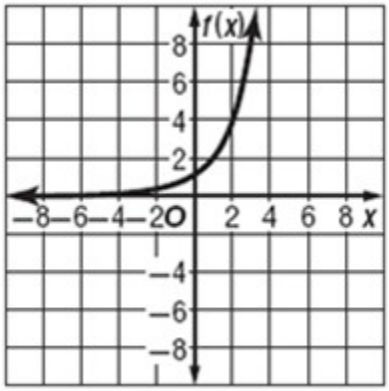
7-1 Graphing Exponential Functions

Graph each function. State the domain and range.

1. $f(x) = 2^x$

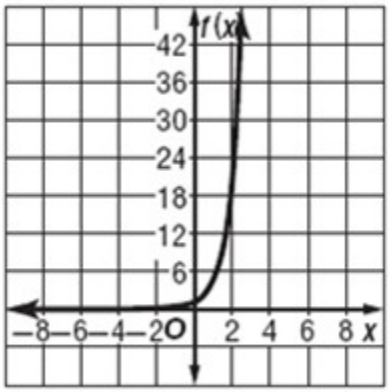
ANSWER:



D = {all real numbers};R = $\{f(x) \mid f(x) > 0\}$

2. $f(x) = 5^x$

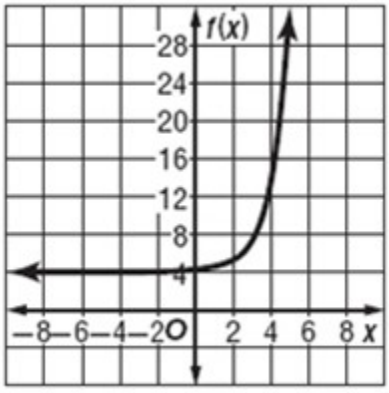
ANSWER:



D = {all real numbers};R = $\{f(x) \mid f(x) > 0\}$

3. $f(x) = 3^{x-2} + 4$

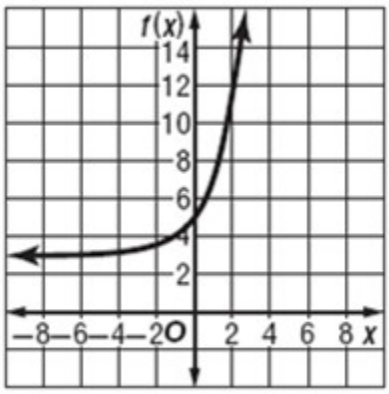
ANSWER:



D = {all real numbers};R = $\{f(x) \mid f(x) > 4\}$

4. $f(x) = 2^{x+1} + 3$

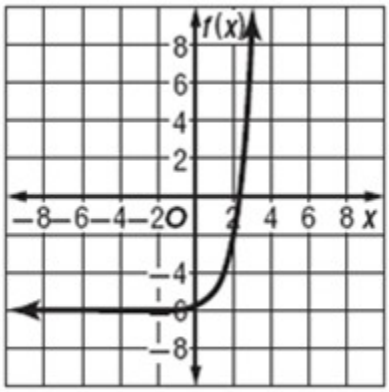
ANSWER:



D = {all real numbers};R = $\{f(x) \mid f(x) > 3\}$

5. $f(x) = 0.25(4)^x - 6$

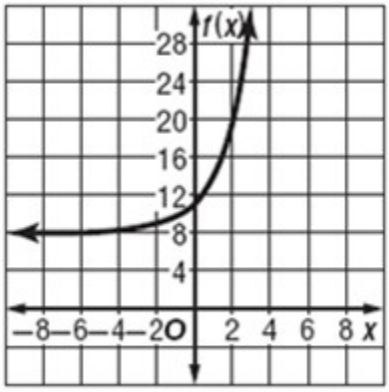
ANSWER:



D = {all real numbers};R = $\{f(x) \mid f(x) > -6\}$

6. $f(x) = 3(2)^x + 8$

ANSWER:



D = {all real numbers};R = $\{f(x) \mid f(x) > 8\}$

7-1 Graphing Exponential Functions

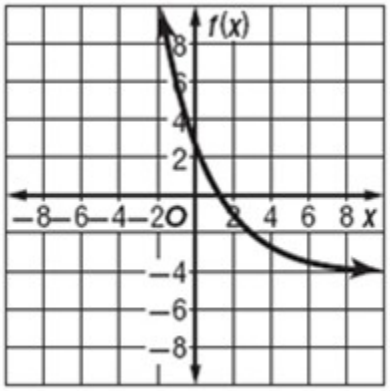
7. **CCSS SENSE-MAKING** A virus spreads through a network of computers such that each minute, 25% more computers are infected. If the virus began at only one computer, graph the function for the first hour of the spread of the virus.

