Use $\log_4 3 \approx 0.7925$ and $\log_4 5 \approx 1.1610$ to approximate the value of each expression.

1. $\log_4 18$

SOLUTION:

$$\begin{aligned}\log_4 18 &= \log_4 (2 \cdot 3 \cdot 3)\\ &= \log_4 2 + \log_4 3 + \log_4 3\\ &\approx 0.5 + 0.7925 + 0.7925\\ &= 2.085\end{aligned}$$

2. $\log_4 15$

SOLUTION:

$$\begin{aligned}\log_4 15 &= \log_4 (3 \cdot 5)\\ &= \log_4 3 + \log_4 5\\ &\approx 0.7925 + 1.1610\\ &= 1.9535\end{aligned}$$

3. $\log_4 \frac{5}{3}$

SOLUTION:

$$\begin{aligned}\log_4 \frac{5}{3} &= \log_4 5 - \log_4 3\\ &\approx 1.1610 - 0.7925\\ &= 0.3685\end{aligned}$$

4. $\log_4 \frac{3}{4}$

SOLUTION:

$$\begin{aligned}\log_4 \frac{3}{4} &= \log_4 3 - \log_4 4\\ &\approx 0.7925 - 1\\ &= -0.2075\end{aligned}$$

5. **MOUNTAIN CLIMBING** As elevation increases, the atmospheric air pressure decreases. The formula for pressure based on elevation is $a = 15{,}500(5 - \log_{10} P)$, where a is the altitude in meters and P is the pressure in pascals (1 psi ≈ 6900 pascals). What is the air pressure at the summit in pascals for each mountain listed in the table at the right?

Mountain	Country	Height (m)
Everest	Nepal/Tibet	8850
Trisuli	India	7074
Bonete	Argentina/Chile	6872
McKinley	United States	6194
Logan	Canada	5959

SOLUTION:

Substitute 8850 for a, then evaluate P.

$$\begin{aligned}a &= 15500(5 - \log_{10} P)\\ 8850 &= 15500(5 - \log_{10} P)\\ (5 - \log_{10} P) &= \frac{8850}{15500}\\ \log_{10} P &= 5 - \frac{8850}{15500}\\ P &= 10^{5 - \frac{8850}{15500}}\\ &\approx 26855.44\end{aligned}$$

The air pressure at the summit of Mt. Everest is about 26,855.44 pascals.

Substitute 7074 for a, then evaluate P.

$$\begin{aligned}a &= 15500(5 - \log_{10} P)\\ 7074 &= 15500(5 - \log_{10} P)\\ (5 - \log_{10} P) &= \frac{7074}{15500}\\ \log_{10} P &= 5 - \frac{7074}{15500}\\ P &= 10^{5 - \frac{7074}{15500}}\\ &\approx 34963.34\end{aligned}$$

The air pressure at the summit of Mt. Trisuli is about 34963.34 pascals.

Substitute 6872 for a, then evaluate P.

$$\begin{aligned} a &= 15500(5 - \log_{10} P) \\ 6872 &= 15500(5 - \log_{10} P) \\ (5 - \log_{10} P) &= \frac{6872}{15500} \\ \log_{10} P &= 5 - \frac{6872}{15500} \\ P &= 10^{5 - \frac{6872}{15500}} \\ &\approx 36028.42 \end{aligned}$$

The air pressure at the summit of Mt. Bonete is about 36028.42 pascals.
Substitute 6194 for a, then evaluate P.

$$\begin{aligned} a &= 15500(5 - \log_{10} P) \\ 6194 &= 15500(5 - \log_{10} P) \\ (5 - \log_{10} P) &= \frac{6194}{15500} \\ \log_{10} P &= 5 - \frac{6194}{15500} \\ P &= 10^{5 - \frac{6194}{15500}} \\ &\approx 39846.22 \end{aligned}$$

The air pressure at the summit of Mt. McKinley is about 39846.22 pascals.
Substitute 5959 for a, then evaluate P.

$$\begin{aligned} a &= 15500(5 - \log_{10} P) \\ 5959 &= 15500(5 - \log_{10} P) \\ (5 - \log_{10} P) &= \frac{5959}{15500} \\ \log_{10} P &= 5 - \frac{5959}{15500} \\ P &= 10^{5 - \frac{5959}{15500}} \\ &\approx 41261.82 \end{aligned}$$

The air pressure at the summit of Mt. Logan is 41261.82 pascals.

Given $\log_3 5 \approx 1.465$ and $\log_5 7 \approx 1.2091$, approximate the value of each expression.

6. $\log_3 25$

SOLUTION:

$$\begin{aligned} \log_3 25 &= \log_3 5^2 \\ &= 2\log_3 5 \\ &\approx 2(1.465) \\ &= 2.93 \end{aligned}$$

7. $\log_5 49$

SOLUTION:

$$\begin{aligned} \log_5 49 &= \log_5 7^2 \\ &= 2\log_5 7 \\ &\approx 2(1.2091) \\ &= 2.4182 \end{aligned}$$

Solve each equation. Check your solutions.

