7-1 Graphing Exponential Functions

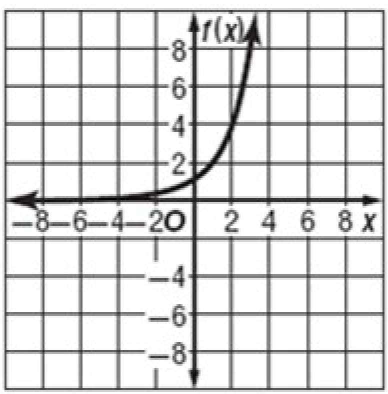
Graph each function. State the domain and range.

1. $f(x) = 2^x$

SOLUTION:

Make a table of values. Then plot the points and sketch the graph.

x	$f(x)$
–2	0.25
–1	0.5
0	1
1	2
2	4
3	8



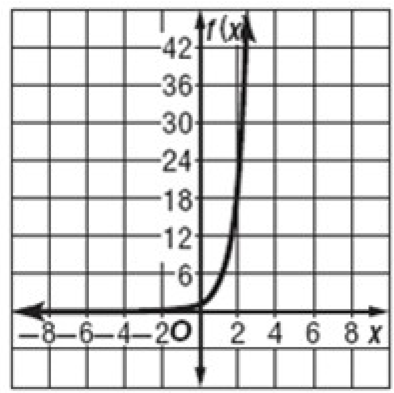
Domain = {all real numbers}; Range = $\{f(x) \mid f(x) > 0\}$

2. $f(x) = 5^x$

SOLUTION:

Make a table of values. Then plot the points and sketch the graph.

x	$f(x)$
–3	0.008
–2	0.04
–1	0.2
0	1
1	5
2	25
3	125



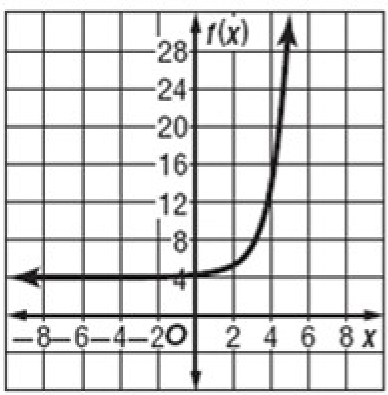
Domain = {all real numbers}; Range = $\{f(x) \mid f(x) > 0\}$

3. $f(x) = 3^{x-2} + 4$

SOLUTION:

Make a table of values. Then plot the points and sketch the graph.

x	$f(x)$
–3	4.004
–2	4.012
–1	4.037
0	4.111
1	4.333
2	5
3	7



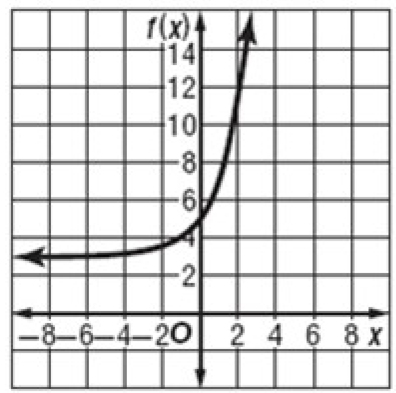
Domain = {all real numbers}; Range = $\{f(x) \mid f(x) > 4\}$

4. $f(x) = 2^{x+1} + 3$

SOLUTION:

Make a table of values. Then plot the points and sketch the graph.

x	$f(x)$
–3	3.25
–2	3.5
–1	4
0	5
1	7



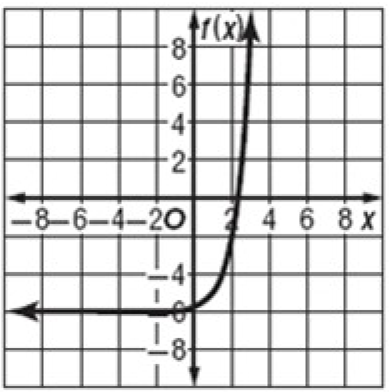
Domain = {all real numbers}; Range = $\{f(x) \mid f(x) > 3\}$

5. $f(x) = 0.25(4)^x - 6$

SOLUTION:

Make a table of values. Then plot the points and sketch the graph.

x	$f(x)$
–2	–5.984
–1	–5.938
0	–5.75
1	–5
2	–2
3	10



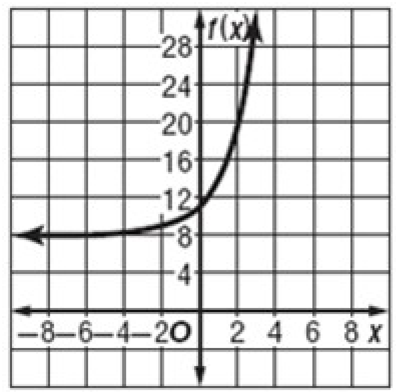
Domain = {all real numbers}; Range = $\{f(x) \mid f(x) > -6\}$

6. $f(x) = 3(2)^x + 8$

SOLUTION:

Make a table of values. Then plot the points and sketch the graph.

x	$f(x)$
–3	8.375
–2	8.75
–1	9.5
0	11
1	14
2	20



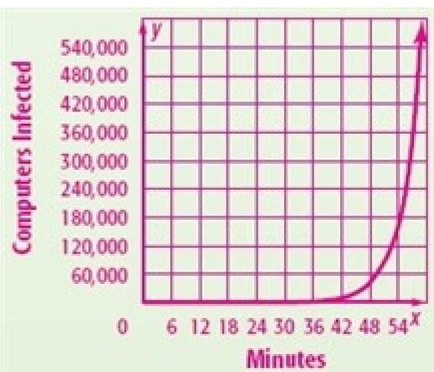
Domain = {all real numbers}; Range = $\{f(x) \mid f(x) > 8\}$

7. **CCSS SENSE-MAKING** A virus spreads through a network of computers such that each minute, 25% more computers are infected. If the virus began at only one computer, graph the function for the first hour of the spread of the virus.