ANSWER:

Graph each function. State the domain and range.

8. $f(x) = 2\left(\frac{2}{3}\right)^{x-3} - 4$

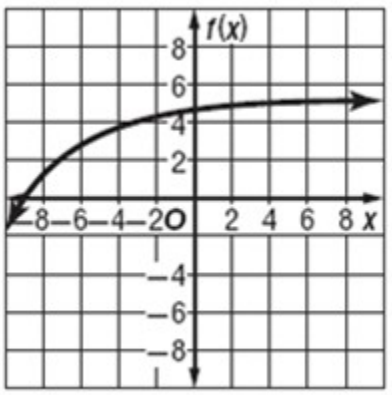
ANSWER:



D = {all real numbers};R = {$f(x) \mid f(x) > -4$}

9. $f(x) = -\frac{1}{2}\left(\frac{3}{4}\right)^{x+1} + 5$

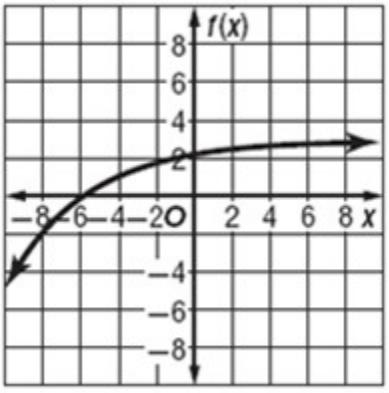
ANSWER:



D = {all real numbers};R = {$f(x) \mid f(x) < 5$}

10. $f(x) = -\frac{1}{3}\left(\frac{4}{5}\right)^{x-4} + 3$

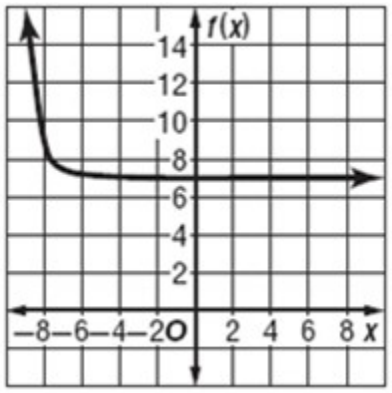
ANSWER:



D = {all real numbers};
R = {$f(x) \mid f(x) < 3$}

11. $f(x) = \frac{1}{8}\left(\frac{1}{4}\right)^{x+6} + 7$

ANSWER:



D = {all real numbers};
R = {$f(x)$ | $f(x) > 7$}

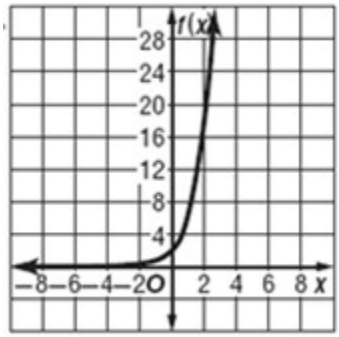
12. **FINANCIAL LITERACY** A new SUV depreciates in value each year by a factor of 15%. Draw a graph of the SUV's value for the first 20 years after the initial purchase.

ANSWER:

Graph each function. State the domain and range.

13. $f(x) = 2(3)^x$

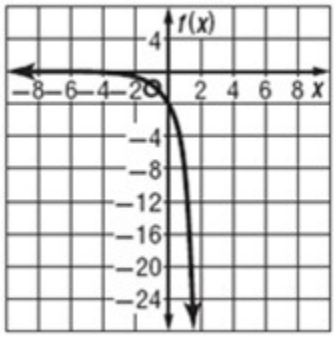
ANSWER:



D = {all real numbers};R = {$f(x)$ | $f(x) > 0$}

14. $f(x) = -2(4)^x$

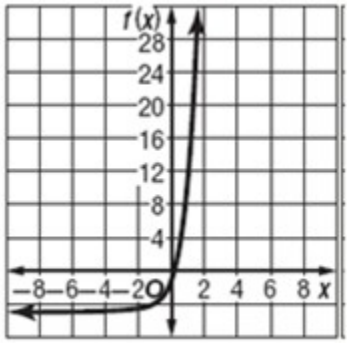
ANSWER:



