Use a calculator to evaluate each expression to the nearest ten-thousandth.

1. log 5

SOLUTION:
KEYSTROKES: LOG 5 ENTER 0.698970043

$$\log 5 \approx 0.6990$$

2. log 21

SOLUTION:
KEYSTROKES: LOG 21 ENTER 1.3222192947

$$\log 21 \approx 1.3222$$

3. log 0.4

SOLUTION:
KEYSTROKES: LOG 0 . 4 ENTER –0.39794000867

$$\log 0.4 \approx -0.3979$$

4. log 0.7

SOLUTION:
KEYSTROKES: LOG 0 . 7 ENTER -0.1549019599857

$$\log 0.7 \approx -0.15490$$

5. **SCIENCE** The amount of energy E in ergs that an earthquake releases is related to its Richter scale magnitude M by the equation $\log E = 11.8 + 1.5M$. Use the equation to find the amount of energy released by the 1960 Chilean earthquake, which measured 8.5 on the Richter scale.

SOLUTION:
Substitute 8.5 for M in the equation and evaluate.

$$\begin{aligned}\log E &= 11.8 + 1.5M \\ &= 11.8 + 1.5(8.5) \\ \log E &= 24.55 \\ E &= 10^{24.55} \\ E &= 3.55 \times 10^{24}\end{aligned}$$

The amount of energy released by the 1960 Chilean earthquake is 3.55×10^{24} ergs.

Solve each equation. Round to the nearest ten-thousandth.

6. $6^x = 40$

SOLUTION:

$$\begin{aligned}6^x &= 40 \\ \log 6^x &= \log 40 \\ x \log 6 &= \log 40 \\ x &= \frac{\log 40}{\log 6} \\ &\approx 2.0588\end{aligned}$$

The solution is about 2.0588.

7. $2.1^{a+2} = 8.25$

SOLUTION:

$$\begin{aligned} 2.1^{a+2} &= 8.25 \\ \log 2.1^{a+2} &= \log 8.25 \\ (a+2)\log 2.1 &= \log 8.25 \\ a+2 &= \frac{\log 8.25}{\log 2.1} \\ a &= \frac{\log 8.25}{\log 2.1} - 2 \\ &\approx 0.8442 \end{aligned}$$

The solution is about 0.8442.

8. $7^{x^2} = 20.42$

SOLUTION:

$$\begin{aligned} 7^{x^2} &= 20.42 \\ \log 7^{x^2} &= \log 20.42 \\ x^2 \log 7 &= \log 20.42 \\ x^2 &= \frac{\log 20.42}{\log 7} \\ x &= \sqrt{\frac{\log 20.42}{\log 7}} \\ &\approx \pm 1.2451 \end{aligned}$$

The solution is about ± 1.2451.

9. $11^{b-3} = 5^b$

SOLUTION:

$$\begin{aligned} 11^{b-3} &= 5^b \\ \log 11^{b-3} &= \log 5^b \\ (b-3)\log 11 &= b \log 5 \\ \frac{b-3}{b} &= \frac{\log 5}{\log 11} \\ 1 - \frac{3}{b} &= \frac{\log 5}{\log 11} \\ \frac{3}{b} &= 1 - \frac{\log 5}{\log 11} \\ &\approx 0.3288 \\ b &\approx 9.1237 \end{aligned}$$

The solution is about 9.1237.

Solve each inequality. Round to the nearest ten-thousandth.

10. $5^{4n} > 33$

SOLUTION:

$$\begin{aligned} 5^{4n} &> 33 \\ \log 5^{4n} &> \log 33 \\ 4n \log 5 &> \log 33 \\ n &> \frac{\log 33}{4 \log 5} \\ &> 0.5431 \end{aligned}$$

The solution region is $\{n \mid n > 0.5431\}$.

11. $6^{p-1} \le 4^p$

SOLUTION:

$$\begin{aligned}
6^{p-1} &\le 4^p \\
\log 6^{p-1} &\le \log 4^p \\
(p-1)\log 6 &\le p\log 4 \\
\frac{p-1}{p} &\le \frac{\log 4}{\log 6} \\
1-\frac{1}{p} &\le \frac{\log 4}{\log 6} \\
\frac{1}{p} &\ge 1-\frac{\log 4}{\log 6} \\
\frac{1}{p} &\ge \frac{\log 6}{\log 6}-\frac{\log 4}{\log 6} \\
p\bullet\frac{1}{p} &\ge \frac{\log 6-\log 4}{\log 6}\bullet p \\
1 &\ge \frac{\log 6-\log 4}{\log 6}\bullet p \\
\frac{1}{\frac{\log 6-\log 4}{\log 6}} &\ge p \\
4.4190 &\ge p \\
p &\le 4.4190
\end{aligned}$$

The solution region is $\{p \mid p \le 4.4190\}$.

Express each logarithm in terms of common logarithms. Then approximate its value to the nearest ten-thousandth.

12. $\log_3 7$

SOLUTION:

$$\begin{aligned}
\log_3 7 &= \frac{\log 7}{\log 3} \\
&\approx 1.7712
\end{aligned}$$

13. $\log_4 23$

SOLUTION:

$$\begin{aligned}
\log_4 23 &= \frac{\log 23}{\log 4} \\
&\approx 2.2618
\end{aligned}$$

14. $\log_9 13$

SOLUTION:

$$\begin{aligned}
\log_9 13 &= \frac{\log 13}{\log 9} \\
&\approx 1.1674
\end{aligned}$$

15. $\log_2 5$

SOLUTION:

$$\begin{aligned}
\log_2 5 &= \frac{\log 5}{\log 2} \\
&\approx 2.3219
\end{aligned}$$

Use a calculator to evaluate each expression to the nearest ten-thousandth.