SOLUTION:

a = 1 and r = 0.25
So, the equation that represents the situation is $y = 1.25^x$.
Make a table of values. Then plot the points and sketch the graph.

x	$f(x)$
10	$y = 1.25^{10} \approx 9$
20	$y = 1.25^{20} \approx 87$
30	$y = 1.25^{30} \approx 808$
40	$y = 1.25^{40} \approx 7523$
50	$y = 1.25^{50} \approx 70065$
60	$y = 1.25^{60} \approx 652530$



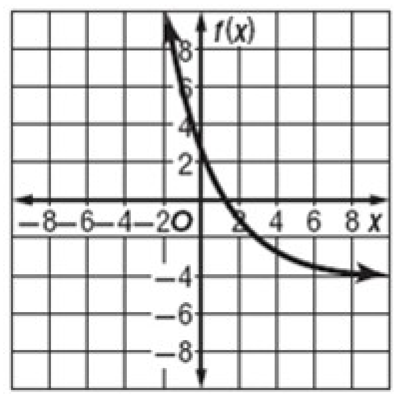
Graph each function. State the domain and range.

8. $f(x) = 2\left(\frac{2}{3}\right)^{x-3} - 4$

SOLUTION:

Make a table of values. Then plot the points and sketch the graph.

x	$f(x)$
−1	6.125
0	2.75
1	0.5
3	−2
6	−3.407
9	−3.824



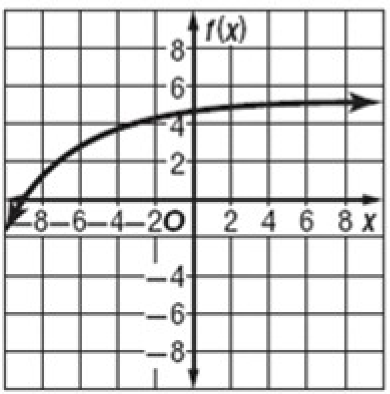
Domain = {all real numbers}; Range = {$f(x)$ | $f(x) > -4$}

9. $f(x) = -\frac{1}{2}\left(\frac{3}{4}\right)^{x+1} + 5$

SOLUTION:

Make a table of values. Then plot the points and sketch the graph.

x	$f(x)$
–8	1.254
–4	3.815
0	4.625
4	4.881
8	4.963



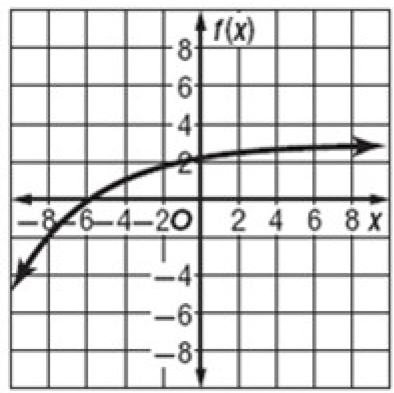
Domain = {all real numbers}; Range = {*f(x)* |*f(x)*< 5}

10. $f(x) = -\frac{1}{3}\left(\frac{4}{5}\right)^{x-4} + 3$

SOLUTION:

Make a table of values. Then plot the points and sketch the graph.

x	$f(x)$
–8	–1.851
–4	1.013
0	2.186
2	2.479
4	2.667
8	2.864



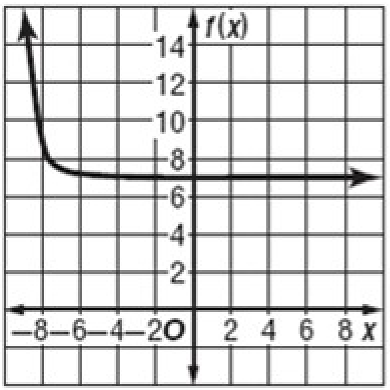
Domain = {all real numbers}; Range = {*f(x)* |*f(x)* < 3}

11. $f(x)=\frac{1}{8}\left(\frac{1}{4}\right)^{x+6}+7$

SOLUTION:

Make a table of values. Then plot the points and sketch the graph.

x	$f(x)$
−8	9
−4	7.008
0	7
4	7
8	7



Domain = {all real numbers}; Range = {$f(x)$ | $f(x) > 7$}

12. **FINANCIAL LITERACY** A new SUV depreciates in value each year by a factor of 15%. Draw a graph of the SUV's value for the first 20 years after the initial purchase.

SOLUTION:

a = 20,000 and r = 0.15.
So, the equation that represents the situation is $y=20000(0.85)^x$.

Make a table of values. Then plot the points and sketch the graph.

x	y
0	$y=20000(0.85)^0=20000$
4	$y=20000(0.85)^4\approx 10440$
8	$y=20000(0.85)^8\approx 5450$
12	$y=20000(0.85)^{12}\approx 2845$
16	$y=20000(0.85)^{16}\approx 1485$
20	$y=20000(0.85)^{20}\approx 775$

Graph of the SUV's value for the first 20 years after the initial purchase:

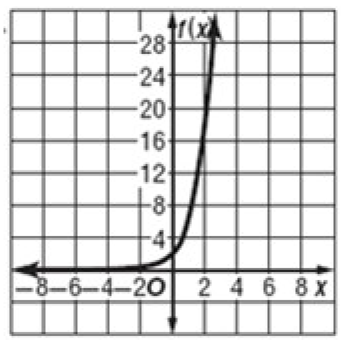
Graph each function. State the domain and range.

