7-4 Solving Logarithmic Equations and Inequalities

Solve each equation.

1. $\log_8 x = \frac{4}{3}$

SOLUTION:

$$\begin{aligned} \log_8 x &= \frac{4}{3} \\ x &= 8^{\frac{4}{3}} \\ &= \left(2^3\right)^{\frac{4}{3}} \\ &= 2^4 \\ &= 16 \end{aligned}$$

2. $\log_{16} x = \frac{3}{4}$

SOLUTION:

$$\begin{aligned} \log_{16} x &= \frac{3}{4} \\ x &= 16^{\frac{3}{4}} \\ &= \left(2^4\right)^{\frac{3}{4}} \\ &= 2^3 \\ &= 8 \end{aligned}$$

3. **MULTIPLE CHOICE** Solve $\log_5 (x^2 - 10) = \log_5 3x$.

A 10
B 2
C 5
D 2, 5

SOLUTION:

$$\begin{aligned} \log_5\left(x^2 - 10\right) &= \log_5 3x \\ x^2 - 10 &= 3x \\ x^2 - 3x - 10 &= 0 \\ (x-5)(x+2) &= 0 \end{aligned}$$

$$\begin{aligned} x - 5 = 0 \quad &\text{or} \quad x + 2 = 0 \\ x = 5 \quad &\quad\quad x = -2 \end{aligned}$$

Substitute each value into the original equation.

$$\begin{array}{ll} x = 5 & x = -2 \\ \log_5\left(5^2 - 10\right) \stackrel{?}{=} \log_5 3(5) & \log_5\left((-2)^2 - 10\right) \stackrel{?}{=} \log_5 3(-2) \\ \log_5 15 = \log_5 15 \checkmark & \log_5 -6 \stackrel{?}{=} \log_5 -6 \times \end{array}$$

The domain of a logarithmic function cannot be 0, so $\log_5 (-6)$ is undefined and –2 is an extraneous solution.

C is the correct option.

Solve each inequality.

4. $\log_5 x > 3$

SOLUTION:

$$\begin{aligned} \log_5 x &> 3 \\ x &> 5^3 \\ x &> 125 \end{aligned}$$

Thus, solution set is $\{x \mid x > 125\}$.

5. $\log_8 x \le -2$

SOLUTION:

$$\begin{aligned}\log_8 x &\le -2\\ x &\le 8^{-2}\\ x &\le \frac{1}{64}\end{aligned}$$

Thus, solution set is $\left\{x \mid 0 < x \le \frac{1}{64}\right\}$.

6. $\log_4 (2x + 5) \le \log_4 (4x - 3)$

SOLUTION:

$$\begin{aligned}\log_4(2x+5) &\le \log_4(4x-3)\\ 2x+5 &\le 4x-3\\ 2x &\ge 8\\ x &\ge 4\end{aligned}$$

Thus, solution set is $\{x \mid x \ge 4\}$.

7. $\log_8 (2x) > \log_8 (6x - 8)$

SOLUTION:

$$\begin{aligned}\log_8(2x) &> \log_8(6x-8)\\ 2x &> 6x-8\\ 4x &< 8\\ x &< 2\end{aligned}$$

Exclude all values of x for which $2x \le 0$ or $6x - 8 \le 0$.

So, $x > 0, x > \frac{4}{3}$ and $x < 2$.

Thus, solution set is $\left\{x \mid 2 > x > \frac{4}{3}\right\}$.

CCSS STRUCTURE Solve each equation.

8. $\log_{81} x = \frac{3}{4}$

SOLUTION:

$$\begin{aligned}\log_{81} x &= \frac{3}{4}\\ x &= 81^{\frac{3}{4}}\\ &= \left(3^4\right)^{\frac{3}{4}}\\ &= 3^3\\ &= 27\end{aligned}$$

9. $\log_{25} x = \frac{5}{2}$

SOLUTION:

$$\begin{aligned}\log_{25} x &= \frac{5}{2}\\ x &= 25^{\frac{5}{2}}\\ &= \left(5^2\right)^{\frac{5}{2}}\\ &= 5^5\\ &= 3125\end{aligned}$$