8. $\log_4 48 - \log_4 n = \log_4 6$

SOLUTION:

$$\begin{aligned} \log_4 48 - \log_4 n &= \log_4 6 \\ \log_4 (4 \cdot 4 \cdot 3) - \log_4 n &= \log_4 (3 \cdot 2) \\ \log_4 4 + \log_4 4 + \log_4 3 - \log_4 n &= \log_4 3 + \log_4 2 \\ 1 + 1 + \log_4 3 - \log_4 n &= \log_4 3 + 0.5 \\ \log_4 n &= 1.5 \\ n &= 4^{1.5} \\ n &= 8 \end{aligned}$$

9. $\log_3 2x + \log_3 7 = \log_3 28$

SOLUTION:

$$\begin{aligned} \log_3 2x + \log_3 7 &= \log_3 28 \\ \log_3 2 + \log_3 x + \log_3 7 &= \log_3 (2 \cdot 2 \cdot 7) \\ \log_3 2 + \log_3 x + \log_3 7 &= \log_3 2 + \log_3 2 + \log_3 7 \\ \log_3 x &= \log_3 2 \\ x &= 2 \end{aligned}$$

10. $3 \log_2 x = \log_2 8$

SOLUTION:

$$\begin{aligned} 3\log_2 x &= \log_2 8 \\ 3\log_2 x &= \log_2 2^3 \\ 3\log_2 x &= 3\log_2 2 \\ \log_2 x &= \log_2 2 \\ x &= 2 \end{aligned}$$

11. $\log_{10} a + \log_{10} (a - 6) = 2$

SOLUTION:

$$\begin{aligned} \log_{10} a + \log_{10}(a-6) &= 2 \\ \log_{10} a(a-6) &= 2 \\ a(a-6) &= 10^2 \\ a^2 - 6a &= 100 \\ a^2 - 6a - 100 &= 0 \end{aligned}$$

By quadratic formula:

$$\begin{aligned} a &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-100)}}{2(1)} \\ &= 13.4403 \text{ or } -7.4403 \end{aligned}$$

The logarithm is not defined for negative values. Therefore, the solution is 13.4403.

Use $\log_4 2 = 0.5$, $\log_4 3 \approx 0.7925$ and $\log_4 5 = 1.1610$ to approximate the value of each expression.

12. $\log_4 30$

SOLUTION:

$$\begin{aligned} \log_4 30 &= \log_4(2 \cdot 3 \cdot 5) \\ &= \log_4 2 + \log_4 3 + \log_4 5 \\ &\approx 0.5 + 0.7925 + 1.1610 \\ &= 2.4535 \end{aligned}$$

13. $\log_4 20$

SOLUTION:

$$\begin{aligned} \log_4 20 &= \log_4 2 \cdot 2 \cdot 5 \\ &= \log_4 2 + \log_4 2 + \log_4 5 \\ &\approx 0.5 + 0.5 + 1.1610 \\ &= 2.1610 \end{aligned}$$

14. $\log_4 \frac{2}{3}$

SOLUTION:

$$\begin{aligned} \log_4 \frac{2}{3} &= \log_4 2 - \log_4 3 \\ &\approx 0.5 - 0.7925 \\ &= -0.2925 \end{aligned}$$

15. $\log_4 \frac{4}{3}$

SOLUTION:

$$\begin{aligned} \log_4 \frac{4}{3} &= \log_4 \frac{2 \cdot 2}{3} \\ &= \log_4 2 + \log_4 2 - \log_4 3 \\ &\approx 0.5 + 0.5 - 0.7925 \\ &= 0.2075 \end{aligned}$$

16. $\log_4 9$

SOLUTION:

$$\begin{aligned} \log_4 9 &= \log_4 3^2 \\ &= 2\log_4 3 \\ &\approx 2(0.7925) \\ &= 1.585 \end{aligned}$$

17. $\log_4 8$

SOLUTION:

$$\begin{aligned} \log_4 8 &= \log_4 2^3 \\ &= 3\log_4 2 \\ &= 3(0.5) \\ &= 1.5 \end{aligned}$$

18. **SCIENCE** The magnitude M of an earthquake is measured on the Richter scale using the formula $M = \log_{10} x$, where x is the intensity of the seismic wave causing the ground motion. In 2007, an

earthquake near San Francisco registered approximately 5.6 on the Richter scale. The famous San Francisco earthquake of 1906 measured 8.3 in magnitude.
a. How many times as intense was the 1906 earthquake as the 2007 earthquake?
b. Richter himself classified the 1906 earthquake as having a magnitude of 8.3. More recent research indicates it was most likely a 7.9. How many times as great was the intensity of Richter's measure of the earthquake?

SOLUTION:

a. The magnitude of an earthquake is measured on a logarithmic scale called the Richter scale. The magnitude M is given by $M = \log_{10} x$, where x represents the amplitude of the seismic wave causing ground motion.
Substitute 8.3 and 5.6 for M, then evaluate the corresponding values of x.

$$M = \log_{10} x$$
1906:
$$8.3 = \log_{10} x$$
$$10^{8.3} = x$$
2007:
$$5.6 = \log_{10} x$$
$$10^{5.6} = x$$

The ratio between the magnitudes is $\frac{10^{8.3}}{10^{5.6}} = 10^{2.7}$.

The 1906 earthquake was $10^{2.7}$ or about 500 times more intense than the 2007 earthquake.
b. Substitute 8.3 and 7.9 for M then evaluate the corresponding values of x.

$$\begin{aligned} M &= \log_{10} x \\ 8.3 &= \log_{10} x \\ 10^{8.3} &= x \\ 7.9 &= \log_{10} x \\ 10^{7.9} &= x \end{aligned}$$

The ratio between the magnitudes is $\frac{10^{8.3}}{10^{7.9}} = 10^{0.4}$.

Richter thought the earthquake was $10^{0.4}$ or about $2\frac{1}{2}$ times more intense than it actually was.

Given $\log_6 8 \approx 1.1606$ and $\log_7 9 \approx 1.1292$, approximate the value of each expression.