13. $f(x) = 2(3)^x$

SOLUTION:

Make a table of values. Then plot the points and sketch the graph.

x	$f(x)$
−2	0.22
−1	0.67
0	2
1	6
2	18
3	54



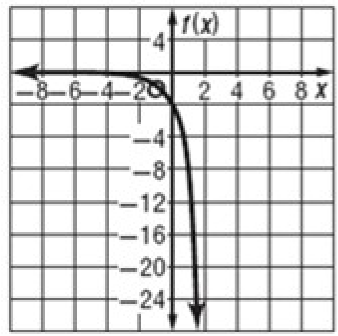
Domain = {all real numbers}; Range = {$f(x) \mid f(x) > 0$}

14. $f(x) = -2(4)^x$

SOLUTION:

Make a table of values. Then plot the points and sketch the graph.

x	$f(x)$
−3	−0.03
−2	−0.125
−1	−0.5
0	−2
1	−8
2	−32



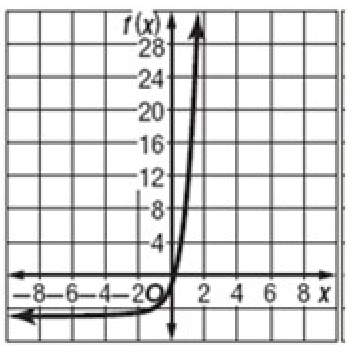
Domain = {all real numbers}; Range = {$f(x) \mid f(x) < 0$}

15. $f(x) = 4^{x+1} - 5$

SOLUTION:

Make a table of values. Then plot the points and sketch the graph.

x	$f(x)$
–3	–4.94
–2	–4.75
–1	–4
0	–1
1	11
2	59



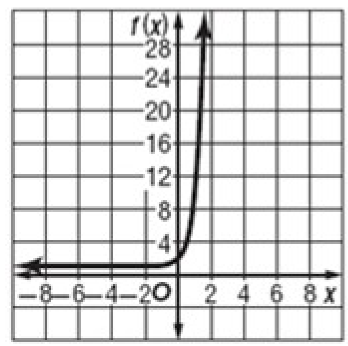
Domain = {all real numbers}; Range = {*f(x)* | *f(x)* > –5}

16. $f(x) = 3^{2x} + 1$

SOLUTION:

Make a table of values. Then plot the points and sketch the graph.

x	$f(x)$
–3	1
–2	1.012
–1	1.111
0	2
1	10
2	82



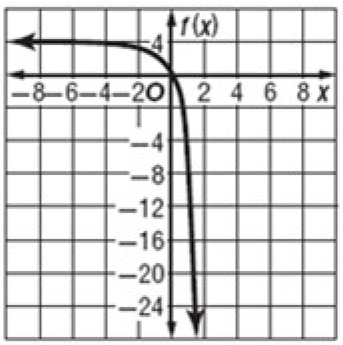
Domain = {all real numbers}; Range = {*f(x)* | *f(x)* > 1}

17. $f(x) = -0.4(3)^{x+2} + 4$

SOLUTION:

Make a table of values. Then plot the points and sketch the graph.

x	$f(x)$
−3	3.867
−2	3.6
−1	2.8
0	0.4
1	−6.8
2	−28.4



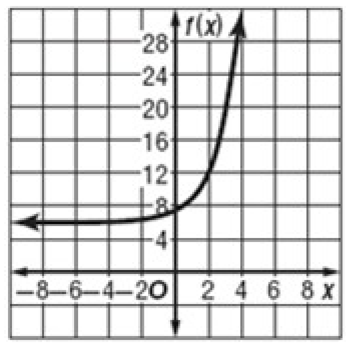
Domain = {all real numbers}; Range = {*f(x)* |*f(x)* < 4}

18. $f(x) = 1.5(2)^x + 6$

SOLUTION:

Make a table of values. Then plot the points and sketch the graph.

x	$f(x)$
−2	6.375
−1	6.75
0	7.5
1	9
2	12
3	18



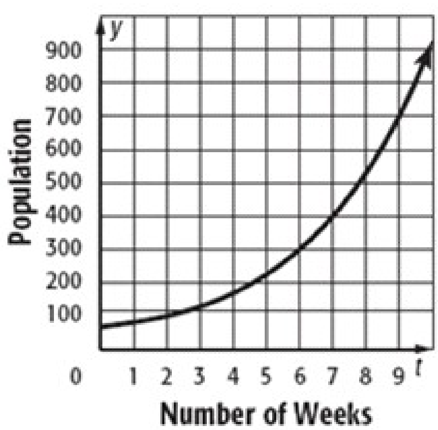
Domain = {all real numbers}; Range = {*f(x)* |*f(x)* > 6}

19. **SCIENCE** The population of a colony of beetles grows 30% each week for 10 weeks. If the initial population is 65 beetles, graph the function that represents the situation.

SOLUTION:

a = 65 and r = 0.3.
So, the equation that represents the situation is $y = 65(1.3)^t$.
Make a table of values. Then plot the points and sketch the graph.

t	y
0	$y = 65(1.3)^0 = 65$
2	$y = 65(1.3)^2 \approx 110$
4	$y = 65(1.3)^4 \approx 186$
6	$y = 65(1.3)^6 \approx 314$
8	$y = 65(1.3)^8 \approx 530$
10	$y = 65(1.3)^{10} \approx 896$



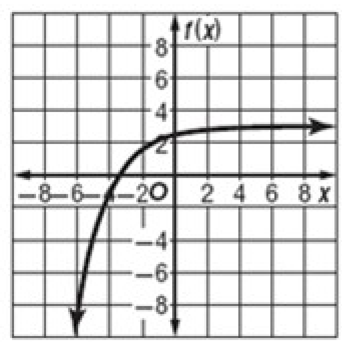
Graph each function. State the domain and range.