10. $\log_8 \frac{1}{2} = x$

SOLUTION:

$$\begin{aligned}\log_8 \frac{1}{2} &= x\\ 8^x &= \frac{1}{2}\\ 2^{3x} &= 2^{-1}\\ 3x &= -1\\ x &= -\frac{1}{3}\end{aligned}$$

7-4 Solving Logarithmic Equations and Inequalities

11. $\log_6 \frac{1}{36} = x$

SOLUTION:

$$\log_6 \frac{1}{36} = x$$
$$6^x = \frac{1}{36}$$
$$6^x = 6^{-2}$$
$$x = -2$$

12. $\log_x 32 = \frac{5}{2}$

SOLUTION:

$$\log_x 32 = \frac{5}{2}$$
$$x^{\frac{5}{2}} = 32$$
$$\left(x^{\frac{1}{2}}\right)^5 = 2^5$$
$$x^{\frac{1}{2}} = 2$$
$$x = 4$$

13. $\log_x 27 = \frac{3}{2}$

SOLUTION:

$$\log_x 27 = \frac{3}{2}$$
$$x^{\frac{3}{2}} = 27$$
$$\left(x^{\frac{1}{2}}\right)^3 = 3^3$$
$$x^{\frac{1}{2}} = 3$$
$$x = 9$$

14. $\log_3 (3x + 8) = \log_3 (x^2 + x)$

SOLUTION:

$$\log_3(3x+8) = \log_3(x^2+x)$$
$$3x+8 = x^2+x$$
$$x^2-2x-8=0$$
$$(x-4)(x+2)=0$$

$$x-4=0 \text{ or } x+2=0$$
$$x=4 \qquad x=-2$$

Substitute each value into the original equation.

$$x=4 \qquad\qquad x=-2$$
$$\log_3(3\cdot4+8)\stackrel{?}{=}\log_3(4^2+4) \qquad \log_3(3\cdot-2+8)\stackrel{?}{=}\log_3\left((-2)^2+(-2)\right)$$
$$\log_3 20 = \log_3 20 \checkmark \qquad \log_3 2 = \log_3 2 \checkmark$$

Thus, $x = –2$ or 4.

15. $\log_{12} (x^2 - 7) = \log_{12} (x + 5)$

SOLUTION:

$$\log_{12}(x^2-7) = \log_{12}(x+5)$$
$$x^2-7 = x+5$$
$$x^2-x-12=0$$
$$(x-4)(x+3)=0$$

$$x-4=0 \text{ or } x+3=0$$
$$x=4 \qquad x=-3$$

Substitute each value into the original equation.

$$x=4 \qquad\qquad x=-3$$
$$\log_{12}(4^2-7)\stackrel{?}{=}\log_{12}(4+5) \qquad \log_{12}\left((-3)^2-7\right)\stackrel{?}{=}\log_{12}(-3+5)$$
$$\log_{12} 9 = \log_{12} 9 \checkmark \qquad \log_{12} 2 = \log_{12} 2 \checkmark$$

Thus, $x = –3$ or 4.

16. $\log_6 (x^2 - 6x) = \log_6 (-8)$

SOLUTION:

$$\log_6\left(x^2-6x\right)=\log_6(-8)$$
$$x^2-6x=-8$$
$$x^2-6x+8=0$$
$$(x-4)(x-2)=0$$

$$x-4=0 \quad \text{or} \quad x-2=0$$
$$x=4 \qquad\qquad x=2$$

Substitute each value into the original equation.

$x=4$: $\log_6\left(4^2-6(4)\right)\stackrel{?}{=}\log_6(-8)$, $\log_6(-8)\stackrel{?}{=}\log_6(-8)$ ✗

$x=2$: $\log_6\left(2^2-6(2)\right)\stackrel{?}{=}\log_6(-8)$, $\log_6(-8)\stackrel{?}{=}\log_6(-8)$ ✗

$\log_6 (–8)$ is undefined, so 4 and 2 are extraneous solutions.
Thus, no solution.

17. $\log_9 (x^2 - 4x) = \log_9 (3x - 10)$

SOLUTION:

$$\log_9\left(x^2-4x\right)=\log_9(3x-10)$$
$$x^2-4x=3x-10$$
$$x^2-7x+10=0$$
$$(x-5)(x-2)=0$$

$$x-5=0 \quad \text{or} \quad x-2=0$$
$$x=5 \qquad\qquad x=2$$

Substitute each value into the original equation.

