7-2 Solving Exponential Equations and Inequalities

Solve each equation.

1. $3^{5x} = 27^{2x-4}$

ANSWER:
12

2. $16^{2y-3} = 4^{y+1}$

ANSWER:
$\frac{7}{3}$

3. $2^{6x} = 32^{x-2}$

ANSWER:
−10

4. $49^{x+5} = 7^{8x-6}$

ANSWER:
$\frac{8}{3}$

5. **SCIENCE** Mitosis is a process in which one cell divides into two. The *Escherichia coli* is one of the fastest growing bacteria. It can reproduce itself in 15 minutes.
a. Write an exponential function to represent the number of cells c after t minutes.
b. If you begin with one *Escherichia coli* cell, how many cells will there be in one hour?

ANSWER:
a. $c = 2^{\frac{t}{15}}$
b. 16 cells

6. A certificate of deposit (CD) pays 2.25% annual interest compounded biweekly. If you deposit $500 into this CD, what will the balance be after 6 years?

ANSWER:
$572.23

Solve each inequality.

7. $4^{2x+6} \leq 64^{2x-4}$

ANSWER:
$x \geq 4.5$

8. $25^{y-3} \leq \left(\frac{1}{125}\right)^{y+2}$

ANSWER:
$\{y | y \leq 0\}$

Solve each equation.

9. $8^{4x+2} = 64$

ANSWER:
0

10. $5^{x-6} = 125$

ANSWER:
9

11. $81^{a+2} = 3^{3a+1}$

ANSWER:
−7

12. $256^{b+2} = 4^{2-2b}$

ANSWER:
−1

13. $9^{3c+1} = 27^{3c-1}$

ANSWER:
$\frac{5}{3}$

14. $8^{2y+4} = 16^{y+1}$

ANSWER:
−4

15. **CCSS MODELING** In 2009, My-Lien received $10,000 from her grandmother. Her parents invested all of the money, and by 2021, the amount will have grown to $16,960.
a. Write an exponential function that could be used to model the money y. Write the function in terms of x, the number of years since 2009.
b. Assume that the amount of money continues to grow at the same rate. What would be the balance in the account in 2031?

ANSWER:

a. $y = 10{,}000(1.045)^x$
b. about $26,336.52

Write an exponential function for the graph that passes through the given points.

16. (0, 6.4) and (3, 100)

ANSWER:

$y = 6.4(2.5)^x$

17. (0, 256) and (4, 81)

ANSWER:

$y = 256(0.75)^x$

18. (0, 128) and (5, 371,293)

ANSWER:

$y = 128(4.926)^x$

19. (0, 144), and (4, 21,609)

ANSWER:

$y = 144(3.5)^x$

20. Find the balance of an account after 7 years if $700 is deposited into an account paying 4.3% interest compounded monthly.

ANSWER:
$945.34

21. Determine how much is in a retirement account after 20 years if $5000 was invested at 6.05% interest compounded weekly.

ANSWER:
$16,755.63

22. A savings account offers 0.7% interest compounded bimonthly. If $110 is deposited in this account, what will the balance be after 15 years?

ANSWER:
$122.17

23. A college savings account pays 13.2% annual interest compounded semiannually. What is the balance of an account after 12 years if $21,000 was initially deposited?

ANSWER:
$97,362.61

Solve each inequality.