20. $f(x) = -4\left(\frac{3}{5}\right)^{x+4} + 3$

SOLUTION:

Make a table of values. Then plot the points and sketch the graph.

x	$f(x)$
–4	–1
–2	1.56
0	2.481
2	2.813
4	2.932
6	2.976



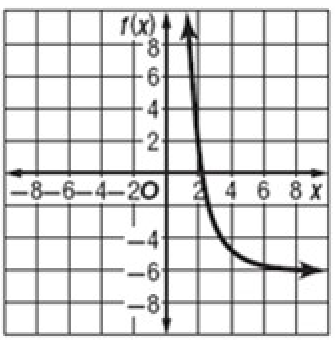
Domain = {all real numbers}; Range = {$f(x)$ | $f(x) < 3$}

21. $f(x) = 3\left(\frac{2}{5}\right)^{x-3} - 6$

SOLUTION:

Make a table of values. Then plot the points and sketch the graph.

x	$f(x)$
2	1.5
4	–4.8
6	–5.81
8	–5.97



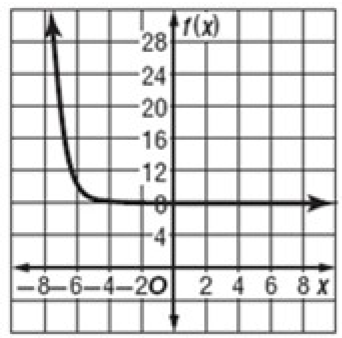
Domain = {all real numbers}; Range = {*f(x)* |*f(x)* > –6}

22. $f(x) = \frac{1}{2}\left(\frac{1}{5}\right)^{x+5} + 8$

SOLUTION:

Make a table of values. Then plot the points and sketch the graph.

x	$f(x)$
–8	70.5
–6	10.5
–4	8.1
–2	8.004
0	8



Domain = {all real numbers}; Range = {*f(x)* |*f(x)* > 8}

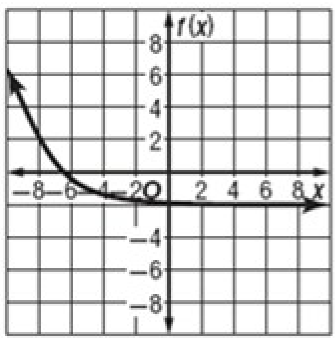
7-1 Graphing Exponential Functions

23. $f(x) = \frac{3}{4}\left(\frac{2}{3}\right)^{x+4} - 2$

SOLUTION:

Make a table of values. Then plot the points and sketch the graph.

x	$f(x)$
–8	1.80
–6	–0.3125
–4	–1.25
–2	–1.67
0	–1.851



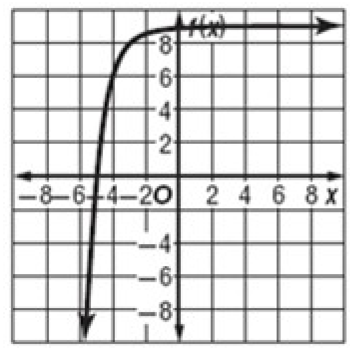
Domain = {all real numbers}; Range = {*f(x)* |*f(x)* > –2}

24. $f(x) = -\frac{1}{2}\left(\frac{3}{8}\right)^{x+2} + 9$

SOLUTION:

Make a table of values. Then plot the points and sketch the graph.

x	$f(x)$
–6	–16.28
–4	5.44
–2	8.5
0	8.93
2	8.99

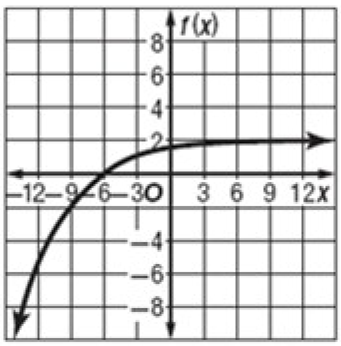


Domain = {all real numbers}; Range = {*f(x)* |*f(x)* < 9}

25. $f(x) = -\frac{5}{4}\left(\frac{4}{5}\right)^{x+4} + 2$

SOLUTION:
Make a table of values. Then plot the points and sketch the graph.

x	$f(x)$
−9	−1.81
−6	0.047
−3	1
0	1.488
3	1.74
6	1.87



Domain = {all real numbers}; Range = {*f(x)* |*f(x)* < 2}

26. **ATTENDANCE** The attendance for a basketball team declined at a rate of 5% per game throughout a losing season. Graph the function modeling the attendance if 15 home games were played and 23,500 people were at the first game.

SOLUTION:
a = 23,500 and *r* = 0.05.
So, the equation that represents the situation is $y = 23500(0.95)^x$.
Make a table of values. Then plot the points and sketch the graph.

x	y
0	$y = 23500(0.95)^0 = 23500$
3	$y = 23500(0.95)^3 \approx 20148$
6	$y = 23500(0.95)^6 \approx 17275$
9	$y = 23500(0.95)^9 \approx 14811$
12	$y = 23500(0.95)^{12} \approx 12698$
15	$y = 23500(0.95)^{15} \approx 10887$

27. **PHONES** The function $P(x) = 2.28(0.9^x)$ can be used to model the number of pay phones in millions x years since 1999.
a. Classify the function representing this situation as either exponential *growth* or *decay*, and identify the growth or decay factor. Then graph the function.
b. Explain what the $P(x)$-intercept and the asymptote represent in this situation.

SOLUTION:

a. Since the number of pay phones is decreasing, this is an example of exponential decay. The rate of decay is 0.9 because it is the base of the exponential expression.

Make a table of values for $P(x) = 2.28(0.9)^x$. Then plot the points and sketch the graph.

x	$P(x)$
0	2.28
2	1.845
4	1.496
6	1.212
8	0.981

b. The $P(x)$-intercept represents the number of pay phones in 1999. The asymptote is the x-axis. The number of pay phones can approach 0, but will never equal 0. This makes sense as there will probably always be a need for some pay phones.