$x=5$: $\log_9\left(5^2-4(5)\right)\stackrel{?}{=}\log_9(3(5)-10)$, $\log_9 5=\log_9 5$ ✓

$x=2$: $\log_9\left(2^2-4(2)\right)\stackrel{?}{=}\log_9(3(2)-10)$, $\log_9(-4)\stackrel{?}{=}\log_9(-4)$ ✗

$\log_9 (–4)$ is undefined and 2 is extraneous solution.
Thus, $x = 5$.

18. $\log_4 (2x^2 + 1) = \log_4 (10x - 7)$

SOLUTION:

$$\log_4\left(2x^2+1\right)=\log_4(10x-7)$$
$$2x^2+1=10x-7$$
$$2x^2-10x+8=0$$
$$(x-4)(x-1)=0$$

$$x-4=0 \quad \text{or} \quad x-1=0$$
$$x=4 \qquad\qquad x=1$$

Substitute each value into the original equation.

$x=4$: $\log_4\left(2(4)^2+1\right)\stackrel{?}{=}\log_4(10(4)-7)$, $\log_4 33=\log_4 33$ ✓

$x=1$: $\log_4\left(2(1)^2+1\right)\stackrel{?}{=}\log_4(10(1)-7)$, $\log_4 3=\log_4 3$ ✓

Thus, $x = 1$ or 4.

19. $\log_7 (x^2 - 4) = \log_7 (-x + 2)$

SOLUTION:

$$\log_7\left(x^2-4\right)=\log_7(-x+2)$$
$$x^2-4=-x+2$$
$$x^2+x-6=0$$
$$(x+3)(x-2)=0$$

$$x+3=0 \quad \text{or} \quad x-2=0$$
$$x=-3 \qquad\qquad x=2$$

Substitute each value into the original equation.

$x=-3$: $\log_7\left((-3)^2-4\right)\stackrel{?}{=}\log_7(-(-3)+2)$, $\log_7 5=\log_7 5$ ✓

$x=2$: $\log_7\left(2^2-4\right)\stackrel{?}{=}\log_7(-2+2)$, $\log_7 0=\log_7 0$

Since you can not have a log of 0, $x = -3$ is the solution.

SCIENCE The equation for wind speed *w*, in miles per hour, near the center of a tornado is $w = 93 \log_{10} d + 65$, where *d* is the distance in miles that the tornado travels.

20. Write this equation in exponential form.

SOLUTION:

$$\begin{aligned} w &= 93\log_{10} d + 65 \\ w - 65 &= 93\log_{10} d \\ \frac{w-65}{93} &= \log_{10} d \\ d &= 10^{\left(\frac{w-65}{93}\right)} \end{aligned}$$

21. In May of 1999, a tornado devastated Oklahoma City with the fastest wind speed ever recorded. If the tornado traveled 525 miles, estimate the wind speed near the center of the tornado.

SOLUTION:

Substitute 525 for *d* in the equation and simplify.

$$\begin{aligned} w &= 93\log_{10} 525 + 65 \\ &\approx 318 \text{ mph} \end{aligned}$$

Solve each inequality.

22. $\log_6 x < -3$

SOLUTION:

$$\begin{aligned} \log_6 x &< -3 \\ x &< 6^{-3} \\ x &< \frac{1}{216} \end{aligned}$$

The solution set is $\left\{x \mid 0 < x < \frac{1}{216}\right\}$.

23. $\log_4 x \geq 4$

SOLUTION:

$$\begin{aligned} \log_4 x &\geq 4 \\ x &\geq 4^4 \\ x &\geq 256 \end{aligned}$$

The solution set is $\{x \mid x \geq 256\}$.

24. $\log_3 x \geq -4$

SOLUTION:

$$\begin{aligned} \log_3 x &\geq -4 \\ x &\geq 3^{-4} \\ x &\geq \frac{1}{81} \end{aligned}$$

The solution set is $\left\{x \middle| x \geq \frac{1}{81}\right\}$.

