6-7 Solving Radical Equations and Inequalities

Solve each equation.

1. $\sqrt{x-4}+6=10$

ANSWER:
20

2. $\sqrt{x+13}-8=-2$

ANSWER:
23

3. $8-\sqrt{x+12}=3$

ANSWER:
13

4. $\sqrt{x-8}+5=7$

ANSWER:
12

5. $\sqrt[3]{x-2}=3$

ANSWER:
29

6. $(x-5)^{\frac{1}{3}}-4=-2$

ANSWER:
13

7. $(4y)^{\frac{1}{3}}+3=5$

ANSWER:
2

8. $\sqrt[3]{n+8}-6=-3$

ANSWER:
19

9. $\sqrt{y}-7=0$

ANSWER:
49

10. $2+4z^{\frac{1}{2}}=0$

ANSWER:
No solution

11. $5+\sqrt{4y-5}=12$

ANSWER:
$\frac{27}{2}$

12. $\sqrt{2t-7}=\sqrt{t+2}$

ANSWER:
9

13. **CCSS REASONING** The time T in seconds that it takes a pendulum to make a complete swing back and forth is given by the formula $T = 2\pi\sqrt{\frac{L}{g}}$, where L is the length of the pendulum in feet and g is the acceleration due to gravity, 32 feet per second squared.

a. In Tokyo, Japan, a huge pendulum in the Shinjuku building measures 73 feet 9.75 inches. How long does it take for the pendulum to make a complete swing?

b. A clockmaker wants to build a pendulum that takes 20 seconds to swing back and forth. How long should the pendulum be?

ANSWER:

a. about 9.5 seconds

b. about 324 ft

14. **MULTIPLE CHOICE** Solve $(2y+6)^{\frac{1}{4}} - 2 = 0$.

A $y = 1$

B $y = 5$

C $y = 11$

D $y = 15$

ANSWER:

B

Solve each inequality.

15. $\sqrt{3x+4} - 5 \le 4$

ANSWER:

$-\frac{4}{3} \le x \le \frac{77}{3}$

16. $\sqrt{b-7} + 6 \le 12$

ANSWER:

$7 \le b \le 43$

17. $2 + \sqrt{4y-4} \le 6$

ANSWER:

$1 \le y \le 5$

18. $\sqrt{3a+3} - 1 \le 2$

ANSWER:

$-1 \le a < 2$

19. $1 + \sqrt{7x-3} > 3$

ANSWER:

$x > 1$

20. $\sqrt{3x+6} + 2 \le 5$

ANSWER:

$-2 \le x \le 1$

21. $-2+\sqrt{9-5x} \geq 6$

ANSWER:
$x \leq -11$

22. $6-\sqrt{2y+1} < 3$

ANSWER:
$y > 4$

Solve each equation. Confirm by using a graphing calculator.

23. $\sqrt{2x+5}-4=3$

ANSWER:
22

24. $6+\sqrt{3x+1}=11$

ANSWER:
8

25. $\sqrt{x+6}=5-\sqrt{x+1}$

ANSWER:
3

26. $\sqrt{x-3}=\sqrt{x+4}-1$

ANSWER:
12

27. $\sqrt{x-15}=3-\sqrt{x}$

ANSWER:
no real solution

28. $\sqrt{x-10}=1-\sqrt{x}$

ANSWER:
no real solution

29. $6+\sqrt{4x+8}=9$

ANSWER:
$\frac{1}{4}$

30. $2+\sqrt{3y-5}=10$

ANSWER:
23

31. $\sqrt{x-4}=\sqrt{2x-13}$

ANSWER:
9

32. $\sqrt{7a-2}=\sqrt{a+3}$

ANSWER:
$\frac{5}{6}$

33. $\sqrt{x-5}-\sqrt{x}=-2$

ANSWER:
$\frac{81}{16}$

34. $\sqrt{b-6}+\sqrt{b}=3$

ANSWER:
$\frac{25}{4}$

35. **CCSS SENSE-MAKING** Isabel accidentally dropped her keys from the top of a Ferris wheel. The formula $t=\frac{1}{4}\sqrt{d-h}$ describes the time *t* in seconds at which the keys are *h* meters above the ground and Isabel is *d* meters above the ground. If Isabel was 65 meters high when she dropped the keys, how many meters above the ground will the keys be after 2 seconds?