24. $625 \geq 5^{a+8}$

ANSWER:
$\{a \mid a \leq -4\}$

25. $10^{5b+2} > 1000$

ANSWER:
$\left\{b \mid b > \frac{1}{5}\right\}$

26. $\left(\frac{1}{64}\right)^{c-2} < 32^{2c}$

ANSWER:
$\left\{c \mid c > \frac{3}{4}\right\}$

27. $\left(\frac{1}{27}\right)^{2d-2} \leq 81^{d+4}$

ANSWER:
$\{d \mid d \geq -1\}$

28. $\left(\frac{1}{9}\right)^{3t+5} \geq \left(\frac{1}{243}\right)^{t-6}$

ANSWER:
$\{t \mid t \leq -40\}$

29. $\left(\frac{1}{36}\right)^{w+2} < \left(\frac{1}{216}\right)^{4w}$

ANSWER:
$\left\{w \mid w < \frac{2}{5}\right\}$

30. **SCIENCE** A mug of hot chocolate is 90°C at time $t = 0$. It is surrounded by air at a constant temperature of 20°C. If stirred steadily, its temperature in Celsius after t minutes will be $y(t) = 20 + 70(1.071)^{-t}$.
a. Find the temperature of the hot chocolate after 15 minutes.
b. Find the temperature of the hot chocolate after 30 minutes.
c. The optimum drinking temperature is 60°C. Will the mug of hot chocolate be at or below this temperature after 10 minutes?

ANSWER:
a. 45.02° C
b. 28.94° C
c. below

31. **ANIMALS** Studies show that an animal will defend a territory, with area in square yards, that is directly proportional to the 1.31 power of the animal's weight in pounds.
a. If a 45-pound beaver will defend 170 square yards, write an equation for the area a defended by a beaver weighing w pounds.
b. Scientists believe that thousands of years ago, the beaver's ancestors were 11 feet long and weighed 430 pounds. Use your equation to determine the area defended by these animals.

ANSWER:
a. $a = 1.16w^{1.31}$
b. about 3268 yd^2

Solve each equation.

32. $\left(\frac{1}{2}\right)^{4x+1} = 8^{2x+1}$

ANSWER:
$-\frac{2}{5}$

33. $\left(\frac{1}{5}\right)^{x-5} = 25^{3x+2}$

ANSWER:
$\frac{1}{7}$

34. $216 = \left(\frac{1}{6}\right)^{x+3}$

ANSWER:
−6

35. $\left(\frac{1}{8}\right)^{3x+4} = \left(\frac{1}{4}\right)^{-2x+4}$

ANSWER:
$-\frac{4}{13}$

36. $\left(\frac{2}{3}\right)^{5x+1} = \left(\frac{27}{8}\right)^{x-4}$

ANSWER:
$\frac{11}{8}$

37. $\left(\frac{25}{81}\right)^{2x+1} = \left(\frac{729}{125}\right)^{-3x+1}$

ANSWER:
1

38. **CCSS MODELING** In 1950, the world population was about 2.556 billion. By 1980, it had increased to about 4.458 billion.

a. Write an exponential function of the form $y = ab^x$ that could be used to model the world population y in billions for 1950 to 1980. Write the equation in terms of x, the number of years since 1950. (Round the value of b to the nearest ten-thousandth.)

b. Suppose the population continued to grow at that rate. Estimate the population in 2000.

c. In 2000, the population of the world was about 6.08 billion. Compare your estimate to the actual population.

d. Use the equation you wrote in part **a** to estimate the world population in the year 2020. How accurate do you think the estimate is? Explain your reasoning.

ANSWER:

a. $y = 2.556(1.0187)^x$

b. 6.455 billion

c. The prediction was about 375 million greater than the actual.

d. About 9.3498 billion; because the prediction for 2000 was greater than the actual population, this prediction is probably even higher than the actual population will be at the time.

39. **TREES** The diameter of the base of a tree trunk in centimeters varies directly with the $\frac{3}{2}$ power of its height in meters.

a. A young sequoia tree is 6 meters tall, and the diameter of its base is 19.1 centimeters. Use this information to write an equation for the diameter d of the base of a sequoia tree if its height is h meters high

b. The General Sherman Tree in Sequoia National Park, California, is approximately 84 meters tall. Find the diameter of the General Sherman Tree at its base.