25. $\log_2 x \leq -2$

SOLUTION:

$$\begin{aligned} \log_2 x &\leq -2 \\ x &\leq 2^{-2} \\ x &\leq \frac{1}{4} \end{aligned}$$

The solution set is $\left\{x \middle| 0 < x \leq \frac{1}{4}\right\}$.

26. $\log_5 x > 2$

SOLUTION:

$$\begin{aligned} \log_5 x &> 2 \\ x &> 5^2 \\ x &> 25 \end{aligned}$$

The solution set is $\{x | x > 25\}$.

27. $\log_7 x < -1$

SOLUTION:

$$\begin{aligned} \log_7 x &< -1 \\ x &< 7^{-1} \\ x &< \frac{1}{7} \end{aligned}$$

The solution set is $\left\{x \middle| 0 < x < \frac{1}{7}\right\}$.

28. $\log_2 (4x - 6) > \log_2 (2x + 8)$

SOLUTION:

$$\begin{aligned}\log_2(4x-6) &> \log_2(2x+8)\\ 4x-6 &> 2x+8\\ 2x &> 14\\ x &> 7\end{aligned}$$

The solution set is $\{x \mid x > 7\}$.

29. $\log_7 (x + 2) \geq \log_7 (6x - 3)$

SOLUTION:

$$\begin{aligned}\log_7(x+2) &\geq \log_7(6x-3)\\ x+2 &\geq 6x-3\\ -5x &\geq -5\\ x &\leq 1\end{aligned}$$

Exclude all values of x for which $x+2 \leq 0$ or $6x-3 \leq 0$.

So, $x > -2, x > \frac{1}{2}$ and $x \leq 1$.

The solution set is $\left\{x \middle| \frac{1}{2} < x \leq 1\right\}$.

30. $\log_3 (7x - 6) < \log_3 (4x + 9)$

SOLUTION:

$$\begin{aligned}\log_3(7x-6) &< \log_3(4x+9)\\ 7x-6 &< 4x+9\\ 3x &< 15\\ x &< 5\end{aligned}$$

Exclude all values of x for which $7x-6 \leq 0$ or $4x+9 \leq 0$.

So, $x > \frac{6}{7}, x > -\frac{9}{4}$ and $x < 5$.

The solution set is $\left\{x \middle| \frac{6}{7} < x < 5\right\}$.

31. $\log_5 (12x + 5) \leq \log_5 (8x + 9)$

SOLUTION:

$$\begin{aligned}\log_5(12x+5) &\leq \log_5(8x+9)\\ 12x+5 &\leq 8x+9\\ 4x &\leq 4\\ x &\leq 1\end{aligned}$$

Exclude all values of x for which $12x+5 \leq 0$ or $8x+9 \leq 0$.

So, $x > -\frac{5}{12}, x > -\frac{9}{8}$ and $x \leq 1$.

The solution set is $\left\{x \middle| -\frac{5}{12} < x \leq 1\right\}$.

32. $\log_{11} (3x - 24) \geq \log_{11} (-5x - 8)$

SOLUTION:

$$\begin{aligned}\log_{11}(3x-24) &\geq \log_{11}(-5x-8)\\ 3x-24 &\geq -5x-8\\ 8x &\geq 16\\ x &\geq 2\end{aligned}$$

The solution set is $\{x \mid x \geq 2\}$.

33. $\log_9 (9x + 4) \leq \log_9 (11x - 12)$

SOLUTION:

$$\begin{aligned}\log_9(9x+4) &\leq \log_9(11x-12)\\ 9x+4 &\leq 11x-12\\ -2x &\leq -16\\ x &\geq 8\end{aligned}$$

The solution set is $\{x \mid x \geq 8\}$.

34. **CCSS MODELING** The magnitude of an earthquake is measured on a logarithmic scale called the Richter scale. The magnitude M is given by $M = \log_{10} x$, where x represents the amplitude of the seismic wave causing ground motion.
a. How many times as great is the amplitude caused by an earthquake with a Richter scale rating of 8 as an aftershock with a Richter scale rating of 5?
b. In 1906, San Francisco was almost completely destroyed by a 7.8 magnitude earthquake. In 1911, an earthquake estimated at magnitude 8.1 occurred along the New Madrid fault in the Mississippi River Valley. How many times greater was the New Madrid earthquake than the San Francisco earthquake?