19. $\log_6 48$

SOLUTION:

$$\begin{aligned} \log_6 48 &= \log_6 (6 \cdot 8) \\ &= \log_6 6 + \log_6 8 \\ &\approx 1 + 1.1606 \\ &= 2.1606 \end{aligned}$$

20. $\log_7 81$

SOLUTION:

$$\begin{aligned} \log_7 81 &= \log_7 9^2 \\ &= 2\log_7 9 \\ &\approx 2(1.1292) \\ &= 2.2584 \end{aligned}$$

21. $\log_6 512$

SOLUTION:

$$\begin{aligned} \log_6 512 &= \log_6 \left(8^3\right) \\ &= \log_6 8^3 \\ &= 3\log_6 8 \\ &\approx 3(1.1606) \\ &= 3.4818 \end{aligned}$$

22. $\log_7 729$

SOLUTION:

$$\begin{aligned} \log_7 729 &= \log_7 \left(9^3\right) \\ &= 3\log_7 9 \\ &\approx 3(1.1292) \\ &= 3.3876 \end{aligned}$$

CCSS PERSEVERANCE Solve each equation. Check your solutions.

23. $\log_3 56 - \log_3 n = \log_3 7$

SOLUTION:

$$\begin{aligned}\log_3 56 - \log_3 n &= \log_3 7\\ \log_3(8\cdot 7) - \log_3 n &= \log_3 7\\ \log_3 8 + \log_3 7 - \log_3 n &= \log_3 7\\ \log_3 n &= \log_3 8\\ n &= 8\end{aligned}$$

24. $\log_2 (4x) + \log_2 5 = \log_2 40$

SOLUTION:

$$\begin{aligned}\log_2 4x + \log_2 5 &= \log_2 40\\ \log_2 4 + \log_2 x + \log_2 5 &= \log_2(4\cdot 5\cdot 2)\\ \log_2 4 + \log_2 x - \log_2 5 &= \log_2 4 + \log_2 5 + \log_2 2\\ \log_2 x &= \log_2 2\\ x &= 2\end{aligned}$$

25. $5 \log_2 x = \log_2 32$

SOLUTION:

$$\begin{aligned}5\log_2 x &= \log_2 32\\ 5\log_2 x &= \log_2 2^5\\ 5\log_2 x &= 5\log_2 2\\ \log_2 x &= \log_2 2\\ x &= 2\end{aligned}$$

26. $\log_{10} a + \log_{10} (a + 21) = 2$

SOLUTION:

$$\begin{aligned}\log_{10} a + \log_{10}(a+21) &= 2\\ \log_{10} a(a+21) &= 2\\ a(a+21) &= 10^2\\ a^2 + 21a - 100 &= 0\\ (a+25)(a-4) &= 0\end{aligned}$$

By the Zero Product Property:

$$\begin{aligned}a + 25 &= 0 \quad\text{or}\quad a - 4 = 0\\ a &= -25 \quad\text{or}\quad a = 4\end{aligned}$$

The logarithm is not defined for negative values. Therefore, the solution is 4.

27. **PROBABILITY** In the 1930s, Dr. Frank Benford demonstrated a way to determine whether a set of numbers has been randomly chosen or manually chosen. If the sets of numbers were not randomly chosen, then the Benford formula, $P = \log_{10}\left(1 + \frac{1}{d}\right)$, predicts the probability of a digit d being the first digit of the set. For example, there is a 4.6% probability that the first digit is 9.

a. Rewrite the formula to solve for the digit if given the probability.

b. Find the digit that has a 9.7% probability of being selected.

c. Find the probability that the first digit is 1 ($\log_{10} 2 \approx 0.30103$).

SOLUTION:

a. Rewrite the function d in terms of P.

$$\begin{aligned}P &= \log_{10}\left(1 + \frac{1}{d}\right)\\ 10^P &= 1 + \frac{1}{d}\\ \frac{1}{d} &= 10^P - 1\\ d &= \frac{1}{10^P - 1}\end{aligned}$$

b. Substitute 0.097 for P and evaluate.

$$\begin{aligned}d &= \frac{1}{10^{0.097} - 1}\\ &\approx 4\end{aligned}$$

c. Substitute 1 for d in the formula $P = \log_{10}\left(1 + \frac{1}{d}\right)$ and evaluate.

$$\begin{aligned}P &= \log_{10}\left(1 + \frac{1}{1}\right)\\ &= \log_{10} 2\\ &\approx 0.30103 = 30.1\%\end{aligned}$$

Use $\log_5 3 \approx 0.6826$ and $\log_5 4 \approx 0.8614$ to approximate the value of each expression.

28. $\log_5 40$

SOLUTION:

$$\begin{aligned}\log_5 40 &= \log_5(2\cdot 4\cdot 5)\\ &= \log_5 2 + \log_5 4 + \log_5 5\\ &\approx \frac{1}{2}\log_5 4 + 0.8614 + 1\\ &\approx 0.4307 + 1.8614\\ &= 2.2921\end{aligned}$$

29. $\log_5 30$

SOLUTION:

$$\begin{aligned}\log_5 30 &= \log_5(2\cdot 3\cdot 5)\\ &= \log_5 2 + \log_5 3 + \log_5 5\\ &= \frac{1}{2}\log_5 4 + 0.6826 + 1\\ &\approx 0.4307 + 1.6826\\ &= 2.1133\end{aligned}$$

30. $\log_5 \frac{3}{4}$

SOLUTION:

$$\begin{aligned}\log_5 \frac{3}{4} &= \log_5 3 - \log_5 4\\ &\approx 0.6826 - 0.8614\\ &= -0.1788\end{aligned}$$

31. $\log_5 \frac{4}{3}$

SOLUTION:

$$\begin{aligned}\log_5 \frac{4}{3} &= \log_5 4 - \log_5 3\\ &\approx 0.8614 - 0.6826\\ &= 0.1788\end{aligned}$$

32. $\log_5 9$

SOLUTION:

$$\begin{aligned}\log_5 9 &= \log_5 3^2\\ &= 2\log_5 3\\ &\approx 2(0.6826)\\ &= 1.3652\end{aligned}$$

33. $\log_5 16$

SOLUTION:

$$\begin{aligned}\log_5 16 &= \log_5 4^2\\ &= 2\log_5 4\\ &\approx 2(0.8614)\\ &= 1.7228\end{aligned}$$

34. $\log_5 12$

SOLUTION:

$$\begin{aligned}\log_5 12 &= \log_5(4\cdot 3)\\ &= \log_5 4 + \log_5 3\\ &\approx 0.8614 + 0.6826\\ &= 1.544\end{aligned}$$

35. $\log_5 27$

SOLUTION:

$$\begin{aligned}\log_5 27 &= \log_5 3^3\\ &= 3\log_5 3\\ &\approx 3(0.6826)\\ &= 2.0478\end{aligned}$$

Solve each equation. Check your solutions.