ANSWER:
1 m

Solve each equation.

36. $(5n-6)^{\frac{1}{3}}+3=4$

ANSWER:
$\frac{7}{5}$

37. $(5p-7)^{\frac{1}{3}}+3=5$

ANSWER:
3

38. $(6q+1)^{\frac{1}{4}}+2=5$

ANSWER:
$\frac{40}{3}$

39. $(3x+7)^{\frac{1}{4}}-3=1$

ANSWER:
83

40. $(3y-2)^{\frac{1}{5}}+5=6$

ANSWER:
1

41. $(4z-1)^{\frac{1}{5}}-1=2$

ANSWER:
61

42. $2(x-10)^{\frac{1}{3}}+4=0$

ANSWER:
2

43. $3(x+5)^{\frac{1}{3}}-6=0$

ANSWER:
3

44. $\sqrt[3]{5x+10}-5=0$

ANSWER:
23

45. $\sqrt[3]{4n-8}-4=0$

ANSWER:
18

46. $\frac{1}{7}(14a)^{\frac{1}{3}}=1$

ANSWER:
24.5

47. $\frac{1}{4}(32b)^{\frac{1}{3}}=1$

ANSWER:
2

48. **MULTIPLE CHOICE** Solve $\sqrt[4]{y+2}+9=14$.

A 23

B 53

C 123

D 623

ANSWER:
D

49. **MULTIPLE CHOICE** Solve $(2x-1)^{\frac{1}{4}}-2=1$.

F 41

G 28

H 13

J 1

ANSWER:
F

Solve each inequality.

50. $1+\sqrt{5x-2}>4$

ANSWER:
$x>\frac{11}{5}$

51. $\sqrt{2x+14}-6\geq 4$

ANSWER:
$x\geq 43$

52. $10-\sqrt{2x+7}\leq 3$

ANSWER:
$x\geq 21$

53. $6+\sqrt{3y+4}<6$

ANSWER:
no real solution

54. $\sqrt{2x+5}-\sqrt{9+x}>0$

ANSWER:
$x>4$

55. $\sqrt{d+3}+\sqrt{d+7}>4$

ANSWER:
$d>-\frac{3}{4}$

56. $\sqrt{3x+9}-2<7$

ANSWER:
$-3\le x<24$

57. $\sqrt{2y+5}+3\le 6$

ANSWER:
$-\frac{5}{2}\le y\le 2$

58. $-2+\sqrt{8-4z}\ge 8$

ANSWER:
$z\le -23$

59. $-3+\sqrt{6a+1}>4$

ANSWER:
$a>8$

60. $\sqrt{2}-\sqrt{b+6}\le -\sqrt{b}$

ANSWER:
$0\le b\le 2$

61. $\sqrt{c+9}-\sqrt{c}>\sqrt{3}$

ANSWER:
$0\le c<3$

62. **PENDULUMS** The formula $s=2\pi\sqrt{\frac{\ell}{32}}$ represents the swing of a pendulum, where s is the time in seconds to swing back and forth, and ℓ is the length of the pendulum in feet. Find the length of a pendulum that makes one swing in 1.5 seconds.

ANSWER:
about 1.82 ft

63. **FISH** The relationship between the length and mass of certain fish can be approximated by the equation $L=0.46\sqrt[3]{M}$, where L is the length in meters and M is the mass in kilograms. Solve this equation for M.

ANSWER:
$M=\left(\frac{L}{0.46}\right)^3$

64. **HANG TIME** Refer to the information at the beginning of the lesson regarding hang time. Describe how the height of a jump is related to the amount of time in the air. Write a step-by-step explanation of how to determine the height of Jordan's 0.98-second jump.

ANSWER:
If the height of a person's jump and the amount of time he or she is in the air are related by an equation involving radicals, then the hang time associated with a given height can be found by solving a radical equation.