ANSWER:

a. $d = 1.30h^{\frac{3}{2}}$

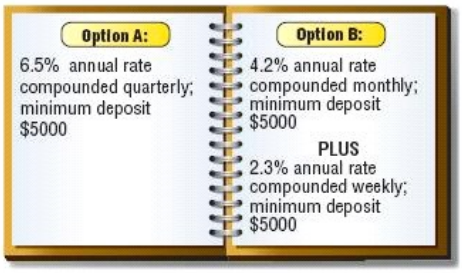
b. about 1001 cm

40. **FINANCIAL LITERACY** Mrs. Jackson has two different retirement investment plans from which to choose.

a. Write equations for Option A and Option B given the minimum deposits.

b. Draw a graph to show the balances for each investment option after t years.

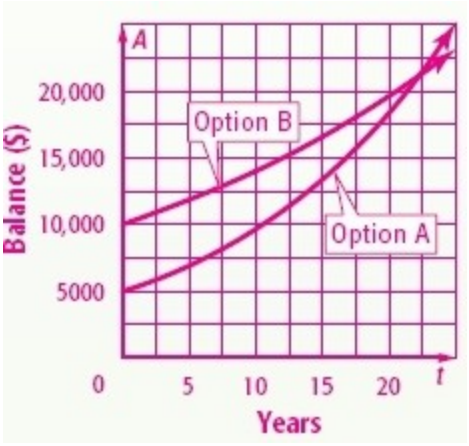
c. Explain whether Option A or Option B is the better investment choice.



ANSWER:

a. $A = 5000\left(\frac{4.065}{4}\right)^{4t}$; $A = 5000\left[\left(\frac{12.042}{12}\right)^{12t} + \left(\frac{52.023}{52}\right)^{52t}\right]$

b.



c. Sample answer: During the first 22 years, Option B is the better choice because the total is greater than that of Option A. However, after about 22 years, the balance of Option A exceeds that of Option B, so Option A is the better choice.

41. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the rapid increase of an exponential function. A large sheet of paper is cut in half, and one of the resulting pieces is placed on top of the other. Then the pieces in the stack are cut in half and placed on top of each other. Suppose this procedure is repeated several times.
a. CONCRETE Perform this activity and count the number of sheets in the stack after the first cut. How many pieces will there be after the second cut? How many pieces after the third cut? How many pieces after the fourth cut?
b. TABULAR Record your results in a table.
c. SYMBOLIC Use the pattern in the table to write an equation for the number of pieces in the stack after x cuts.
d. ANALYTICAL The thickness of ordinary paper is about 0.003 inch. Write an equation for the thickness of the stack of paper after x cuts.
e. ANALYTICAL How thick will the stack of paper be after 30 cuts?

ANSWER:

a. 2, 4, 8, 16
b.

Cuts	Pieces
1	2
2	4
3	8
4	16

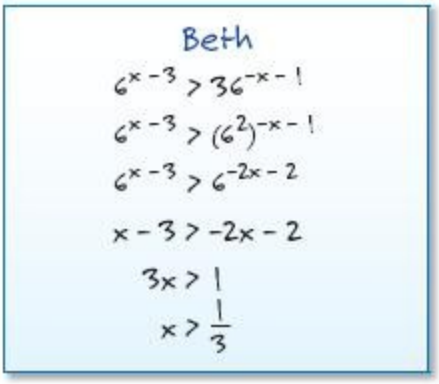
c. $y = 2^x$
d. $y = 0.003(2)^x$
e. about 3,221,225.47 in.

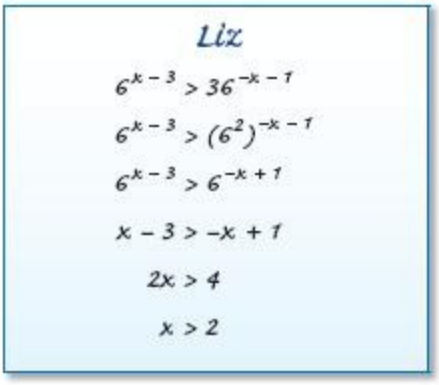
42. **WRITING IN MATH** In a problem about compound interest, describe what happens as the compounding period becomes more frequent while the principal and overall time remain the same.