36. $\log_3 6 + \log_3 x = \log_3 12$

SOLUTION:

$$\begin{aligned}\log_3 6 + \log_3 x &= \log_3 12\\ \log_3 6 + \log_3 x &= \log_3(6\cdot 2)\\ \log_3 6 + \log_3 x &= \log_3 6 + \log_3 2\\ \log_3 x &= \log_3 2\\ x &= 2\end{aligned}$$

7-5 Properties of Logarithms

37. $\log_4 a + \log_4 8 = \log_4 24$

SOLUTION:

$$\begin{aligned}
\log_4 a + \log_4 8 &= \log_4 24 \\
\log_4 a + \log_4 8 &= \log_4 (8 \cdot 3) \\
\log_4 a + \log_4 8 &= \log_4 8 + \log_3 3 \\
\log_4 a &= \log_4 3 \\
a &= 3
\end{aligned}$$

38. $\log_{10} 18 - \log_{10} 3x = \log_{10} 2$

SOLUTION:

$$\begin{aligned}
\log_{10} 18 - \log_{10} 3x &= \log_{10} 2 \\
\log_{10} 18 - \log_{10} 2 &= \log_{10} 3x \\
\log_{10} \frac{18}{2} &= \log_{10} 3 + \log_{10} x \\
\log_{10} 9 &= \log_{10} 3 + \log_{10} x \\
\log_{10} (3 \cdot 3) &= \log_{10} 3 + \log_{10} x \\
\log_{10} 3 + \log_{10} 3 &= \log_{10} 3 + \log_{10} x \\
\log_{10} 3 &= \log_{10} x \\
x &= 3
\end{aligned}$$

39. $\log_7 100 - \log_7 (y + 5) = \log_7 10$

SOLUTION:

$$\begin{aligned}
\log_7 100 - \log_7 (y+5) &= \log_7 10 \\
\log_7 100 - \log_7 10 &= \log_7 (y+5) \\
\log_7 \frac{100}{10} &= \log_7 (y+5) \\
\log_7 10 &= \log_7 (y+5) \\
y + 5 &= 10 \\
y &= 5
\end{aligned}$$

40. $\log_2 n = \frac{1}{3}\log_2 27 + \log_2 36$

SOLUTION:

$$\begin{aligned}
\log_2 n &= \frac{1}{3}\log_2 27 + \log_2 36 \\
\log_2 n &= \log_2 (27)^{1/3} + \log_2 36 \\
\log_2 n &= \log_2 3 + \log_2 36 \\
\log_2 n &= \log_2 (3 \cdot 36) \\
\log_2 n &= \log_2 108 \\
n &= 108
\end{aligned}$$

41. $3\log_{10} 8 - \frac{1}{2}\log_{10} 36 = \log_{10} x$

SOLUTION:

$$\begin{aligned}
3\log_{10} 8 - \frac{1}{2}\log_{10} 36 &= \log_{10} x \\
\log_{10} 8^3 - \log_{10} 36^{1/2} &= \log_{10} x \\
\log_{10} 512 - \log_{10} 6 &= \log_{10} x \\
\log_{10} \frac{512}{6} &= \log_{10} x \\
x = \frac{512}{6} &= 85\frac{1}{3}
\end{aligned}$$

Solve for *n*.

42. $\log_a 6n - 3\log_a x = \log_a x$

SOLUTION:

$$\begin{aligned}
\log_a 6n - 3\log_a x &= \log_a x \\
\log_a 6n &= \log_a x + 3\log_a x \\
\log_a 6n &= \log_a x + \log_a x^3 \\
\log_a 6n &= \log_a (x \cdot x^3) \\
6n &= x^4 \\
n &= \frac{x^4}{6}
\end{aligned}$$

43. $2 \log_b 16 + 6 \log_b n = \log_b (x - 2)$

SOLUTION:

$$\begin{aligned} 2\log_b 16 + 6\log_b n &= \log_b (x-2) \\ \log_b 16^2 + \log_b n^6 &= \log_b (x-2) \\ \log_b 256 n^6 &= \log_b (x-2) \\ 256 n^6 &= x - 2 \\ n^6 &= \frac{x-2}{256} \\ n &= \left(\frac{x-2}{256}\right)^{\frac{1}{6}} \end{aligned}$$

Solve each equation. Check your solutions.

44. $\log_{10} z + \log_{10} (z + 9) = 1$

SOLUTION:

$$\begin{aligned} \log_{10} z + \log_{10}(z+9) &= 1 \\ \log_{10}(z(z+9)) &= 1 \\ z(z+9) &= 10^1 \\ z^2 + 9z &= 10 \\ z^2 + 9z - 10 &= 0 \\ (z+10)(z-1) &= 0 \\ z &= -10 \text{ or } 1 \end{aligned}$$

45. $\log_3 (a^2 + 3) + \log_3 3 = 3$

SOLUTION:

$$\begin{aligned} \log_3\left(a^2+3\right) + \log_3 3 &= 3 \\ \log_3\left(a^2+3\right) + 1 &= 3 \\ \log_3\left(a^2+3\right) &= 2 \\ a^2 + 3 &= 3^2 \\ a^2 + 3 &= 9 \\ a^2 &= 6 \\ a &= \pm\sqrt{6} \end{aligned}$$