D = {all real numbers};R = {$f(x)$ | $f(x) < 0$}

15. $f(x) = 4^{x+1} - 5$

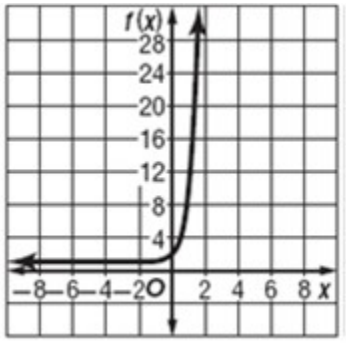
ANSWER:



D = {all real numbers};R = {$f(x)$ | $f(x) > -5$}

16. $f(x) = 3^{2x} + 1$

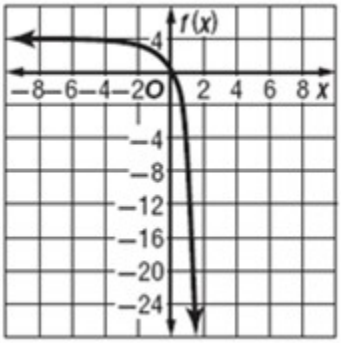
ANSWER:



D = {all real numbers};R = {$f(x)$ | $f(x) > 1$}

17. $f(x) = -0.4(3)^{x+2} + 4$

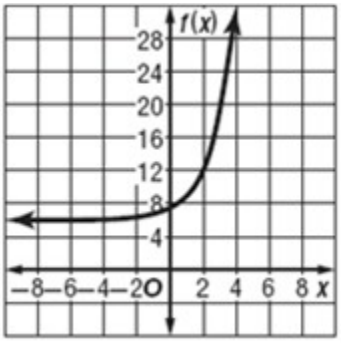
ANSWER:



D = {all real numbers};R = {$f(x)$ | $f(x) < 4$}

18. $f(x) = 1.5(2)^{x} + 6$

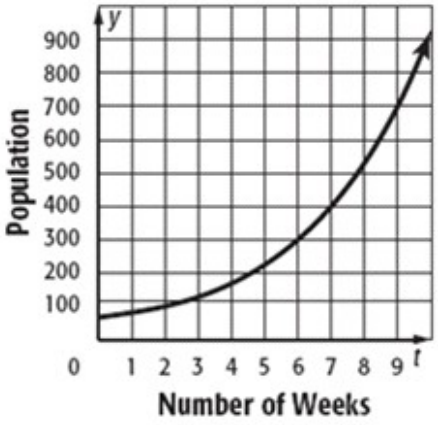
ANSWER:



D = {all real numbers};R = {$f(x)$ | $f(x) > 6$}

19. **SCIENCE** The population of a colony of beetles grows 30% each week for 10 weeks. If the initial population is 65 beetles, graph the function that represents the situation.

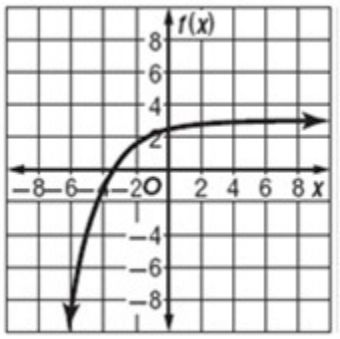
ANSWER:



Graph each function. State the domain and range.

20. $f(x) = -4\left(\frac{3}{5}\right)^{x+4} + 3$

ANSWER:

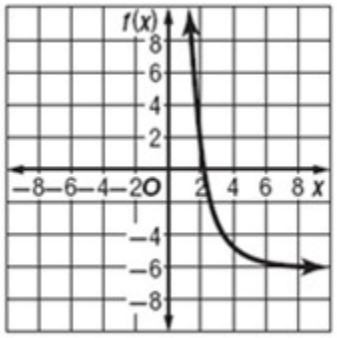


D = {all real numbers};R = {$f(x)$ | $f(x) < 3$}

7-1 Graphing Exponential Functions

21. $f(x)=3\left(\frac{2}{5}\right)^{x-3}-6$

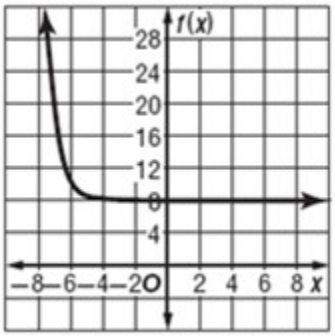
ANSWER:



D = {all real numbers};R = {$f(x)$ |$f(x) > -6$}

22. $f(x)=\frac{1}{2}\left(\frac{1}{5}\right)^{x+5}+8$

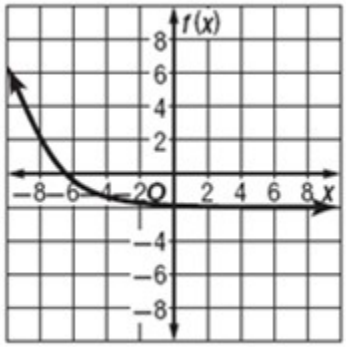
ANSWER:



D = {all real numbers};R = {$f(x)$ |$f(x) > 8$}

23. $f(x)=\frac{3}{4}\left(\frac{2}{3}\right)^{x+4}-2$

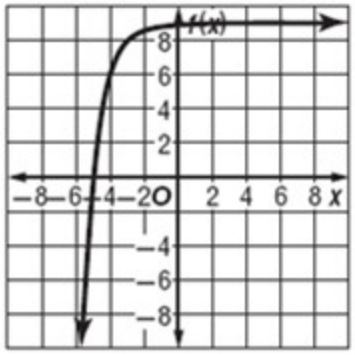
ANSWER:



D = {all real numbers};R = {$f(x)$ |$f(x) > -2$}

24. $f(x)=-\frac{1}{2}\left(\frac{3}{8}\right)^{x+2}+9$

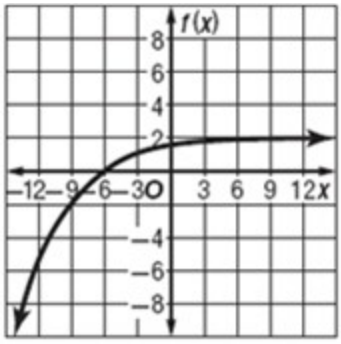
ANSWER:



D = {all real numbers};R = {$f(x)$ |$f(x) < 9$}

25. $f(x)=-\frac{5}{4}\left(\frac{4}{5}\right)^{x+4}+2$

ANSWER:



D = {all real numbers};R = {$f(x)$ |$f(x) < 2$}

26. **ATTENDANCE** The attendance for a basketball team declined at a rate of 5% per game throughout a losing season. Graph the function modeling the attendance if 15 home games were played and 23,500 people were at the first game.

ANSWER:

27. **PHONES** The function $P(x) = 2.28(0.9^x)$ can be used to model the number of pay phones in millions x years since 1999.
a. Classify the function representing this situation as either exponential *growth* or *decay*, and identify the growth or decay factor. Then graph the function.
b. Explain what the $P(x)$-intercept and the asymptote represent in this situation.

ANSWER:

a. decay; 0.9

b. The $P(x)$-intercept represents the number of pay phones in 1999. The asymptote is the x-axis. The number of pay phones can approach 0, but will never equal 0. This makes sense as there will probably always be a need for some pay phones.

28. **HEALTH** Each day, 10% of a certain drug dissipates from the system.
a. Classify the function representing this situation as either exponential *growth* or *decay*, and identify the growth or decay factor. Then graph the function.
b. How much of the original amount remains in the system after 9 days?
c. If a second dose should not be taken if more than 50% of the original amount is in the system, when should the label say it is safe to redose? Design the label and explain your reasoning.

ANSWER:

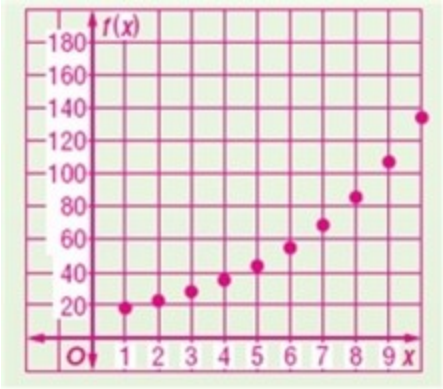
a. decay; 0.9

b. a little less than 40%
c. Sample answer: The 7th day; see students' work.

29. **CCSS REASONING** A sequence of numbers follows a pattern in which the next number is 125% of the previous number. The first number in the pattern is 18.
a. Write the function that represents the situation.
b. Classify the function as either exponential *growth* or *decay*, and identify the growth or decay factor. Then graph the function for the first 10 numbers.
c. What is the value of the tenth number? Round to the nearest whole number.

