6-6 Rational Exponents

Write each expression in radical form, or write each radical in exponential form.

1. $10^{\frac{1}{4}}$

ANSWER:
$\sqrt[4]{10}$

2. $x^{\frac{3}{5}}$

ANSWER:
$\sqrt[5]{x^3}$

3. $\sqrt[3]{15}$

ANSWER:
$15^{\frac{1}{3}}$

4. $\sqrt[4]{7x^6y^9}$

ANSWER:
$7^{\frac{1}{4}}x^{\frac{3}{2}}y^{\frac{9}{4}}$

Evaluate each expression.

5. $343^{\frac{1}{3}}$

ANSWER:
7

6. $125^{\frac{2}{3}}$

ANSWER:
25

7. $32^{-\frac{1}{5}}$

ANSWER:
$\dfrac{1}{2}$

8. $\dfrac{24}{4^{\frac{3}{2}}}$

ANSWER:
3

9. **GARDENING** If the area A of a square is known, then the lengths of its sides ℓ can be computed using $\ell = A^{\frac{1}{2}}$. You have purchased a 169 ft^2 share in a community garden for the season. What is the length of one side of your square garden?

ANSWER:
13 ft

CCSS PRECISION Simplify each expression.

10. $a^{\frac{3}{4}} \cdot a^{\frac{1}{2}}$

ANSWER:
$a^{\frac{5}{4}}$

6-6 Rational Exponents

11. $\dfrac{x^{\frac{4}{5}}}{x^{\frac{1}{5}}}$

   ANSWER:
   $x^{\frac{3}{5}}$

12. $\dfrac{b^3}{c^{\frac{1}{2}}} \cdot \dfrac{c}{b^{\frac{1}{3}}}$

   ANSWER:
   $b^{\frac{8}{3}}c^{\frac{1}{2}}$

13. $\sqrt[4]{9g^2}$

   ANSWER:
   $\sqrt{3g}$

14. $\dfrac{\sqrt[5]{64}}{\sqrt[5]{4}}$

   ANSWER:
   $2^{\frac{4}{5}}$ or $\sqrt[5]{16}$

15. $\dfrac{g^{\frac{1}{2}}-1}{g^{\frac{1}{2}}+1}$

   ANSWER:
   $\dfrac{g-2g^{\frac{1}{2}}+1}{g-1}$

Write each expression in radical form, or write each radical in exponential form.

16. $8^{\frac{1}{5}}$

   ANSWER:
   $\sqrt[5]{8}$

17. $4^{\frac{2}{7}}$

   ANSWER:
   $\sqrt[7]{16}$

18. $a^{\frac{3}{4}}$

   ANSWER:
   $\sqrt[4]{a^3}$

19. $\left(x^3\right)^{\frac{3}{2}}$

   ANSWER:
   $\sqrt{x^9}$

20. $\sqrt{17}$

   ANSWER:
   $17^{\frac{1}{2}}$

21. $\sqrt[4]{63}$

   ANSWER:
   $63^{\frac{1}{4}}$

6-6 Rational Exponents

22. $\sqrt[3]{5xy^2}$

    ANSWER:
    $5^{\frac{1}{3}} x^{\frac{1}{3}} y^{\frac{2}{3}}$

23. $\sqrt[4]{625x^2}$

    ANSWER:
    $5x^{\frac{1}{2}}$

    Evaluate each expression.

24. $27^{\frac{1}{3}}$

    ANSWER:
    3

25. $256^{\frac{1}{4}}$

    ANSWER:
    4

26. $16^{-\frac{1}{2}}$

    ANSWER:
    $\dfrac{1}{4}$

27. $81^{-\frac{1}{4}}$

    ANSWER:
    $\dfrac{1}{3}$

28. **CCSS SENSE-MAKING** A women's regulation-sized basketball is slightly smaller than a men's basketball. The radius r of the ball that holds V cubic units of air is $\left(\dfrac{3V}{4\pi}\right)^{\frac{1}{3}}$.

    Men's 455 in³   Women's 413 in³

    a. Find the radius of a women's basketball.

    b. Find the radius of a men's basketball.

    ANSWER:
    a. about 4.62 in.

    b. about 4.77 in.

29. **GEOMETRY** The radius r of a sphere with volume V is given by $r = \left(\dfrac{3V}{4\pi}\right)^{\frac{1}{3}}$. Find the radius of a ball with a volume of 77 cm^3.

    ANSWER:
    about 2.64 cm

Simplify each expression.