65. **CONCERTS** The organizers of a concert are preparing for the arrival of 50,000 people in the open field where the concert will take place. Each person is allotted 5 square feet of space, so the organizers rope off a circular area of 250,000 square feet. Using the formula $A = \pi r^2$, where A represents the area of the circular region and r represents the radius of the region, find the radius of this region.

ANSWER:
about 282 ft

66. **WEIGHTLIFTING** The formula $M = 512 - 146.230B^{-\frac{8}{5}}$ can be used to estimate the maximum total mass that a weightlifter of mass B kilograms can lift using the snatch and the clean and jerk. According to the formula, how much does a person weigh who can lift at most 470 kilograms?

ANSWER:
163 kg

67. **CCSS ARGUMENTS** Which equation does not have a solution?

$\sqrt{x-1}+3=4$

$\sqrt{x-2}+7=10$

$\sqrt{x+1}+3=4$

$\sqrt{x+2}-7=-10$

ANSWER:
$\sqrt{x+2}-7=-10$

68. **CHALLENGE** Lola is working to solve $(x+5)^{\frac{1}{4}}=-4$. She said that she could tell there was no real solution without even working the problem. Is Lola correct? Explain your reasoning.

ANSWER:
Yes; since $\sqrt[4]{x+5} \geq 0$, the left side of the equation is nonnegative. Therefore, the left side of the equation cannot equal –4. Thus the equation has no solution.

69. **REASONING** Determine whether $\frac{\sqrt{(x^2)^2}}{-x} = x$ is *sometimes, always,* or *never* true when *x* is a real number. Explain your reasoning.

ANSWER:
never;

$$\frac{\sqrt{(x^2)^2}}{-x} = x$$
$$\frac{\sqrt{(x^2)^2}}{-x} = x$$
$$\frac{x^2}{-x} = x$$
$$x^2 = (x)(-x)$$
$$x^2 \neq -x^2$$

70. **OPEN ENDED** Select a whole number. Now work backward to write two radical equations that have that whole number as solutions. Write one square root equation and one cube root equation. You may need to experiment until you find a whole number you can easily use.

ANSWER:

Sample answer using $6: \sqrt{x-2} = 2, (x+21)^{\frac{1}{3}} = 3$

71. **WRITING IN MATH** Explain the relationship between the index of the root of a variable in an equation and the power to which you raise each side of the equation to solve the equation.

ANSWER:
They are the same number.

72. **OPEN ENDED** Write an equation that can be solved by raising each side of the equation to the given power.

a. $\frac{3}{2}$ power

b. $\frac{5}{4}$ power

c. $\frac{7}{8}$ power

ANSWER:

a. Sample answer: $0 = 6x^{\frac{2}{3}} - 5$

b. Sample answer: $0 = x^{\frac{4}{5}} - 9$

c. Sample answer: $10x^{\frac{8}{7}} = -1$

73. **CHALLENGE** Solve $7^{3x-1} = 49^{x+1}$ for *x*. (Hint: $b^x = b^y$ if and only if $x = y$.)

ANSWER:
3

REASONING Determine whether the following statements are *sometimes, always,* or *never* true for $x^{\frac{1}{n}} = a$. Explain your reasoning.

74. If *n* is odd, there will be extraneous solutions.

ANSWER:
never; Sample answer: The radicand can be negative.

75. If n is even, there will be extraneous solutions.

ANSWER:
sometimes; Sample answer: when the radicand is negative, then there will be extraneous roots.

76. What is an equivalent form of $\frac{4}{5+i}$?

A $\frac{10-2i}{13}$

B $\frac{5-i}{6}$

C $\frac{6-i}{6}$

D $\frac{6-i}{13}$

ANSWER:
A

77. Which set of points describes a function?

F {(3, 0), (–2, 5), (2, –1), (2, 9)}

G {(–3, 5), (–2, 3), (–1, 5), (0, 7)}

H {(2, 5), (2, 4), (2, 3), (2, 2)}

J {(3, 1), (–3, 2), (3, 3), (–3, 4)}

ANSWER:
G

78. **SHORT RESPONSE** The perimeter of an isosceles triangle is 56 inches. If one leg is 20 inches long, what is the measure of the base of the triangle?