ANSWER:

Sample answer: The more frequently interest is compounded, the higher the account balance becomes.

43. **ERROR ANALYSIS** Beth and Liz are solving $6^{x-3} > 36^{-x-1}$. Is either of them correct? Explain your reasoning.





ANSWER:

Sample answer: Beth; Liz added the exponents instead of multiplying them when taking the power of a power.

44. **CHALLENGE** Solve for x: $16^{18} + 16^{18} + 16^{18} + 16^{18} + 16^{18} + 16^{18} = 4^x$.

ANSWER:

37.1610

45. **OPEN ENDED** What would be a more beneficial change to a 5-year loan at 8% interest compounded monthly: reducing the term to 4 years or reducing the interest rate to 6.5%?

ANSWER:

Reducing the term will be more beneficial. The multiplier is 1.3756 for the 4-year and 1.3828 for the 6.5%.

46. **CCSS ARGUMENTS** Determine whether the following statements are *sometimes*, *always*, or *never* true. Explain your reasoning.
a. $2^x > -8^{20x}$ for all values of x.
b. The graph of an exponential growth equation is increasing.
c. The graph of an exponential decay equation is increasing.

ANSWER:

a. Always; 2^x will always be positive, and -8^{20x} will always be negative.
b. Always; by definition the graph will always be increasing even if it is a small increase.
c. Never; by definition the graph will always be decreasing even if it is a small decrease.

47. **OPEN ENDED** Write an exponential inequality with a solution of $x \leq 2$.

ANSWER:

Sample answer: $4^x \leq 4^2$

48. **PROOF** Show that $27^{2x} \cdot 81^{x+1} = 3^{2x+2} \cdot 9^{4x+1}$.

ANSWER:

$27^{2x} \cdot 81^{x+1} = 3^{2x+2} \cdot (9)^{4x+1}$ Original equation

$\left(3^3\right)^{2x} \cdot \left(3^4\right)^{x+1} = 3^{2x+2} \cdot \left(3^2\right)^{4x+1}$ $3^2 = 9,\ 3^3 = 27,\ and\ 3^4 = 81$

$3^{6x} \cdot 3^{4x+4} = 3^{2x+2} \cdot 3^{8x+2}$ Power of a Power

$3^{10x+4} = 3^{10x+4}$ Product of Powers

$10x + 4 = 10x + 4$ Property of Equality for Exponential Functions

$10x = 10x$ Subtract 4 from each side.

$x = x$ Divide each side by 10.

49. **WRITING IN MATH** If you were given the initial and final amounts of a radioactive substance and the amount of time that passes, how would you determine the rate at which the amount was increasing or decreasing in order to write an equation?

ANSWER:

Sample answer: Divide the final amount by the initial amount. If n is the number of time intervals that pass, take the nth root of the answer.

50. $3 \times 10^{-4} =$
A −30,000
B 0.0003
C −120
D 0.00003

ANSWER:

B

51. Which of the following could *not* be a solution to $5 - 3x < -3$?
F 2.5
G 3
H 3.5
J 4

ANSWER:

F

52. **GRIDDED RESPONSE** The three angles of a triangle are $3x$, $x + 10$, and $2x - 40$. Find the measure of the smallest angle in the triangle.

ANSWER:

30

53. **SAT/ACT** Which of the following is equivalent to $(x)(x)(x)(x)$ for all x?

A $x + 4$
B $4x$
C $2x^2$
D $4x^2$
E x^4

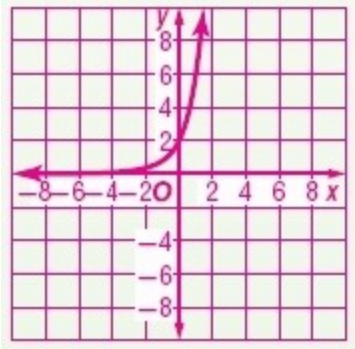
ANSWER:

E

Graph each function.