28. **HEALTH** Each day, 10% of a certain drug dissipates from the system.
a. Classify the function representing this situation as either exponential *growth* or *decay*, and identify the growth or decay factor. Then graph the function.
b. How much of the original amount remains in the system after 9 days?
c. If a second dose should not be taken if more than 50% of the original amount is in the system, when should the label say it is safe to redose? Design the label and explain your reasoning.

SOLUTION:

a. Since the amount of the drug in the system is decreasing, this is an example of exponential decay. Use the equation form $y = a(1 - r)^x$ with $a = 100$ and $r = 0.1$ to model the amount of drug still in the system. Then the equation that represents the situation is $y = 100(0.9)^x$. The rate of decay is 0.9 because it is the base of the exponential expression.

Make a table of values. Then plot the points and sketch the graph.

x	y
0	$y = 100(0.9)^0 = 100$
2	$y = 100(0.9)^2 = 81$
4	$y = 100(0.9)^4 \approx 66$
6	$y = 100(0.9)^6 \approx 53$
8	$y = 100(0.9)^8 \approx 43$
10	$y = 100(0.9)^{10} \approx 35$

b. After the 9th day a little less than 40% of the original amount remains in the system.
c. Sample answer: From the graph, a little less than 50% of the original amount is still in the system after

7 days. So, it is safe to redose on the 7th day.
.

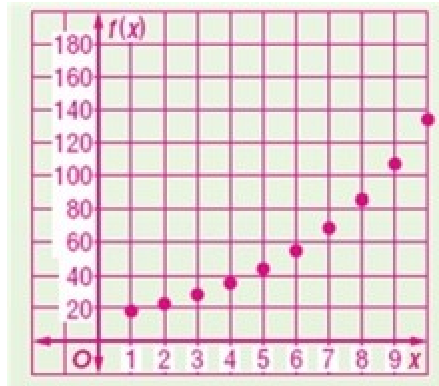
29. **CCSS REASONING** A sequence of numbers follows a pattern in which the next number is 125% of the previous number. The first number in the pattern is 18.
a. Write the function that represents the situation.
b. Classify the function as either exponential *growth* or *decay*, and identify the growth or decay factor. Then graph the function for the first 10 numbers.
c. What is the value of the tenth number? Round to the nearest whole number.

SOLUTION:
a. Write an exponential function that has an initial value of 18, a base of 1.25, and an exponent of x – 1 where x is the position of the number in the list. So, the function representing this situation is $f(x) = 18(1.25)^{x-1}$.

b. Since the numbers will be increasing, this is an example of exponential growth. The rate of growth is 1.25 because it is the base of the exponent.

Make a table of values of $f(x) = 18(1.25)^{x-1}$. Then plot the points and sketch the graph.

x	$f(x)$
0	$y = 18(1.25)^{0-1} \approx 14$
2	$y = 18(1.25)^{2-1} \approx 23$
4	$y = 18(1.25)^{4-1} \approx 35$
6	$y = 18(1.25)^{6-1} \approx 55$
8	$y = 18(1.25)^{8-1} \approx 86$
9	$y = 18(1.25)^{9-1} \approx 107$

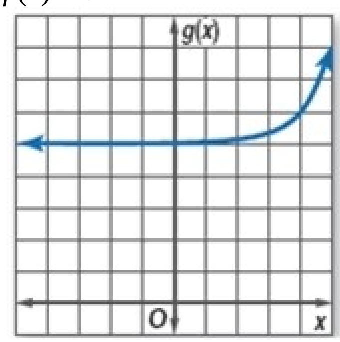


c. Substitute $x = 10$ in the function and simplify.

$$f(10) = 18(1.25)^{10-1}$$
$$= 18(1.25)^9$$
$$\approx 134$$

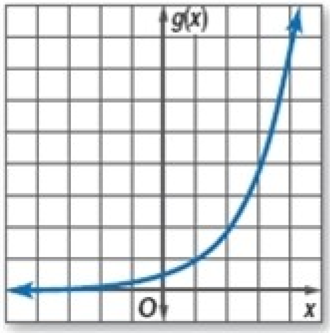
For each graph, $f(x)$ is the parent function and $g(x)$ is a transformation of $f(x)$. Use the graph to determine the equation of $g(x)$.

30. $f(x) = 3^x$



SOLUTION:
The graph of $f(x)$ is translated 5 units up and 4 units right. Here, $k = 5$ and $h = 4$.
So, $g(x) = 3^{x-4} + 5$.

31. $f(x) = 2^x$

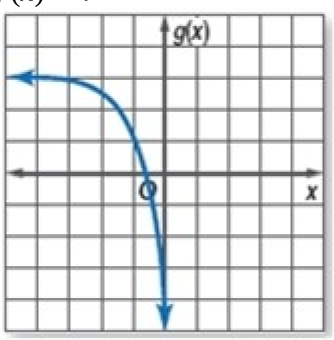


SOLUTION:

The graph of $f(x)$ is compressed 4 units and translated 3 units right. Here, $a = 4$ and $h = 3$.

So, $g(x) = 4(2)^{x-3}$ or $g(x) = \frac{1}{2}(2^x)$.

32. $f(x) = 4^x$



SOLUTION:

The graph of $f(x)$ is reflected in the x-axis and expanded.

The graph is translated one unit left and 3 units up.

Here, $a = –2$, $h = –1$ and $k = 3$.

So, $g(x) = -2(4)^{x+1} + 3$.