30. $x^{\frac{1}{3}} \cdot x^{\frac{2}{5}}$

ANSWER:

$x^{\frac{11}{15}}$

31. $a^{\frac{4}{9}} \cdot a^{\frac{1}{4}}$

ANSWER:

$a^{\frac{25}{36}}$

32. $b^{-\frac{3}{4}}$

ANSWER:

$\dfrac{b^{\frac{1}{4}}}{b}$

33. $y^{-\frac{4}{5}}$

ANSWER:

$\dfrac{y^{\frac{1}{5}}}{y}$

34. $\dfrac{\sqrt[8]{81}}{\sqrt[6]{3}}$

ANSWER:

$\sqrt[3]{3}$

35. $\dfrac{\sqrt[4]{27}}{\sqrt[4]{3}}$

ANSWER:

$\sqrt{3}$

36. $\sqrt[4]{25x^2}$

ANSWER:

$\sqrt{5x}$

37. $\sqrt[6]{81g^3}$

ANSWER:

$\sqrt[3]{9} \cdot \sqrt{g}$

38. $\dfrac{h^{\frac{1}{2}} + 1}{h^{\frac{1}{2}} - 1}$

ANSWER:
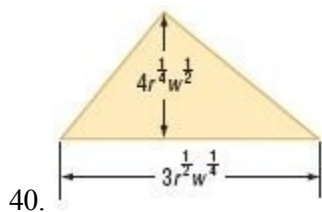
$\dfrac{h + 2h^{\frac{1}{2}} + 1}{h - 1}$

39. $\dfrac{x^{\frac{1}{4}} + 2}{x^{\frac{1}{4}} - 2}$

ANSWER:

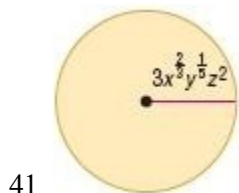$\dfrac{x + 4x^{\frac{3}{4}} + 8x^{\frac{1}{2}} + 16x^{\frac{1}{4}} + 16}{x - 16}$

6-6 Rational Exponents

GEOMETRY Find the area of each figure.

40.

ANSWER:

$6r^{\frac{3}{4}}w^{\frac{3}{4}}$ units^2

41.

ANSWER:

about $28.27x^{\frac{4}{3}}y^{\frac{3}{5}}z^4$ unit^2

42. Find the simplified form of $18^{\frac{1}{2}} + 2^{\frac{1}{2}} - 32^{\frac{1}{2}}$.

ANSWER:
0

43. What is the simplified form of $64^{\frac{1}{3}} - 32^{\frac{1}{3}} + 8^{\frac{1}{3}}$?

ANSWER:

$6 - 2 \cdot 4^{\frac{1}{3}}$

Simplify each expression.

44. $a^{\frac{7}{4}} \cdot a^{\frac{5}{4}}$

ANSWER:
a^3

45. $x^{\frac{2}{3}} \cdot x^{\frac{8}{3}}$

ANSWER:
$x^{\frac{10}{3}}$

46. $\left(b^{\frac{3}{4}} \right)^{\frac{1}{3}}$

ANSWER:
$b^{\frac{1}{4}}$

47. $\left(y^{-\frac{3}{5}} \right)^{-\frac{1}{4}}$

ANSWER:
$y^{\frac{3}{20}}$

48. $\sqrt[4]{64}$

ANSWER:
$2\sqrt{2}$

49. $\sqrt[6]{216}$

    ANSWER:

    $\sqrt{6}$

50. $d^{-\frac{5}{6}}$

    ANSWER:

    $\dfrac{d^{\frac{1}{6}}}{d}$

51. $w^{-\frac{7}{8}}$

    ANSWER:

    $\dfrac{w^{\frac{1}{8}}}{w}$

52. **WILDLIFE** A population of 100 deer is reintroduced to a wildlife preserve. Suppose the population does extremely well and the deer population doubles in two years. Then the number D of deer after t years is given by $D = 100 \cdot 2^{\frac{t}{2}}$.

    a. How many deer will there be after $4\dfrac{1}{2}$ years?

    b. Make a table that charts the population of deer every year for the next five years.

    c. Make a graph using your table.

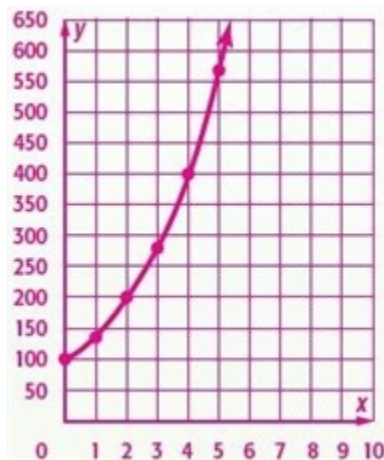   **d.** Using your table and graph, decide whether this is a reasonable trend over the long term. Explain.

    ANSWER:

    a. about 475

    b.