SOLUTION:

a.
The amplitude of the seismic wave with a Richter scale rating of 8 and 5 are 10^8 and 10^5 respectively. Divide 10^8 by 10^5.

$$\frac{10^8}{10^5} = 10^{8-5}$$
$$= 10^3$$

The scale rating of 8 is 10^3 or 1000 times greater than the scale rating of 5.

b.
The amplitudes of San Francisco earthquake and New Madrid earthquake were $10^{7.8}$ and $10^{8.1}$ respectively.

Divide $10^{8.1}$ by $10^{7.8}$.

$$\frac{10^{8.1}}{10^{7.8}} = 10^{8.1-7.8}$$
$$= 10^{0.3}$$

The New Madrid earthquake was $10^{0.3}$ or about 2 times greater than the San Francisco earthquake.

35. **MUSIC** The first key on a piano keyboard corresponds to a pitch with a frequency of 27.5 cycles per second. With every successive key, going up the black and white keys, the pitch multiplies by a constant. The formula for the frequency of the pitch sounded when the nth note up the keyboard is played is given by $n = 1 + 12 \log_2 \frac{f}{27.5}$.
a. A note has a frequency of 220 cycles per second. How many notes up the piano keyboard is this?
b. Another pitch on the keyboard has a frequency of 880 cycles per second. After how many notes up the keyboard will this be found?

SOLUTION:

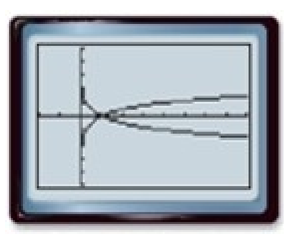
a.
Substitute 220 for f in the formula and solve for n.

$$n = 1 + 12 \log_2 \frac{220}{27.5}$$
$$= 1 + 12 \log_2 2^3$$
$$= 1 + 12(3)$$
$$= 37$$

b.
Substitute 880 for f in the formula and solve for n.

$$n = 1 + 12 \log_2 \frac{880}{27.5}$$
$$= 1 + 12 \log_2 2^5$$
$$= 1 + 12(5)$$
$$= 61$$

36. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the graphs shown: $y = \log_4 x$ and $y = \log_{\frac{1}{4}} x$.



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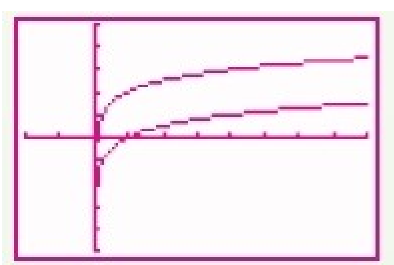
a. ANALYTICAL How do the shapes of the graphs compare? How do the asymptotes and the x-intercepts of the graphs compare?

b. VERBAL Describe the relationship between the graphs.
c. GRAPHICAL Use what you know about transformations of graphs to compare and contrast the graph of each function and the graph of $y = \log_4 x$.

1. $y = \log_4 x + 2$
2. $y = \log_4 (x + 2)$
3. $y = 3 \log_4 x$

d. ANALYTICAL Describe the relationship between $y = \log_4 x$ and $y = -1(\log_4 x)$. What are a reasonable domain and range for each function?
e. ANALYTICAL Write an equation for a function for which the graph is the graph of $y = \log_3 x$ translated 4 units left and 1 unit up.

SOLUTION:
a.
The shapes of the graphs are the same. The asymptote for each graph is the y-axis and the x-intercept for each graph is 1.
b.
The graphs are reflections of each other over the x-axis.
c.
1. The second graph is the same as the first, except it is shifted horizontally to the left 2 units.



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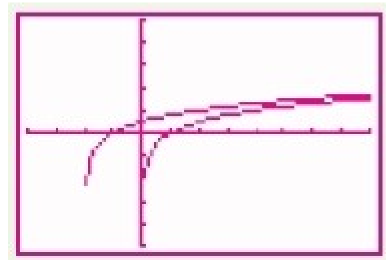
2. The second graph is the same as the first, except it is shifted vertically up 2 units.