46. $\log_2 (15b - 15) - \log_2 (-b^2 + 1) = 1$

SOLUTION:

$$\begin{aligned} \log_2(15b - 15) - \log_2\left(-b^2+1\right) &= 1 \\ \log_2\left(\frac{15b-15}{1-b^2}\right) &= 1 \\ \frac{15b-15}{1-b^2} &= 2^1 \\ 15b - 15 &= 2\left(1-b^2\right) \\ 2b^2 + 15b - 17 &= 0 \\ (b-1)(2b+17) &= 0 \\ b &= 1 \text{ or } -8.5 \end{aligned}$$

Substitute each value into the original equation.

$$x = 1 \qquad\qquad x = -8.5$$
$$\log_2[15\cdot(1) - 15] - \log_2\left[-(1)^2+1\right] \stackrel{?}{=} 1 \quad \log_2[15(-8.5) - 15] - \log_2\left[-(-8.5)^2+1\right] \stackrel{?}{=} 1$$
$$\log_2 0 - \log_2 0 \neq 1 \qquad\qquad \log_2(-142.5) - \log_2(-71.25) \neq 1$$

$\log_2 0$, $\log_2(-142.5)$, and $\log_2(-71.25)$ are undefined, so 1 and –8.5 are extraneous solutions. Therefore, the equation has no solution.

47. $\log_4 (2y + 2) - \log_4 (y - 2) = 1$

SOLUTION:

$$\begin{aligned} \log_4(2y+2) - \log_4(y-2) &= 1 \\ \log_4\left(\frac{2y+2}{y-2}\right) &= 1 \\ \frac{2y+2}{y-2} &= 4^1 \\ 2y + 2 &= 4(y-2) \\ 2y &= 10 \\ y &= 5 \end{aligned}$$

48. $\log_6 0.1 + 2\log_6 x = \log_6 2 + \log_6 5$

SOLUTION:

$$\log_6 0.1 + 2\log_6 x = \log_6 2 + \log_6 5$$
$$\log_6 0.1 + \log_6 x^2 = \log_6 2(5)$$
$$\log_6 0.1x^2 = \log_6 10$$
$$0.1x^2 = 10$$
$$x^2 = 100$$
$$x = \pm 10$$

Logarithms are not defined for negative values. Therefore, the solution is 10.

49. $\log_7 64 - \log_7 \frac{8}{3} + \log_7 2 = \log_7 4p$

SOLUTION:

$$\log_7 64 - \log_7 \frac{8}{3} + \log_7 2 = \log_7 4p$$
$$\log_7 \frac{64 \cdot 2}{\frac{8}{3}} = \log_7 4p$$
$$4p = 48$$
$$p = 12$$

50. **CCSS REASONING** The humpback whale is an endangered species. Suppose there are 5000 humpback whales in existence today, and the population decreases at a rate of 4% per year.
a. Write a logarithmic function for the time in years based upon population.
b. After how long will the population drop below 1000? Round your answer to the nearest year.

SOLUTION:

a. It's more natural to write an exponential function of population as a function of time. That would be:
$p = 5000t^{0.96}$

Rewrite this as a log function.

$$t = \log_{0.96}\left(\frac{p}{5000}\right)$$

b. Substitute 1000 for p.

$$t = \log_{0.96}\left(\frac{1000}{5000}\right)$$
$$t = \log_{0.96}(0.2)$$
$$t \approx 39$$

It will take about 39 years for the population to drop below 1000.

State whether each equation is *true* or *false*.

51. $\log_8 (x - 3) = \log_8 x - \log_8 3$

SOLUTION:

$$\log_8 (x-3) \neq \log_8 x - \log_8 3$$

Therefore, the equation is false.

52. $\log_5 22x = \log_5 22 + \log_5 x$

SOLUTION:

$$\log_5 22x = \log_5 22 + \log_5 x$$

Therefore, the equation is true.

53. $\log_{10} 19k = 19 \log_{10} k$

SOLUTION:

$$\log_{10} 19k \neq 19 \log_{10} k$$

Therefore, the equation is false.

54. $\log_2 y^5 = 5 \log_2 y$

SOLUTION:

$$\log_2 y^5 = 5 \log_2 y$$

Therefore, the equation is true.

55. $\log_7 \frac{x}{3} = \log_7 x - \log_7 3$

SOLUTION:

$$\log_7 \frac{x}{3} = \log_7 x - \log_7 3$$

Therefore, the equation is true.

56. $\log_4 (z + 2) = \log_4 z + \log_4 2$

SOLUTION:

$$\log_4 (z+2) \neq \log_4 z + \log_4 2$$

Therefore, the equation is false.

57. $\log_8 p^4 = (\log_8 p)^4$

SOLUTION:

$$\log_8 p^4 \neq (\log_8 p)^4$$

Therefore, the equation is false.

58. $\log_9 \frac{x^2 y^3}{z^4} = 2 \log_9 x + 3 \log_9 y - 4 \log_9 z$

SOLUTION:

$$\begin{aligned}\log_9 \frac{x^2 y^3}{z^4} &= \log_9 x^2 + \log_9 y^3 - \log_9 z^4 \\ &= 2 \log_9 x + 3 \log_9 y - 4 \log_9 z\end{aligned}$$

Therefore, the equation is true.

59. **PARADE** An equation for loudness L, in decibels, is $L = 10 \log_{10} R$, where R is the relative intensity of the sound.
a. Solve $120 = 10 \log_{10} R$ to find the relative intensity of the Macy's Thanksgiving Day Parade with a loudness of 120 decibels depending on how close you are.
b. Some parents with young children want the decibel level lowered to 80. How many times less intense would this be? In other words, find the ratio of their intensities.

