $36\sqrt{5}|a|b^4$

ANSWER:
C

Simplify.

70. $\sqrt{81x^6}$

ANSWER:
$9|x^3|$

71. $\sqrt[3]{729a^3b^9}$

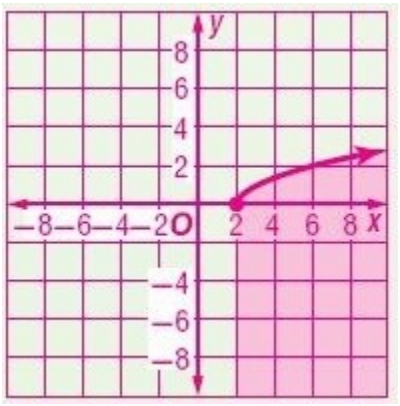
ANSWER:
$9ab^3$

72. $\sqrt{(g+5)^2}$

ANSWER:
$|g+5|$

73. Graph $y \le \sqrt{x-2}$.

ANSWER:



Solve each equation.

74. $x^4 - 34x^2 + 225 = 0$

ANSWER:
$-5, -3, 3, 5$

75. $x^4 - 15x^2 - 16 = 0$

ANSWER:
$-4, 4, -i, i$

76. $x^4 + 6x^2 - 27 = 0$

ANSWER:
$-\sqrt{3}, \sqrt{3}, -3i, 3i$

77. $x^3 + 64 = 0$

ANSWER:
$-4, 2 + 2i\sqrt{3}, 2 - 2i\sqrt{3}$

78. $27x^3 + 1 = 0$

ANSWER:
$-\frac{1}{3}, \frac{1 + i\sqrt{3}}{6}, \frac{1 - i\sqrt{3}}{6}$

79. $8x^3 - 27 = 0$

ANSWER:
$\frac{3}{2}, \frac{-3 + 3i\sqrt{3}}{4}, \frac{-3 - 3i\sqrt{3}}{4}$

80. **MODELS** A model car builder is building a display table for model cars. He wants the perimeter of the table to be 26 feet, but he wants the area of the table to be no more than 30 square feet. What could be the width of the table?

ANSWER:
between 0 and 3 ft or between 10 and 13 ft

81. **CONSTRUCTION** Cho charges $1500 to build a small deck and $2500 to build a large deck. During the spring and summer, she built 5 more small decks than large decks. If she earned $23,500 how many of each type of deck did she build?

ANSWER:
9 small, 4 large

82. **FOOD** The Hot Dog Grille offers the lunch combinations shown. Assume that the price of a combo meal is the same price as purchasing each item separately. Find the prices for a hot dog, a soda, and a bag of potato chips.

Lunch Combo Meals
1. Two hot dogs, one soda $5.40
2. One hot dog, potato chips, one soda $4.35
3. Two hot dogs, two bags of chips $5.70

ANSWER:
hot dog, $1.95; soda, $1.50; potato chips, $0.90

Evaluate each expression.

83. $2\left(\frac{1}{6}\right)$

ANSWER:
$\frac{1}{3}$

84. $3\left(\frac{1}{8}\right)$

ANSWER:
$\frac{3}{8}$

85. $\frac{1}{4}+\frac{1}{3}$

ANSWER:
$\frac{7}{12}$

86. $\frac{1}{2}+\frac{3}{8}$

ANSWER:
$\frac{7}{8}$

87. $\frac{2}{3}-\frac{1}{4}$

ANSWER:
$\frac{5}{12}$

88. $\frac{5}{6}-\frac{2}{5}$

ANSWER:

$\frac{13}{30}$