Year	0	1	2	3	4	5
Deer	100	141	200	282	400	565

    c.

    d. Sample answer: No; it is not reasonable to think that the population will continue to grow without bounds. This does not take into account the death rate of the deer.

Simplify each expression.

53. $\dfrac{f^{-\frac{1}{4}}}{4f^{\frac{1}{2}} \cdot f^{-\frac{1}{3}}}$

ANSWER:

$\dfrac{f^{\frac{7}{12}}}{4f}$

54. $\dfrac{g^{\frac{5}{2}}}{g^{\frac{1}{2}}+2}$

ANSWER:

$\dfrac{g^3 - 2g^{\frac{5}{2}}}{g-4}$

55. $\dfrac{c^{\frac{2}{3}}}{c^{\frac{1}{6}}}$

ANSWER:

$c^{\frac{1}{2}}$

56. $\dfrac{z^{\frac{4}{5}}}{z^{\frac{1}{2}}}$

ANSWER:

$z^{\frac{3}{10}}$

57. $\sqrt{23} \cdot \sqrt[3]{23^2}$

ANSWER:

$23\sqrt[6]{23}$

58. $\sqrt[8]{36h^4 j^4}$

ANSWER:

$6^{\frac{1}{4}} h^{\frac{1}{2}} j^{\frac{1}{2}}$

59. $\sqrt{\sqrt{81}}$

ANSWER:
3

60. $\sqrt[4]{\sqrt{256}}$

ANSWER:
2

61. $\dfrac{ab}{\sqrt{c}}$

ANSWER:

$\dfrac{ab\sqrt{c}}{c}$

62. $\dfrac{xy}{\sqrt[3]{z}}$

ANSWER:

$\dfrac{xy\sqrt[3]{z^2}}{z}$

63. $\dfrac{8^{\frac{1}{6}} - 9^{\frac{1}{4}}}{\sqrt{3} + \sqrt{2}}$

ANSWER:

$2\sqrt{6} - 5$

64. $\dfrac{x^{\frac{5}{3}} - x^{\frac{1}{3}} z^{\frac{4}{3}}}{x^{\frac{2}{3}} + z^{\frac{2}{3}}}$

ANSWER:

$x - x^{\frac{1}{3}} z^{\frac{2}{3}}$

65. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the functions

$f(x) = x^3$ and $g(x) = x^{\frac{1}{3}}$.

x	f(x)	g(x)
−2		
−1		
0		
1		
2		

a. TABULAR Copy and complete the table to the right.

b. GRAPHICAL Graph $f(x)$ and $g(x)$.

c. VERBAL Explain the transformation between $f(x)$ and $g(x)$.

ANSWER:
a.

x	f(x)	g(x)
−2	−8	−1.26
−1	−1	−1
0	0	0
1	1	1
2	8	1.26

b.
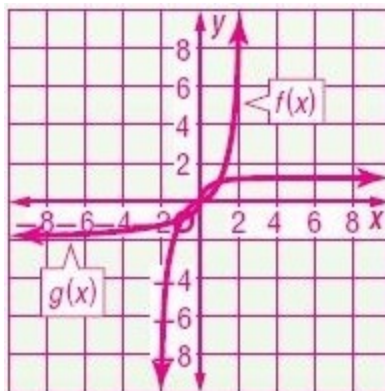
c. It is a reflection of the line $y = x$.

66. **REASONING** Determine whether $-x^{-2} = (-x)^{-2}$ is *always, sometimes,* or *never* true. Explain your reasoning.

ANSWER:
Never; the quantities are not the same. When the negative is enclosed inside of the parentheses and the base is raised to an even power, the answer is positive. When the negative is not enclosed inside of the parentheses and the base is raised to an even power, the answer is negative.

67. **CHALLENGE** Consider $\sqrt[4]{(-16)^3}$.

    a. Explain why the expression is not a real number.

    b. Find n such that $n\sqrt[4]{(-16)^3}$ is a real number.

    ANSWER:

    a. Sample answer: $\sqrt[4]{(-16)^3} = \sqrt[4]{-4096}$; there is no real number that when raised to the forth power results in a negative number.

    b. Sample answer: $\sqrt[4]{-1}$

68. **OPEN ENDED** Find two different expressions that equal 2 in the form $x^{\frac{1}{a}}$.

    ANSWER:

    Sample answer: $4^{\frac{1}{2}}$ and $16^{\frac{1}{4}}$

69. **WRITING IN MATH** Explain how it might be easier to simplify an expression using rational exponents rather than using radicals.

    ANSWER:

    Sample answer: It may be easier to simplify an expression when it has rational exponents because all the properties of exponents apply. We do not have as many properties dealing directly with radicals. However, we can convert all radicals to rational exponents, and then use the properties of exponents to simplify.