SOLUTION:

a. Solve for R.

$$\begin{aligned}120 &= 10 \log_{10} R \\ 12 &= \log_{10} R \\ 10^{12} &= R\end{aligned}$$

b. Substitute 80 for L and solve for R.

$$\begin{aligned}80 &= 10 \log_{10} R \\ 8 &= \log_{10} R \\ 10^{8} &= R\end{aligned}$$

The ratio of their intensities is $\frac{10^{12}}{10^8} = 10^4$.

Therefore, the ratio is 10^4 or about 10,000 times.

60. **FINANCIAL LITERACY** The average American carries a credit card debt of approximately \$8600 with an annual percentage rate (APR) of 18.3%.

The formula $m = \frac{b\left(\frac{r}{n}\right)}{1 - \left(1 + \frac{r}{n}\right)^{-nt}}$ can be used to compute the monthly payment m that is necessary to pay off a credit card balance b in a given number of years t, where r is the annual percentage rate and n is the number of payments per year.
a. What monthly payment should be made in order to pay off the debt in exactly three years? What is the total amount paid?
b. The equation $t = \frac{\log\left(1 - \frac{br}{mn}\right)}{-n \log\left(1 + \frac{r}{n}\right)}$ can be used to calculate the number of years necessary for a given payment schedule. Copy and complete the table.

c. Graph the information in the table from part b.
d. If you could only afford to pay $100 a month, will you be able to pay off the debt? If so, how long will it take? If not, why not?
e. What is the minimum monthly payment that will work toward paying off the debt?

SOLUTION:

a. Substitute 8600, 0.183, 3 and 12 for *b*, *r*, *t* and *n* respectively then evaluate.

$$m = \frac{8600\left(\frac{0.183}{12}\right)}{1-\left(1+\frac{0.183}{12}\right)^{-3(12)}}$$
$$= 312.21$$

The monthly payment should be $312.21.
The total amount paid is $312.21 \times 36 =$ $11,239.56.

b. Substitute 50, 100, 150, 200, 250 and 300 for *m* then solve for *t*.

Payment (m)	Years (t)
$50	non-real
$100	non-real
$150	11.42
$200	5.87
$250	4.09
$300	3.16

c. Graph the information in the table from part b.

d. Logarithm is not defined for negative values.

So, $1-\frac{br}{mn} > 0$

$$1 > \frac{br}{mn}$$
$$m > \frac{br}{n}$$
$$m > \frac{8600 \times 0.183}{12} = 131.15$$

No. The monthly interest is $131.15, so the payments do not even cover the interest.

e. Since $m > 131.15$, the minimum monthly payment should be $131.16.

61. **OPEN ENDED** Write a logarithmic expression for each condition. Then write the expanded expression.
a. a product and a quotient
b. a product and a power
c. a product, a quotient, and a power

SOLUTION:

a. Sample answer: $\log_b \frac{xz}{5} = \log_b x + \log_b z - \log_b 5$

b. Sample answer: $\log_b m^4 p^6 = 4 \log_b m + 6 \log_b p$

c. Sample answer:

$$\log_b \frac{j^8 k}{h^5} = 8 \log_b j + \log_b k - 5 \log_b h$$

62. **CCSS ARGUMENTS** Use the properties of exponents to prove the Power Property of Logarithms.

SOLUTION:

$$m^p = m^p$$
$$\left(b^{\log_b m}\right)^p = b^{\log_b (m^p)}$$
$$b^{\log_b m p} = b^{\log_b (m^p)}$$
$$\log_b m p = \log_b (m^p)$$
$$p \log_b m = \log_b (m^p)$$

63. **WRITING IN MATH** Explain why the following are true.

a. $\log_b 1 = 0$

b. $\log_b b = 1$

c. $\log_b b^x = x$

SOLUTION:

a. $\log_b 1 = 0$, because $b^0 = 1$.

b. $\log_b b = 1$, because $b^1 = b$.

c. $\log_b b^x = x$, because $b^x = b^x$.

64. **CHALLENGE** Simplify $\log_{\sqrt{a}}\left(a^2\right)$ to find an exact numerical value.

SOLUTION:

$$\log_{\sqrt{a}}\left(a^2\right) = x$$
$$\left(\sqrt{a}\right)^x = a^2$$
$$\left(a^{\frac{1}{2}}\right)^x = a^2$$
$$a^{\frac{x}{2}} = a^2$$
$$\frac{x}{2} = 2$$
$$x = 4$$

65. **WHICH ONE DOESN'T BELONG?** Find the expression that does not belong. Explain.

$\log_b 24 = \log_b 2 + \log_b 12$

$\log_b 24 = \log_b 8 + \log_b 3$

$\log_b 24 = \log_b 20 + \log_b 4$

$\log_b 24 = \log_b 4 + \log_b 6$

SOLUTION:

$\log_b 24 \neq \log_b 20 + \log_b 4$

All other choices are equal to $\log_b 24$.

66. **REASONING** Use the properties of logarithms to prove that $\log_a \frac{1}{x} = -\log_a x$

SOLUTION:

$\log_a \frac{1}{x} = -\log_a x$	Original equation
$\log_a x^{-1} = -\log_a x$	Definiton of negative exponents
$\log_a x^{-1} = (-1)\log_a x$	Power Property of Logarithms
$\log_a \frac{1}{x} = -\log_a x$	Simplify

67. Simplify $x^{3\log_x 2 - \log_x 5}$ to find an exact numerical value.

SOLUTION:

$$x^{3\log_x 2-\log_x 5} = x^{\log_x 2^3-\log_x 5}$$
$$= x^{\log_x 8-\log_x 5}$$
$$= x^{\log_x \frac{8}{5}}$$
$$= \frac{8}{5}$$

68. **WRITING IN MATH** Explain how the properties of exponents and logarithms are related. Include examples like the one shown at the beginning of the lesson illustrating the Product Property, but with the Quotient Property and Power Property of Logarithms.