ANSWER:
16 in.

79. **SAT/ACT** If $\sqrt{x+5}+1=4$, what is the value of x?

A 4

B 10

C 11

D 12

E 20

ANSWER:
A

Evaluate.

80. $27^{-\frac{2}{3}}$

ANSWER:
$\frac{1}{9}$

81. $9^{\frac{1}{3}} \cdot 9^{\frac{5}{3}}$

ANSWER:
81

82. $\left(\frac{8}{27}\right)^{-\frac{2}{3}}$

ANSWER:

$\frac{9}{4}$

83. **GEOMETRY** The measures of the legs of a right triangle can be represented by the expressions $4x^2y^2$ and $8x^2y^2$. Use the Pythagorean Theorem to find a simplified expression for the measure of the hypotenuse.

ANSWER:

$4x^2y^2\sqrt{5}$

Find the inverse of each function.

84. $y = 3x - 4$

ANSWER:

$y = \frac{x+4}{3}$

85. $y = -2x - 3$

ANSWER:

$y = \frac{-x-3}{2}$

86. $y = x^2$

ANSWER:

$y = \pm\sqrt{x}$

87. $y = (2x + 3)^2$

ANSWER:

$y = \pm\frac{1}{2}\sqrt{x} - \frac{3}{2}$

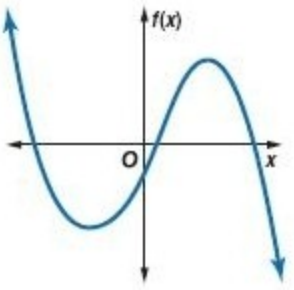
For each graph,

a. describe the end behavior,

b. determine whether it represents an odd-degree or an even-degree polynomial function, and

c. state the number of real zeros.

88.



ANSWER:

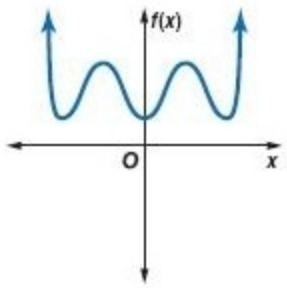
a.

$f(x) \to -\infty \text{ as } x \to +\infty,$
$f(x) \to +\infty \text{ as } x \to -\infty;$

b. odd;

c. 3

89.



ANSWER:

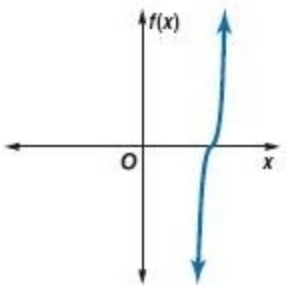
a.

$f(x) \to +\infty \text{ as } x \to +\infty,$
$f(x) \to +\infty \text{ as } x \to -\infty;$

b. even;

c. 0

90.



ANSWER:

a.

$f(x) \to +\infty \text{ as } x \to +\infty,$
$f(x) \to -\infty \text{ as } x \to -\infty;$

b. odd;

c. 1

Solve each equation. Write in simplest form.

91. $\frac{8}{5}x = \frac{4}{15}$

ANSWER:

$\frac{1}{6}$

92. $\frac{27}{14}y = \frac{6}{7}$

ANSWER:

$\frac{4}{9}$

93. $\frac{3}{10} = \frac{12}{25}a$

ANSWER:

$\frac{5}{8}$

94. $\frac{6}{7} = 9m$

ANSWER:

$\frac{2}{21}$

95. $\frac{9}{8}b = 18$

ANSWER:

16

96. $\frac{6}{7}n = \frac{3}{4}$

ANSWER:

$\frac{7}{8}$

97. $\frac{1}{3}p = \frac{5}{6}$

ANSWER:

$2\frac{1}{2}$

98. $\frac{2}{3}q = 7$

ANSWER:

$10\frac{1}{2}$