33. **MULTIPLE REPRESENTATIONS** In this problem, you will use the tables below for exponential functions $f(x)$, $g(x)$, and $h(x)$.

x	−1	0	1	2	3	4	5
f(x)	2.5	2	1	−1	−5	−13	−29

x	−1	0	1	2	3	4	5
g(x)	5	11	23	47	95	191	383

x	−1	0	1	2	3	4	5
h(x)	3	2.5	2.25	2.125	2.0625	2.0313	2.0156

a. GRAPHICAL Graph the functions for $-1 \le x \le 5$ on separate graphs.

b. LOGICAL Which function(s) has a negative coefficient, a? Explain your reasoning.

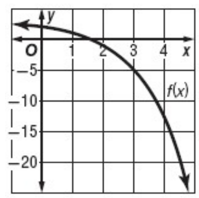
c. LOGICAL Which function(s) is translated to the left?

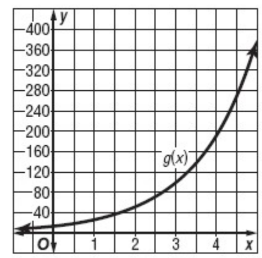
d. ANALYTICAL Determine which functions are growth models and which are decay models.

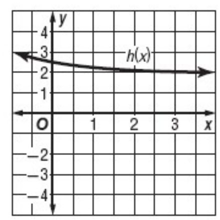
SOLUTION:

a.

Plot the points given in the table and sketch the graph of $f(x)$, $g(x)$ and $h(x)$.







b.

Sample answer: $f(x)$; the graph of $f(x)$ is a reflection along the x-axis and the output values in the table are negative.

c.

$g(x)$ and $h(x)$ are translated to the left.

d.

Sample answer: $f(x)$ and $g(x)$ are growth functions and $h(x)$ is a decay function; The absolute value of the output is increasing for the growth functions and decreasing for the decay function.

34. **REASONING** Determine whether each statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

a. An exponential function of the form $y = ab^{x-h} + k$ has a y-intercept.

b. An exponential function of the form $y = ab^{x-h} + k$ has an x-intercept.

c. The function $f(x) = |b|^x$ is an exponential growth function if b is an integer.

SOLUTION:

a. Always; Sample answer: The domain of exponential functions is all real numbers, so $(0, y)$ always exists.

b. Sometimes; Sample answer: The graph of an exponential function crosses the x-axis when $k < 0$.

c. Sometimes; Sample answer: The function is not exponential if $b = 1$ or -1.

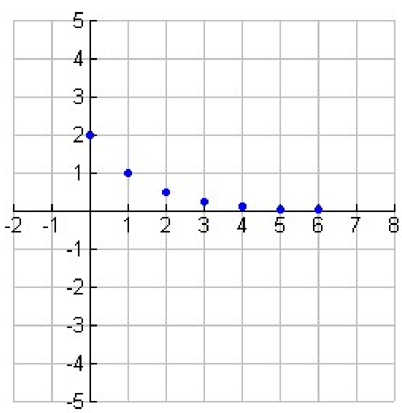
35. **CCSS CRITIQUE** Vince and Grady were asked to graph the following functions. Vince thinks they are the same, but Grady disagrees. Who is correct? Explain your reasoning.

x	y
0	2
1	1
2	0.5
3	0.25
4	0.125
5	0.0625
6	0.03125

an exponential function with rate of decay of $\frac{1}{2}$ and an initial amount of 2

SOLUTION:

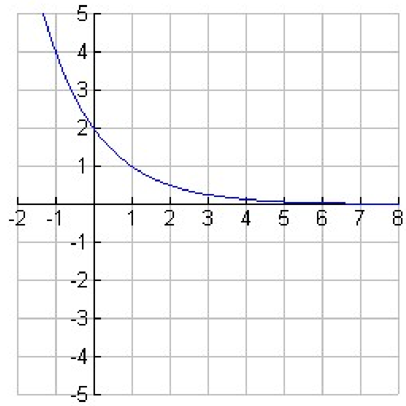
First plot the points in the table.



Next, find and graph an equation that matches the description given: an exponential function with a rate of decay of 1/2 and an initial amount of 2. Exponential decay can be modeled by the function $A(t) = a(1 - r)^t$ where r is the rate of decay and a is the initial amount.

$$A(t) = a(1 - r)^t$$
$$A(t) = 2\left(1 - \frac{1}{2}\right)^t$$

Graph this function on the coordinate plane.



Compare the two graphs. The graph of the exponential decay function is the same as the graph of the ordered pairs. Vince is correct.

36. **CHALLENGE** A substance decays 35% each day. After 8 days, there are 8 milligrams of the substance remaining. How many milligrams were there initially?

SOLUTION:

Substitute 8 for y, 0.35 for r and 8 for x in the equation $y = a(1 - r)^x$ and solve for x.

$$8 = a(1 - 0.35)^8$$
$$8 = a(0.65)^8$$
$$\frac{8}{0.65^8} = a$$
$$251 \approx a$$

There were about 251 mg initially.

37. **OPEN ENDED** Give an example of a value of b for which $f(x) = \left(\frac{8}{b}\right)^x$ represents exponential decay.

SOLUTION:

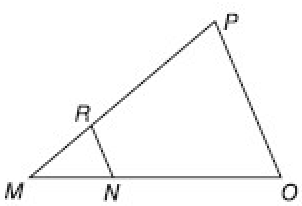
Sample answer: For $b = 10$, the given function represents exponential decay.

38. **WRITING IN MATH** Write the procedure for transforming the graph of $g(x) = b^x$ to the graph $f(x) = ab^{x-h} + k$

SOLUTION:

Sample answer: The parent function, $g(x) = b^x$, is stretched if a is greater than 1 or compressed if a is less than 1 and greater than 0. The parent function is translated up k units if k is positive and down $|k|$ units if k is negative. The parent function is translated h units to the right if h is positive and $|h|$ units to the left if h is negative.