SOLUTION:

Since logarithms are exponents, the properties of logarithms are similar to the properties of exponents. The Product Property states that to multiply two powers that have the same base, add the exponents. Similarly, the logarithm of a product is the sum of the logarithms of its factors.
The Quotient Property states that to divide two powers that have the same base, subtract their exponents. Similarly the logarithm of a quotient is the difference of the logarithms of the numerator and the denominator.
The Power Property states that to find the power of a power, multiply the exponents. Similarly, the logarithm of a power is the product of the logarithm and the exponent. Answers should include the following.

•

Quotient Property:

$\log_2\left(\frac{32}{8}\right) = \log_2\left(\frac{2^5}{2^3}\right)$ Replace 32 with 2^5 and 8 with 2^3.

$= \log_2 2^{(5-3)}$ Quotient of Powers

$= 5 - 3$ or 2 Inverse Property of Exponents and Logarithms

$$\begin{aligned}\log_2 32 - \log_2 8 &= \log_2 2^5 - \log_2 2^3\\ &= 5 - 3\\ &= 2\end{aligned}$$

So, $\log_2\left(\frac{32}{8}\right) = \log_2 32 - \log_2 8$

Power Property: $\log_3 9^4 = \log_3 (3^2)^4$ Replace 9 with 3^2.

$= \log_3 3^{(2 \cdot 4)}$ Power of a Power

$= 2 \cdot 4$ or 8 Inverse Property of Exponents and Logarithms

$4 \log_3 9 = (\log_3 9) \cdot 4$ Commutative Property ($\times$)

$= (\log_3 3^2) \cdot 4$ Replace 9 with 3^2.

$= 2 \cdot 4$ or 8 Inverse Property of Exponents and Logarithms

So, $\log_3 9^4 = 4 \log_3 9$.

• The Product of Powers Property and Product Property of Logarithms both involve the addition of exponents, since logarithms are exponents.

69. Find the mode of the data. 22, 11, 12, 23, 7, 6, 17, 15, 21, 19

A 11
B 15
C 16
D There is no mode.

SOLUTION:

None of the data are repeated more than once. Therefore, option D is correct.

70. **SAT/ACT** What is the effect on the graph of $y = 4x^2$ when the equation is changed to $y = 2x^2$?

F The graph is rotated 90 degrees about the origin.
G The graph is narrower.
H The graph is wider.
J The graph of $y = 2x^2$ is a reflection of the graph $y = 4x^2$ across the x-axis.
K The graph is unchanged.

SOLUTION:

The graph is wider.
Option H is the correct answer.

71. **SHORT RESPONSE** In $y = 6.5(1.07)^x$, x represents the number of years since 2000, and y represents the approximate number of millions of Americans 7 years of age and older who went camping two or more times that year. Describe how the number of millions of Americans who go camping is changing over time.

SOLUTION:

Since x represents the number of years since 2000, the value of the exponent is always positive. Therefore, it is growing exponentially.

72. What are the x-intercepts of the graph of $y = 4x^2 - 3x - 1$?

A. $-\frac{1}{4}$ and $\frac{1}{4}$

B. -1 and $\frac{1}{4}$

C. -1 and 1

D. 1 and $-\frac{1}{4}$

SOLUTION:

Substitute 0 for y and solve for x.

$$\begin{aligned}0 &= 4x^2 - 3x - 1\\ 0 &= 4x^2 - 4x + x - 1\\ 0 &= 4x(x-1) + 1(x-1)\\ 0 &= (4x+1)(x-1)\end{aligned}$$

By the Zero Product Property:

$$\begin{array}{lll} 4x + 1 = 0 & \text{or} & x - 1 = 0 \\ x = -\frac{1}{4} & \text{or} & x = 1 \end{array}$$

Therefore, option D is the correct answer.

Solve each equation. Check your solutions.

73. $\log_5 (3x - 1) = \log_5 (2x^2)$

SOLUTION:

$$\begin{aligned}\log_5(3x-1) &= \log_5(2x^2)\\ 3x - 1 &= 2x^2\\ 2x^2 - 3x + 1 &= 0\\ 2x^2 - 2x - x + 1 &= 0\\ 2x(x-1) - 1(x-1) &= 0\\ (x-1)(2x-1) &= 0\end{aligned}$$

By the Zero Product Property:

$$\begin{array}{lll} 2x - 1 = 0 & \text{or} & x - 1 = 0 \\ x = \frac{1}{2} & \text{or} & x = 1 \end{array}$$

Therefore, the solutions are 1 and $\frac{1}{2}$.

74. $\log_{10} (x^2 + 1) = 1$

SOLUTION:

$$\begin{aligned}\log_{10}(x^2+1) &= 1\\ x^2 + 1 &= 10^1\\ x^2 &= 9\\ x &= \pm 3\end{aligned}$$

Therefore, the solutions are ± 3.

75. $\log_{10} (x^2 - 10x) = \log_{10} (-21)$

SOLUTION:

$\log_{10}(-21)$ is not defined.

Therefore, there is no solution.

7-5 Properties of Logarithms

Evaluate each expression.

76. $\log_{10} 0.001$

SOLUTION:

$$\begin{aligned}\log_{10} 0.001 &= \log_{10} 10^{-3}\\ &= -3\log_{10} 10\\ &= -3(1)\\ &= -3\end{aligned}$$

77. $\log_4 16^x$

SOLUTION:

$$\begin{aligned}\log_4 16^x &= \log_4 \left(4^2\right)^x\\ &= \log_4 4^{2x}\\ &= 2x\log_4 4\\ &= 2x(1)\\ &= 2x\end{aligned}$$

78. $\log_3 27^x$

SOLUTION:

$$\begin{aligned}\log_3 27^x &= \log_3 \left(3^3\right)^x\\ &= \log_3 3^{3x}\\ &= 3x\log_3 3\\ &= 3x(1)\\ &= 3x\end{aligned}$$

79. **ELECTRICITY** The amount of current in amperes *I* that an appliance uses can be calculated using the formula $I = \left(\frac{P}{R}\right)^{\frac{1}{2}}$, where *P* is the power in watts and *R* is the resistance in ohms. How much current does an appliance use if $P = 120$ watts and $R = 3$ ohms? Round to the nearest tenth.