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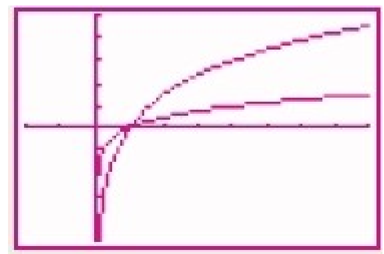
3. Each point on the second graph has a y-coordinate 3 times that of the corresponding point on the first graph.



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d.
The graphs are reflections of each other over the x-axis.
D = $\{x \mid x > 0\}$; R = {all real numbers}
e. $f(x) = a\log(x - h) + k$ where h is the horizontal shift and k is the vertical shift. Since there is a horizontal shift of 4 and vertical shift of 1, $h = 4$ and $k = 1$.
$y = \log_3 (x + 4) + 1$

37. **SOUND** The relationship between the intensity of sound I and the number of decibels β is $\beta = 10 \log_{10}\left(\frac{I}{10^{-12}}\right)$, where I is the intensity of sound in watts per square meter.

a. Find the number of decibels of a sound with an intensity of 1 watt per square meter.
b. Find the number of decibels of sound with an intensity of 10^{-2} watts per square meter.
c. The intensity of the sound of 1 watt per square meter is 100 times as much as the intensity of 10^{-2} watts per square meter. Why are the decibels of sound not 100 times as great?

SOLUTION:

a.
Substitute 1 for I in the given equation and solve for β.

$$\beta = 10 \log_{10}\left(\frac{I}{10^{-12}}\right)$$
$$\beta = 10 \log_{10}\left(\frac{1}{10^{-12}}\right)$$
$$= 10 \log_{10} 10^{12}$$
$$= 120$$

b.
Substitute 10^{-2} for I in the given equation and solve for β.

$$\beta = 10 \log_{10}\left(\frac{I}{10^{-12}}\right)$$
$$\beta = 10 \log_{10}\left(\frac{10^{-2}}{10^{-12}}\right)$$
$$= 10 \log_{10} 10^{10}$$
$$= 100$$

c. Sample answer: The power of the logarithm only changes by 2. The power is the answer to the logarithm. That 2 is multiplied by the 10 before the logarithm. So we expect the decibels to change by 20.

38. **CCSS CRITIQUE** Ryan and Heather are solving $\log_3 x \geq -3$. Is either of them correct? Explain your reasoning.

Ryan

$$\log_3 x \geq -3$$
$$x \geq 3^{-3}$$
$$x \geq \frac{1}{27}$$

Heather

$$\log_3 x \geq -3$$
$$x \geq 3^{-3}$$
$$0 < x \leq \frac{1}{27}$$

SOLUTION:
Sample answer: Ryan; Heather did not need to switch the inequality symbol when raising to a negative power.

39. **CHALLENGE** Find $\log_3 27 + \log_9 27 + \log_{27} 27 + \log_{81} 27 + \log_{243} 27$.

SOLUTION:

$$\log_3 27 + \log_9 27 + \log_{27} 27 + \log_{81} 27 + \log_{243} 27$$
$$= 3 + \frac{\log_3 27}{\log_3 9} + 1 + \frac{\log_3 27}{\log_3 81} + \frac{\log_3 27}{\log_3 243}$$
$$= 3 + \frac{3}{2} + 1 + \frac{3}{4} + \frac{3}{5}$$
$$= 6\frac{17}{20}$$

40. **REASONING** The Property of Inequality for Logarithmic Functions states that when $b > 1$, $\log_b x > \log_b y$ if and only if $x > y$. What is the case for when $0 < b < 1$? Explain your reasoning.

SOLUTION:
Sample answer: When $0 < b < 1$, $\log_b x > \log_b y$ if and only if $x < y$. The inequality symbol is switched because a fraction that is less than 1 becomes smaller when it is taken to a greater power.

41. **WRITING IN MATH** Explain how the domain and range of logarithmic functions are related to the domain and range of exponential functions.

SOLUTION:

The logarithmic function of the form $y = \log_b x$ is the inverse of the exponential function of the form $y = b^x$. The domain of one of the two inverse functions is the range of the other. The range of one of the two inverse functions is the domain of the other.

42. **OPEN ENDED** Give an example of a logarithmic equation that has no solution.