39. **GRIDDED RESPONSE** In the figure, $\overline{PO} \parallel \overline{RN}$, $ON = 12$, $MN = 6$, and $RN = 4$. What is the length of $\overline{PO}$?



SOLUTION:

ΔMRN and ΔMPO are similar triangles.
Find the similarity ratio.

$$\text{Similarity ratio} = \frac{MN}{MO} = \frac{6}{18} = \frac{1}{3}$$

Length of $\overline{PO}$:

$$\frac{1}{3} = \frac{\overline{RN}}{\overline{OP}}$$
$$\frac{1}{3} = \frac{4}{\overline{OP}}$$
$$\overline{OP} = 12$$

40. Ivan has enough money to buy 12 used CDs. If the cost of each CD was \$0.20 less, Ivan could buy 2 more CDs. How much money does Ivan have to spend on CDs?

A \$16.80
B \$16.40
C \$15.80
D \$15.40

SOLUTION:

Let x be the cost of a CD.
The equation that represents the situation is $12x = 14(x - 0.20)$.

$$12x = 14(x - 0.20)$$
$$12x = 14x - 2.8$$
$$-2x = -2.8$$
$$x = 1.4$$
$$\text{The cost of 12 CDs} = 12 \times 1.4 = 16.8$$

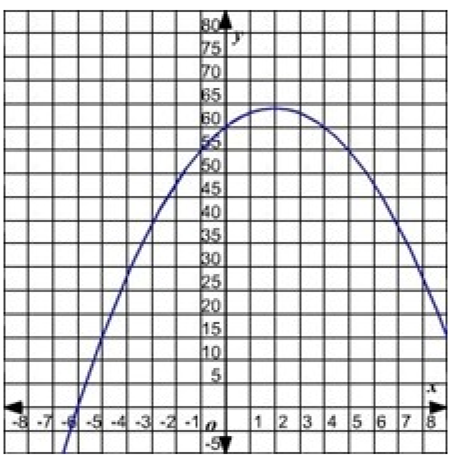
Ivan has to spend \$16.80 on CDs.
A is the correct choice.

41. One hundred students will attend the fall dance if tickets cost $30 each. For each $5 increase in price, 10 fewer students will attend. What price will deliver the maximum dollar sales?
F $30
G $35
H $40
J $45

SOLUTION:

Let $f(x)$ represent the number of students attending the fall dance and x be the price increase, in multiples of $5.
The function that represents the situation is:

$$\begin{aligned} f(x) &= (30+5x)(100-10x) \\ &= 3000 - 300x + 500x - 50x^2 \\ &= -50x^2 + 200x + 3000 \end{aligned}$$



The function attains its maximum at $x = 2$.
So, the price of the ticket that delivers the maximum dollar sale is 30 + 5(2) = $40.
H is the correct choice.

42. **SAT/ACT** Javier mows a lawn in 2 hours. Tonya mows the same lawn in 1.5 hours. About how many minutes will it take to mow the lawn if Javier and Tonya work together?
A 28 minutes
B 42 minutes
C 51 minutes
D 1.2 hours
E 1.4 hours

SOLUTION:

Javier mows $\frac{1}{2}$ the lawn in 1 hour.

Tanya mows $\frac{2}{3}$ the lawn in 1 hour.

Working together, Javier and Tanya mow $\frac{1}{2} + \frac{2}{3} = \frac{7}{6}$ lawns per hour.

So, they mow the lawn in $\frac{1}{\left(\frac{7}{6}\right)} = \frac{6}{7}$ of an hour.

$$\frac{6}{7} \approx 0.857$$

Multiply by 60 to convert to minutes.

$$0.857 \text{ hr} \times \frac{60 \text{ min}}{1 \text{ hr}} \approx 51 \text{ min}$$

The correct answer is C.

Solve each equation or inequality.

43. $\sqrt{y+5} = \sqrt{2y-3}$

SOLUTION:

$$\begin{aligned} \left(\sqrt{y+5}\right)^2 &= \left(\sqrt{2y-3}\right)^2 \\ y+5 &= 2y-3 \\ 8 &= y \end{aligned}$$

44. $\sqrt{y+1}+\sqrt{y-4}=5$

SOLUTION:

$$\begin{aligned}\sqrt{y+1}+\sqrt{y-4}&=5\\ \left(\sqrt{y+1}\right)^2&=\left(5-\sqrt{y-4}\right)^2\\ y+1&=25+y-4-10\sqrt{y-4}\\ (-20)^2&=\left(-10\sqrt{y-4}\right)^2\\ 400&=100y-400\\ 8&=y\end{aligned}$$

45. $10-\sqrt{2x+7}\le 3$

SOLUTION:

$$\begin{aligned}10-\sqrt{2x+7}&\le 3\\ -\sqrt{2x+7}&\le -7\\ \left(\sqrt{2x+7}\right)^2&\ge 7^2\\ 2x+7&\ge 49\\ x&\ge 21\end{aligned}$$

46. $6+\sqrt{3y+4}<6$

SOLUTION:

$$\begin{aligned}6+\sqrt{3y+4}&<6\\ \sqrt{3y+4}&<0\end{aligned}$$

The square root of $3y + 4$ cannot be negative, so there is no solution.

47. $\sqrt{d+3}+\sqrt{d+7}>4$

SOLUTION:

$$\begin{aligned}\sqrt{d+3}+\sqrt{d+7}&>4\\ \left(\sqrt{d+3}\right)^2&>\left(4-\sqrt{d+7}\right)^2\\ d+3&>16+d+7-8\sqrt{d+7}\\ -20&>-8\sqrt{d+7}\\ (20)^2&<\left(8\sqrt{d+7}\right)^2\\ 400&<64(d+7)\\ -48&<64d\\ -\frac{3}{4}&<d\end{aligned}$$

48. $\sqrt{2x+5}-\sqrt{9+x}>0$

SOLUTION:

$$\begin{aligned}\sqrt{2x+5}-\sqrt{9+x}&>0\\ \left(\sqrt{2x+5}\right)^2&>\left(\sqrt{9+x}\right)^2\\ 2x+5&>9+x\\ x&>4\end{aligned}$$

Simplify.