SOLUTION:

Substitute 120 and 3 for *P* and *R* and evaluate.

$$\begin{aligned}I &= \left(\frac{120}{3}\right)^{\frac{1}{2}}\\ &= 40^{\frac{1}{2}}\\ &\approx 6.3\end{aligned}$$

Determine whether each pair of functions are inverse functions. Write *yes* or *no*.

80. $f(x) = x + 73$
$g(x) = x - 73$

SOLUTION:

$$\begin{aligned}[f\circ g](x) &= f(g(x))\\ &= f(x-73)\\ &= (x-73)+73\\ &= x\\ [g\circ f](x) &= g(f(x))\\ &= g(x+73)\\ &= (x+73)-73\\ &= x\end{aligned}$$

Since $[f\circ g](x) = [g\circ f](x) = x$, they are inverse functions.

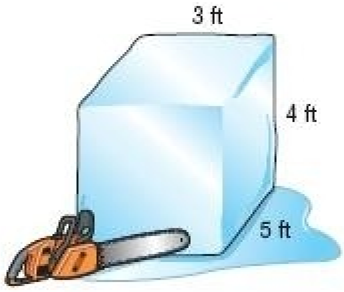
81. $g(x) = 7x - 11$
$h(x) = \frac{1}{7}x + 11$

SOLUTION:

$$\begin{aligned}[h\circ g](x) &= h(g(x))\\ &= h(7x-11)\\ &= \frac{1}{7}(7x-11)+11\\ &= x - \frac{11}{7} + 11\\ &= x + \frac{66}{7}\\ [g\circ h](x) &= g(h(x))\\ &= g\left(\frac{1}{7}x+11\right)\\ &= 7\left(\frac{1}{7}x+11\right)-11\\ &= x+77-11\\ &= x+66\end{aligned}$$

Since $[f\circ g](x) \neq [g\circ f](x)$, they are not inverse functions.

82. **SCULPTING** Antonio is preparing to make an ice sculpture. He has a block of ice that he wants to reduce in size by shaving off the same amount from the length, width, and height. He wants to reduce the volume of the ice block to 24 cubic feet.



a. Write a polynomial equation to model this situation.
b. How much should he take from each dimension?

SOLUTION:

a. The dimensions of the ice block is 3 ft, 4 ft and 5 ft.
Let x be the shaving off the amount of ice in a side.
The equation representing this situation is:

$$(3-x)(4-x)(5-x)=24$$

b. Solve the above equation.

$$\begin{aligned}(3-x)(4-x)(5-x)&=24\\(12-7x+x^2)(5-x)&=24\\-x^3+12x^2-47x+60&=24\\-x^3+12x^2-47x+36&=0\\(x-1)(-x^2+11x-36)&=0\end{aligned}$$

By the Zero Product Property:

$x-1=0$ or $\quad -x^2+11x-36=0$

The expression $-x^2+11x-36$ is a prime.

So, $x = 1$.
He should take 1 ft from each dimension.

Solve each equation or inequality. Check your solution.

83. $3^{4x} = 3^{3-x}$

SOLUTION:

$$\begin{aligned}3^{4x}&=3^{3-x}\\4x&=3-x\\5x&=3\\x&=\frac{3}{5}\end{aligned}$$

The solution is $\frac{3}{5}$.

84. $3^{2n} \le \frac{1}{9}$

SOLUTION:

$$\begin{aligned}3^{2n}&\le\frac{1}{9}\\3^{2n}&\le\frac{1}{3^2}\\3^{2n}&\le3^{-2}\\2n&\le-2\\n&\le-1\end{aligned}$$

The solution region is $n \le -1$.

85. $3^{5x} \cdot 81^{1-x} = 9^{x-3}$

SOLUTION:

$$\begin{aligned}3^{5x}\cdot81^{1-x}&=9^{x-3}\\3^{5x}\cdot(3^4)^{1-x}&=(3^2)^{x-3}\\3^{5x}\cdot3^{4-4x}&=3^{2x-6}\\3^{x+4}&=3^{2x-6}\\x+4&=2x-6\\x&=10\end{aligned}$$

The solution is 10.

86. $49^x = 7^{x^2-15}$

SOLUTION:

$$\begin{aligned} 49^x &= 7^{x^2-15} \\ (7^2)^x &= 7^{x^2-15} \\ 7^{2x} &= 7^{x^2-15} \\ 2x &= x^2 - 15 \\ x^2 - 2x - 15 &= 0 \\ (x-5)(x+3) &= 0 \end{aligned}$$

By the Zero Product Property:

$$\begin{array}{lll} x+3=0 & \text{or} & x-5=0 \\ x=-3 & \text{or} & x=5 \end{array}$$

Therefore, the solutions are –3 and 5.

87. $\log_2 (x + 6) > 5$

SOLUTION:

$$\begin{aligned} \log_2(x+6) &> 5 \\ x+6 &> 2^5 \\ x+6 &> 32 \\ x &> 26 \end{aligned}$$

The solution region is $x > 26$.

88. $\log_5 (4x - 1) = \log_5 (3x + 2)$

SOLUTION:

$$\begin{aligned} \log_5(4x-1) &= \log_5(3x+2) \\ 4x-1 &= 3x+2 \\ x &= 3 \end{aligned}$$

The solution is 3.