ANSWER:

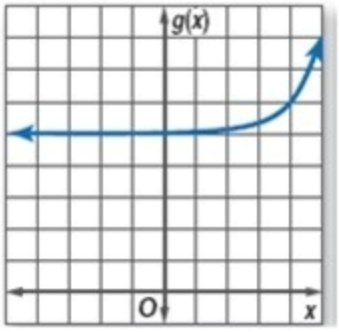
a. $f(x) = 18(1.25)^{x-1}$
b. growth; 1.25



c. 134

For each graph, $f(x)$ is the parent function and $g(x)$ is a transformation of $f(x)$. Use the graph to determine the equation of $g(x)$.

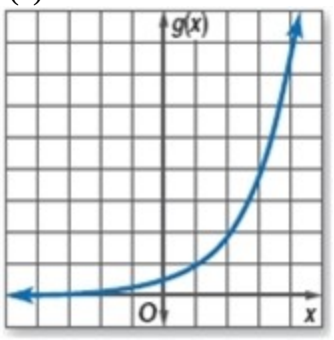
30. $f(x) = 3^x$



ANSWER:

$g(x) = 3^{x-4} + 5$

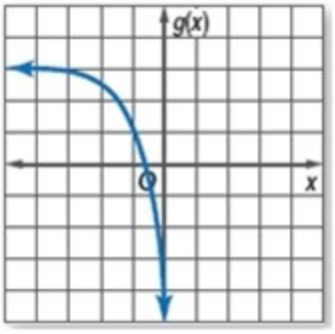
31. $f(x) = 2^x$



ANSWER:

$g(x) = 4(2)^{x-3}$ or $g(x) = \frac{1}{2}(2^x)$

32. $f(x) = 4^x$



ANSWER:

$g(x) = -2(4)^{x+1} + 3$

33. **MULTIPLE REPRESENTATIONS** In this problem, you will use the tables below for exponential functions $f(x)$, $g(x)$, and $h(x)$.

x	−1	0	1	2	3	4	5
$f(x)$	2.5	2	1	−1	−5	−13	−29

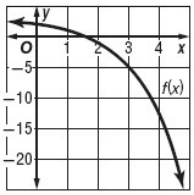
x	−1	0	1	2	3	4	5
$g(x)$	5	11	23	47	95	191	383

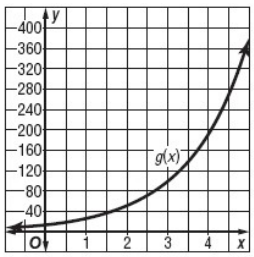
x	−1	0	1	2	3	4	5
$h(x)$	3	2.5	2.25	2.125	2.0625	2.0313	2.0156

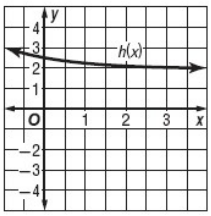
a. GRAPHICAL Graph the functions for $-1 \le x \le 5$ on separate graphs.
b. LOGICAL Which function(s) has a negative coefficient, a? Explain your reasoning.
c. LOGICAL Which function(s) is translated to the left?
d. ANALYTICAL Determine which functions are growth models and which are decay models.

ANSWER:

a.







b. Sample answer: $f(x)$; the graph of $f(x)$ is a reflection along the x-axis and the output values in the table are negative.
c. $g(x)$ and $h(x)$
d. Sample answer: $f(x)$ and $g(x)$ are growth and $h(x)$ is decay; The absolute value of the output is increasing for the growth functions and decreasing for the decay function.

34. **REASONING** Determine whether each statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

a. An exponential function of the form $y = ab^{x-h} + k$ has a y-intercept.

b. An exponential function of the form $y = ab^{x-h} + k$ has an x-intercept.

c. The function $f(x) = |b|^x$ is an exponential growth function if b is an integer.

ANSWER:

a. Always; Sample answer: The domain of exponential functions is all real numbers, so $(0, y)$ always exists.
b. Sometimes; Sample answer: The graph of an exponential function crosses the x-axis when $k < 0$.
c. Sometimes; Sample answer: The function is not exponential if $b = 1$ or -1.

35. **CCSS CRITIQUE** Vince and Grady were asked to graph the following functions. Vince thinks they are the same, but Grady disagrees. Who is correct? Explain your reasoning.

x	y
0	2
1	1
2	0.5
3	0.25
4	0.125
5	0.0625
6	0.03125

an exponential function with rate of decay of $\frac{1}{2}$ and an initial amount of 2

ANSWER:

Vince; the graphs of the function would be the same.

36. **CHALLENGE** A substance decays 35% each day. After 8 days, there are 8 milligrams of the substance remaining. How many milligrams were there initially?