49. $\dfrac{1}{y^{\frac{2}{5}}}$

SOLUTION:

$$\frac{1}{y^{\frac{2}{5}}}\cdot\frac{y^{\frac{3}{5}}}{y^{\frac{3}{5}}}=\frac{y^{\frac{3}{5}}}{1}$$

50. $\dfrac{xy}{\sqrt[3]{z}}$

SOLUTION:

$$\begin{aligned}\frac{xy}{\sqrt[3]{z}}&=\frac{xy}{z^{\frac{1}{3}}}\\ &=\frac{xy}{z^{\frac{1}{3}}}\cdot\frac{z^{\frac{2}{3}}}{z^{\frac{2}{3}}}\\ &=\frac{xyz^{\frac{2}{3}}}{z}\end{aligned}$$

51. $\dfrac{3x+4x^2}{x^{-\frac{2}{3}}}$

SOLUTION:

$$\begin{aligned}&\frac{3x+4x^2}{x^{-\frac{2}{3}}}\cdot\frac{x^{\frac{2}{3}}}{x^{\frac{2}{3}}}\\ &=\frac{3x\left(x^{\frac{2}{3}}\right)+4x^2\left(x^{\frac{2}{3}}\right)}{x^{-\frac{2}{3}+\frac{2}{3}}}\\ &=3x^{\frac{5}{3}}+4x^{\frac{8}{3}}\end{aligned}$$

52. $\sqrt[6]{27x^3}$

SOLUTION:

$$\begin{aligned}\sqrt[6]{27x^3} &= \left(3^3x^3\right)^{\frac{1}{6}}\\ &= 3^{\frac{1}{2}}x^{\frac{1}{2}}\\ &= \sqrt{3x}\end{aligned}$$

53. $\dfrac{\sqrt[4]{27}}{\sqrt[4]{3}}$

SOLUTION:

$$\begin{aligned}\frac{\sqrt[4]{27}}{\sqrt[4]{3}} &= \sqrt[4]{\frac{27}{3}}\\ &= \sqrt[4]{9}\\ &= \left(3^2\right)^{\frac{1}{4}}\\ &= \sqrt{3}\end{aligned}$$

54. $\dfrac{a^{-\frac{1}{2}}}{6a^{\frac{1}{3}}.a^{-\frac{1}{4}}}$

SOLUTION:

$$\begin{aligned}\frac{a^{-\frac{1}{2}}}{6a^{\frac{1}{3}}.a^{-\frac{1}{4}}} &= \frac{a^{-\frac{1}{2}}}{6a^{\frac{1}{3}-\frac{1}{4}}}\\ &= \frac{a^{-\frac{1}{2}}}{6a^{\frac{1}{12}}}\cdot\frac{a^{\frac{11}{12}}}{a^{\frac{11}{12}}}\\ &= \frac{a^{\frac{5}{12}}}{6a}\end{aligned}$$

55. **FOOTBALL** The path of a football thrown across a field is given by the equation $y = -0.005x^2 + x + 5$, where x represents the distance, in feet, the ball has traveled horizontally and y represents the height, in feet, of the ball above ground level. About how far has the ball traveled horizontally when it returns to ground level?

SOLUTION:

Substitute 0 for y in the equation and solve for x.

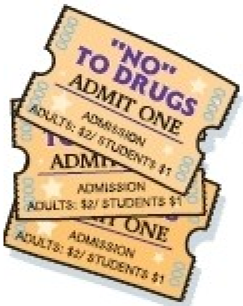
$$-0.005x^2 + x + 5 = 0$$

Substitute these values into the Quadratic Formula and simplify.

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\\ x &= \frac{-1 \pm \sqrt{(1)^2 - 4(-0.005)(5)}}{2(-0.005)}\\ &= \frac{-1 \pm \sqrt{1.1}}{-0.01}\\ &\approx -4.88 \text{ or } 204.88\end{aligned}$$

The distance cannot have negative value. So the ball has traveled about 204.88 ft horizontally.

56. **COMMUNITY SERVICE** A drug awareness program is being presented at a theater that seats 300 people. Proceeds will be donated to a local drug information center. If every two adults must bring at least one student, what is the maximum amount of money that can be raised?



SOLUTION:

Every two adults must bring at least one student, and there are a total of 300 seats available.
So, the maximum occupancy is 200 adults and 100 students.
Maximum amount = 200($2) + 100($1) = $500.

Simplify. Assume that no variable equals 0.

57. $f^{-7} \cdot f^{4}$

SOLUTION:

$$\begin{aligned} f^{-7} \cdot f^{4} &= f^{-7+4} \\ &= f^{-3} \\ &= \frac{1}{f^{3}} \end{aligned}$$

58. $(3x^2)^3$

SOLUTION:

$$\begin{aligned} (3x^2)^3 &= 3^3 \cdot (x^2)^3 \\ &= 27x^6 \end{aligned}$$

59. $(2y)(4xy^3)$

SOLUTION:

$$(2y)(4xy^3) = 8xy^4$$

60. $\left(\frac{3}{5}c^2 f\right)\left(\frac{4}{3}cd\right)^2$

SOLUTION:

$$\begin{aligned} \left(\frac{3}{5}c^2 f\right)\left(\frac{4}{3}cd\right)^2 &= \left(\frac{3}{5}c^2 f\right)\left(\frac{16}{9}c^2 d^2\right) \\ &= \frac{16}{15}c^4 d^2 f \end{aligned}$$