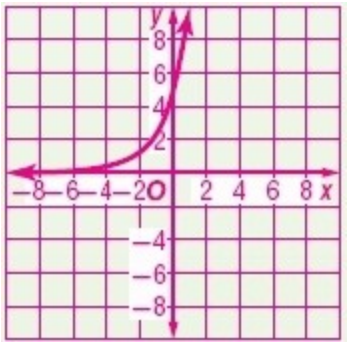
54. $y = 2(3)^x$

ANSWER:



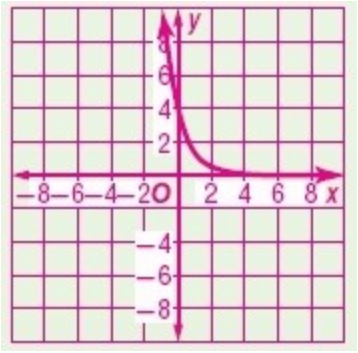
55. $y = 5(2)^x$

ANSWER:



56. $y = 4\left(\frac{1}{3}\right)x$

ANSWER:



Solve each equation.

57. $\sqrt{x+5} - 3 = 0$

ANSWER:
4

58. $\sqrt{3t-5} - 3 = 4$

ANSWER:
18

59. $\sqrt[4]{2x-1} = 2$

ANSWER:
8.5

60. $\sqrt{x-6} - \sqrt{x} = 3$

ANSWER:
no solution

61. $\sqrt[3]{5m+2} = 3$

ANSWER:
5

62. $(6n-5)^{\frac{1}{3}} + 3 = -2$

ANSWER:
−20

63. $(5x+7)^{\frac{1}{5}} + 3 = 5$

ANSWER:
5

64. $(3x-2)^{\frac{1}{5}} + 6 = 5$

ANSWER:

$\frac{1}{3}$

65. $(7x-1)^{\frac{1}{3}} + 4 = 2$

ANSWER:
−1

66. **SALES** A salesperson earns $10 an hour plus a 10% commission on sales. Write a function to describe the salesperson's income. If the salesperson wants to earn $1000 in a 40-hour week, what should his sales be?

ANSWER:
$I(m) = 400 + 0.1m$; $6000

67. **STATE FAIR** A dairy makes three types of cheese—cheddar, Monterey Jack, and Swiss—and sells the cheese in three booths at the state fair. At the beginning of one day, the first booth received x pounds of each type of cheese. The second booth received y pounds of each type of cheese, and the third booth received z pounds of each type of cheese. By the end of the day, the dairy had sold 131 pounds of cheddar, 291 pounds of Monterey Jack, and 232 pounds of Swiss. The table below shows the percent of the cheese delivered in the morning that was sold at each booth. How many pounds of cheddar cheese did each booth receive in the morning?

Type	Booth 1	Booth 2	Booth 3
Cheddar	40%	30%	10%
Monterey Jack	40%	90%	80%
Swiss	30%	70%	70%

ANSWER:
booth 1, 190 lb; booth 2, 150 lb; booth 3, 100 lb

Find $[g \circ h](x)$ and $[h \circ g](x)$.

68. $h(x) = 2x - 1$
$g(x) = 3x + 4$

ANSWER:
$6x + 1$; $6x + 7$

69. $h(x) = x^2 + 2$
$g(x) = x - 3$

ANSWER:
$x^2 - 1$; $x^2 - 6x + 11$

70. $h(x) = x^2 + 1$
$g(x) = -2x + 1$

ANSWER:
$-2x^2 - 1$; $4x^2 - 4x + 2$

71. $h(x) = -5x$
$g(x) = 3x - 5$

ANSWER:
$-15x - 5$; $-15x + 25$

72. $h(x) = x^3$
$g(x) = x - 2$

ANSWER:
$x^3 - 2$; $x^3 - 6x^2 + 12x - 8$

73. $h(x) = x + 4$
$g(x) = |x|$

ANSWER:
$|x + 4|$; $|x| + 4$