SOLUTION:

Sample answer: $\log_3 (x + 4) = \log_3 (2x + 12)$

43. **REASONING** Choose the appropriate term. Explain your reasoning. All logarithmic equations are of the form $y = \log_b x$.

a. If the base of a logarithmic equation is greater than 1 and the value of x is between 0 and 1, then the value for y is (*less than, greater than, equal to*) 0.
b. If the base of a logarithmic equation is between 0 and 1 and the value of x is greater than 1, then the value of y is (*less than, greater than, equal to*) 0.
c. There is/are (*no, one, infinitely many*) solution(s) for b in the equation $y = \log_b 0$.

d. There is/are (*no, one, infinitely many*) solution (s) for b in the equation $y = \log_b 1$.

SOLUTION:

a. less than
b. less than
c. no
d. infinitely many

44. **WRITING IN MATH** Explain why any logarithmic function of the form $y = \log_b x$ has an x-intercept of (1, 0) and no y-intercept.

SOLUTION:

The y-intercept of the exponential function $y = b^x$ is (0, 1). When the x and y coordinates are switched, the y-intercept is transformed to the x-intercept of (1, 0). There was no x-intercept (1, 0) in the exponential function of the form $y = b^x$. So when the x and y-coordinates are switched there would be no point on the inverse of (0, 1), and there is no y-intercept.

45. Find x if $\frac{6.4}{x} = \frac{4}{7}$.

A 3.4
B 9.4
C 11.2
D 44.8

SOLUTION:

$$\frac{6.4}{x} = \frac{4}{7}$$
$$44.8 = 4x$$
$$x = 11.2$$

C is the correct choice.

46. The monthly precipitation in Houston for part of a year is shown.

Month	Precipitation (in.)
April	3.60
May	5.15
June	5.35
July	3.18
August	3.83

Find the median precipitation.

F 3.60 in.
G 4.22 in.
H 3.83 in.
J 4.25 in.

SOLUTION:
Arrange the data in ascending order.
3.18, 3.60, 3.83, 5.15, 5.35
The median is the middle value. So, 3.83 is the median precipitation.
H is the correct choice.

47. Clara received a 10% raise each year for 3 consecutive years. What was her salary after the three raises if her starting salary was $12,000 per year?
A $14,520
B $15,972
C $16,248
D $16,410

SOLUTION:
Use the compound interest formula.
Substitute $12,000 for *P*, 0.10 for *r*, 1 for *n* and 3 for *t* and simplify.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$
$$A = 12000\left(1 + \frac{0.1}{1}\right)^{1(3)}$$
$$\approx \$15,972$$

B is the correct choice.

48. **SAT/ACT** A vendor has 14 helium balloons for sale: 9 are yellow, 3 are red, and 2 are green. A balloon is selected at random and sold. If the balloon sold is yellow, what is the probability that the next balloon, selected at random, is also yellow?

F $\frac{1}{9}$
G $\frac{1}{8}$
H $\frac{36}{91}$
J $\frac{8}{13}$
K $\frac{9}{14}$

SOLUTION:
The probability of selecting an yellow balloon next is:

$$\frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{8}{13}$$

So, the correct answer choice is J.

Evaluate each expression.

49. $\log_4 256$

SOLUTION:
$$\log_4 256 = \log_4 4^4$$
$$= 4$$

50. $\log_2 \frac{1}{8}$

SOLUTION:
$$\log_2 \frac{1}{8} = \log_2 2^{-3}$$
$$= -3$$

51. $\log_6 216$

SOLUTION:
$$\log_6 216 = \log_6 6^3$$
$$= 3$$

52. $\log_3 27$

SOLUTION:

$$\begin{aligned}\log_3 27 &= \log_3 3^3\\ &= 3\end{aligned}$$

53. $\log_5 \frac{1}{125}$

SOLUTION:

$$\begin{aligned}\log_5 \frac{1}{125} &= \log_5 5^{-3}\\ &= -3\end{aligned}$$

54. $\log_7 2401$

SOLUTION:

$$\begin{aligned}\log_7 2401 &= \log_7 7^4\\ &= 4\end{aligned}$$

Solve each equation or inequality. Check your solution.