ANSWER:

about 251 mg

37. **OPEN ENDED** Give an example of a value of b for which $f(x) = \left(\frac{8}{b}\right)^x$ represents exponential decay.

ANSWER:

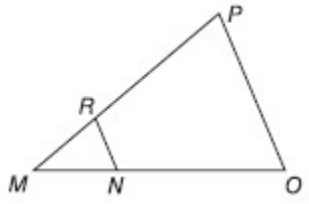
Sample answer: 10

38. **WRITING IN MATH** Write the procedure for transforming the graph of $g(x) = b^x$ to the graph $f(x) = ab^{x-h} + k$

ANSWER:

Sample answer: The parent function, $g(x) = b^x$, is stretched if a is greater than 1 or compressed if a is less than 1. The parent function is translated up k units if k is positive and down $|k|$ units if k is negative. The parent function is translated h units to the right if h is positive and $|h|$ units to the left if h is negative.

39. **GRIDDED RESPONSE** In the figure, $\overline{PO} \parallel \overline{RN}$, $ON = 12$, $MN = 6$, and $RN = 4$. What is the length of $\overline{PO}$?



ANSWER:

12

40. Ivan has enough money to buy 12 used CDs. If the cost of each CD was $0.20 less, Ivan could buy 2 more CDs. How much money does Ivan have to spend on CDs?

A $16.80
B $16.40
C $15.80
D $15.40

ANSWER:

A

41. One hundred students will attend the fall dance if tickets cost $30 each. For each $5 increase in price, 10 fewer students will attend. What price will deliver the maximum dollar sales?

F $30
G $35
H $40
J $45

ANSWER:

H

42. **SAT/ACT** Javier mows a lawn in 2 hours. Tonya mows the same lawn in 1.5 hours. About how many minutes will it take to mow the lawn if Javier and Tonya work together?

A 28 minutes
B 42 minutes
C 51 minutes
D 1.2 hours
E 1.4 hours

ANSWER:

C

Solve each equation or inequality.

43. $\sqrt{y+5} = \sqrt{2y-3}$

ANSWER:

8

44. $\sqrt{y+1} + \sqrt{y-4} = 5$

ANSWER:

8

45. $10 - \sqrt{2x+7} \leq 3$

ANSWER:

$x \geq 21$

46. $6 + \sqrt{3y+4} < 6$

ANSWER:

no solution

47. $\sqrt{d+3} + \sqrt{d+7} > 4$

ANSWER:

$d > -\frac{3}{4}$

48. $\sqrt{2x+5} - \sqrt{9+x} > 0$

ANSWER:

$x > 4$

Simplify.

49. $\frac{1}{y^{\frac{2}{5}}}$

ANSWER:

$\frac{y^{\frac{3}{5}}}{y}$

50. $\frac{xy}{\sqrt[3]{z}}$

ANSWER:

$\frac{xyz^{\frac{2}{3}}}{z}$

51. $\frac{3x+4x^2}{x^{-\frac{2}{3}}}$

ANSWER:

$3x^{\frac{5}{3}}+4x^{\frac{8}{3}}$

52. $\sqrt[6]{27x^3}$

ANSWER:

$\sqrt{3x}$

53. $\frac{\sqrt[4]{27}}{\sqrt[4]{3}}$

ANSWER:

$\sqrt{3}$

54. $\frac{a^{-\frac{1}{2}}}{6a^{\frac{1}{3}}.a^{-\frac{1}{4}}}$

ANSWER:

$\frac{a^{\frac{5}{12}}}{6a}$

55. **FOOTBALL** The path of a football thrown across a field is given by the equation $y = -0.005x^2 + x + 5$, where x represents the distance, in feet, the ball has traveled horizontally and y represents the height, in feet, of the ball above ground level. About how far has the ball traveled horizontally when it returns to ground level?

ANSWER:

about 204.88 ft

56. **COMMUNITY SERVICE** A drug awareness program is being presented at a theater that seats 300 people. Proceeds will be donated to a local drug information center. If every two adults must bring at least one student, what is the maximum amount of money that can be raised?

ANSWER:

$500

Simplify. Assume that no variable equals 0.

57. $f^{-7} \cdot f^4$

ANSWER:

$\frac{1}{f^3}$

58. $\left(3x^2\right)^3$

ANSWER:

$27x^6$

59. $(2y)\left(4xy^3\right)$

ANSWER:

$8xy^4$

60. $\left(\frac{3}{5}c^2f\right)\left(\frac{4}{3}cd\right)^2$

ANSWER:

$\frac{16}{15}c^4d^2f$