70. **CCSS CRITIQUE** Ayana and Kenji are simplifying $\dfrac{x^{\frac{3}{4}}}{x^{\frac{1}{2}}}$. Is either of them correct? Explain your reasoning.

ANSWER:
No; Ayana added the exponents and Kenji divided the exponents. The exponents should have been subtracted.

71. The expression $\sqrt{56-c}$ is equivalent to a positive integer when c is equal to

    A 8

    B –8

    C 56

    D 36

    ANSWER:
    B

6-6 Rational Exponents

72. **SAT/ACT** Which of the following sentences is true about the graphs of $y = 2(x-3)^2 + 1$ and $y = 2(x+3)^2 + 1$?

F Their vertices are maximums.

G The graphs have the same shape with different vertices.

H The graphs have different shapes with the same vertices.

J The graphs have different shapes with different vertices.

K One graph has a vertex that is a maximum while the other graph has a vertex that is a minimum.

ANSWER:
G

73. **GEOMETRY** What is the converse of the statement? *If it is summer, then it is hot outside.*

A If it is not hot outside, then it is not summer.

B If it is not summer, then it is not hot outside.

C If it is hot outside, then it is summer.

D If it is hot outside, it is not summer.

ANSWER:
C

74. **SHORT RESPONSE** If $3^5 \cdot p = 3^3$, then find p.

ANSWER:
3^{-2}

Simplify.

75. $\sqrt{243}$

ANSWER:
$9\sqrt{3}$

76. $\sqrt[3]{16y^3}$

ANSWER:
$2y\sqrt[3]{2}$

77. $3\sqrt[3]{56y^6z^3}$

ANSWER:
$6y^2z\sqrt[3]{7}$

78. **PHYSICS** The speed of sound in a liquid is $s = \sqrt{\dfrac{B}{d}}$, where B is the bulk modulus of the liquid and d is its density. For water, $B = 2.1 \times 10^9$ N/m^2 and $d = 10^3$ kg/m^3. Find the speed of sound in water to the nearest meter per second.

ANSWER:
1449 m/s

Find p (–4) and p (x + h) for each function.

79. $p(x) = x - 2$

ANSWER:
$-6; x + h - 2$

80. $p(x) = -x + 4$

    ANSWER:
    $8; -x - h + 4$

81. $p(x) = 6x + 3$

    ANSWER:
    $-21; 6x + 6h + 3$

82. $p(x) = x^2 + 5$

    ANSWER:
    $21; x^2 + 2xh + h^2 + 5$

83. $p(x) = x^2 - x$

    ANSWER:
    $20; x^2 + 2xh + h^2 - x - h$

84. $p(x) = 2x^3 - 1$

    ANSWER:
    $-129; 2x^3 + 6x^2h + 6xh^2 + 2h^3 - 1$

    Solve each equation by factoring.

85. $x^2 - 11x = 0$

    ANSWER:
    $\{0, 11\}$

86. $x^2 + 6x - 16 = 0$

    ANSWER:
    $\{-8, 2\}$

87. $4x^2 - 13x = 12$

    ANSWER:
    $\left\{ -\dfrac{3}{4}, 4 \right\}$

88. $x^2 - 14x = -49$

    ANSWER:
    $\{7\}$

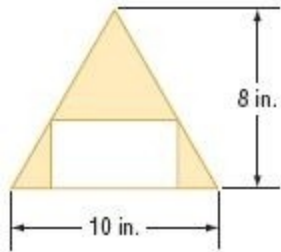89. $x^2 + 9 = 6x$

    ANSWER:
    $\{3\}$

90. $x^2 - 3x = -\dfrac{9}{4}$

    ANSWER:
    $\left\{ \dfrac{3}{2} \right\}$

6-6 Rational Exponents

91. **GEOMETRY** A rectangle is inscribed in an isosceles triangle as shown. Find the dimensions of the inscribed rectangle with maximum area. (Hint: Use similar triangles.)

ANSWER:
5 in. by 4 in.

Find each power.

92. $\left(\sqrt{x-3}\right)^2$

ANSWER:
$x - 3$

93. $\left(\sqrt[3]{3x-4}\right)^3$

ANSWER:
$3x - 4$

94. $\left(\sqrt[4]{7x-1}\right)^4$

ANSWER:
$7x - 1$

95. $\left(\sqrt{x}-4\right)^2$

ANSWER:
$x - 8\sqrt{x} + 16$

96. $\left(2\sqrt{x}-5\right)^2$

ANSWER:
$4x - 20\sqrt{x} + 25$

97. $\left(3\sqrt{x}+1\right)^2$

ANSWER:
$9x + 6\sqrt{x} + 1$