55. $5^{2x+3} \le 125$

SOLUTION:

$$\begin{aligned}5^{2x+3} &\le 125\\ 5^{2x+3} &\le 5^3\\ 2x+3 &\le 3\\ x &\le 0\end{aligned}$$

56. $3^{3x-2} > 81$

SOLUTION:

$$\begin{aligned}3^{3x-2} &> 81\\ 3^{3x-2} &> 3^4\\ 3x-2 &> 4\\ 3x &> 6\\ x &> 2\end{aligned}$$

57. $4^{4a+6} \le 16^a$

SOLUTION:

$$\begin{aligned}4^{4a+6} &\le 16^a\\ 4^{4a+6} &\le 4^{2a}\\ 4a+6 &\le 2a\\ 2a &\le -6\\ a &\le -3\end{aligned}$$

58. $11^{2x+1} = 121^{3x}$

SOLUTION:

$$\begin{aligned}11^{2x+1} &= 121^{3x}\\ 11^{2x+1} &= (11)^{2(3x)}\\ 2x+1 &= 2(3x)\\ 2x+1 &= 6x\\ 1 &= 4x\\ 0.25 &= x\end{aligned}$$

59. $3^{4x-7} = 27^{2x+3}$

SOLUTION:

$$\begin{aligned}3^{4x-7} &= 27^{2x+3}\\ 3^{4x-7} &= 3^{(3)(2x+3)}\\ 4x-7 &= 3(2x+3)\\ 4x-7 &= 6x+9\\ -7 &= 2x+9\\ -16 &= 2x\\ -8 &= x\end{aligned}$$

60. $8^{x-4} \le 2^{4-x}$

SOLUTION:

$$\begin{aligned}8^{x-4} &\le 2^{4-x}\\ 2^{(3)(x-4)} &\le 2^{4-x}\\ (3)(x-4) &\le 4-x\\ 3x-12 &\le 4-x\\ 4x &\le 16\\ x &\le 4\end{aligned}$$

61. **SHIPPING** The height of a shipping cylinder is 4 feet more than the radius. If the volume of the cylinder is 5π cubic feet, how tall is it? Use the formula $V = \pi r^2 h$.

SOLUTION:

Substitute 5π for V and $r + 4$ for h in the formula and simplify.

$$5\pi = \pi r^2 (r + 4)$$
$$r^3 + 4r^2 - 5 = 0$$

The equation has one real root $r = 1$.
Thus, the height of the shipping cylinder is 1 + 4 = 5 ft.

62. **NUMBER THEORY** Two complex conjugate numbers have a sum of 12 and a product of 40. Find the two numbers

SOLUTION:

The equations that represent the situation are:

$$(a + bi) + (a - bi) = 12 \rightarrow (1)$$
$$(a + bi)(a - bi) = 40 \quad \rightarrow (2)$$

Solve equation (1).

$$\begin{aligned}(a + bi) + (a - bi) &= 12 \\ 2a &= 12 \\ a &= 6\end{aligned}$$

Solve equation (2).

$$\begin{aligned}(a + bi)(a - bi) &= 40 \\ a^2 + b^2 &= 40 \quad \text{Substitute 6 for } a \\ 36 + b^2 &= 40 \\ b &= \pm 2\end{aligned}$$

Thus, the two numbers are $6 + 2i$ and $6 - 2i$.

Simplify. Assume that no variable equals zero.

63. $x^5 \cdot x^3$

SOLUTION:

$$\begin{aligned}x^5 \cdot x^3 &= x^{5+3} \\ &= x^8\end{aligned}$$

64. $a^2 \cdot a^6$

SOLUTION:

$$\begin{aligned}a^2 \cdot a^6 &= a^{2+6} \\ &= a^8\end{aligned}$$

65. $(2p^2n)^3$

SOLUTION:

$$(2p^2n)^3 = 8p^6n^3$$

66. $(3b^3c^2)^2$

SOLUTION:

$$(3b^3c^2)^2 = 9b^6c^4$$

67. $\dfrac{x^4y^6}{xy^2}$

SOLUTION:

$$\frac{x^4y^6}{xy^2} = x^3y^4$$

68. $\left(\dfrac{c^9}{d^7}\right)^0$

SOLUTION:

$$\left(\frac{c^9}{d^7}\right)^0 = 1$$