16. log 3

SOLUTION:

KEYSTROKES: LOG 3 ENTER 0.4771212547

$\log 3 \approx 0.4771$

17. log 11

SOLUTION:

KEYSTROKES: LOG 11 ENTER 1.041392685

$\log 11 \approx 1.0414$

18. log 3.2

SOLUTION:

KEYSTROKES: LOG 3 . 2 ENTER 0.50514998

$\log 3.2 \approx 0.5051$

19. log 8.2

SOLUTION:

KEYSTROKES: LOG 8 . 2 ENTER 0.913813852

$\log 8.2 \approx 0.9138$

20. log 0.9

SOLUTION:

KEYSTROKES: LOG 0 . 9 ENTER –0.045757491

$$\log 0.9 \approx -0.0458$$

21. log 0.04

SOLUTION:

KEYSTROKES: LOG 0 . 0 4 ENTER –1.39794001

$$\log 0.04 \approx -1.3979$$

22. **CCSS SENSE-MAKING** Loretta had a new muffler installed on her car. The noise level of the engine dropped from 85 decibels to 73 decibels.

a. How many times the minimum intensity of sound detectable by the human ear was the car with the old muffler, if *m* is defined to be 1?

b. How many times the minimum intensity of sound detectable by the human ear was the car with the new muffler? Find the percent of decrease of the intensity of the sound with the new muffler.

SOLUTION:

a.

$$\begin{aligned} 85 &= 10\log\frac{I}{1} \\ 8.5 &= \log I \\ I &= 10^{8.5} \\ &\approx 316{,}227{,}766 \end{aligned}$$

The old muffler was about 316 million times louder than the minimum intensity detectable by the human ear.

b.

$$\begin{aligned} 73 &= 10\log\frac{I}{1} \\ 7.3 &= \log I \\ I &= 10^{7.3} \\ &\approx 19{,}952{,}623 \end{aligned}$$

The new muffler is about 20 million times louder than the minimum intensity detectable by the human ear.

$$\frac{19{,}952{,}623}{316{,}227{,}766} \approx 0.063$$

The percent of decrease is about $100 - 6.3 = 93.7\%$.

Solve each equation. Round to the nearest ten-thousandth.

23. $8^x = 40$

SOLUTION:

$$\begin{aligned} 8^x &= 40 \\ \log 8^x &= \log 40 \\ x \log 8 &= \log 40 \\ x &= \frac{\log 40}{\log 8} \\ &\approx 1.7740 \end{aligned}$$

The solution is about 1.7740.

24. $5^x = 55$

SOLUTION:

$$\begin{aligned} 5^x &= 55 \\ \log 5^x &= \log 55 \\ x \log 5 &= \log 55 \\ x &= \frac{\log 55}{\log 5} \\ &\approx 2.4899 \end{aligned}$$

The solution is about 2.4899.

25. $2.9^{a-4} = 8.1$

SOLUTION:

$$\begin{aligned} 2.9^{a-4} &= 8.1 \\ \log 2.9^{a-4} &= \log 8.1 \\ (a-4)\log 2.9 &= \log 8.1 \\ a-4 &= \frac{\log 8.1}{\log 2.9} \\ a &= \frac{\log 8.1}{\log 2.9} + 4 \\ &\approx 5.9647 \end{aligned}$$

The solution is about 5.9647.

26. $9^{b-1} = 7^b$

SOLUTION:

$$\begin{aligned} 9^{b-1} &= 7^b \\ \log 9^{b-1} &= \log 7^b \\ (b-1)\log 9 &= b \log 7 \\ \frac{b-1}{b} &= \frac{\log 7}{\log 9} \\ 1 - \frac{1}{b} &= \frac{\log 7}{\log 9} \\ \frac{1}{b} &= 1 - \frac{\log 7}{\log 9} \\ b &= \frac{1}{1 - \frac{\log 7}{\log 9}} \\ &\approx 8.7429 \end{aligned}$$

The solution is about 8.7429.

27. $13^{x^2} = 33.3$

SOLUTION:

$$\begin{aligned} 13^{x^2} &= 33.3 \\ \log 13^{x^2} &= \log 33.3 \\ x^2 \log 13 &= \log 33.3 \\ x^2 &= \frac{\log 33.3}{\log 33} \\ x &= \pm\sqrt{\frac{\log 33.3}{\log 13}} \\ &\approx \pm 1.1691 \end{aligned}$$

The solution is about 1.1691.

28. $15^{x^2} = 110$

SOLUTION:

$$\begin{aligned} 15^{x^2} &= 110 \\ \log 15^{x^2} &= \log 110 \\ x^2 \log 15 &= \log 110 \\ x^2 &= \frac{\log 110}{\log 15} \\ x &= \pm\sqrt{\frac{\log 110}{\log 15}} \\ &\approx \pm 1.3175 \end{aligned}$$

The solution is about ± 1.3175.

Solve each inequality. Round to the nearest ten-thousandth.

29. $6^{3n} > 36$

SOLUTION:

$$\begin{aligned} 6^{3n} &> 36 \\ \log 6^{3n} &> \log 36 \\ 3n \log 6 &> \log 36 \\ n &> \frac{\log 36}{3 \log 6} \\ &> 0.6667 \end{aligned}$$

The solution region is $\{n \mid n > 0.6667\}$.

30. $2^{4x} \le 20$

SOLUTION:

$$\begin{aligned} 2^{4x} &\le 20 \\ \log 2^{4x} &\le \log 20 \\ 4x \log 2 &\le \log 20 \\ 4x &\le \frac{\log 20}{\log 2} \\ x &\le \frac{\log 20}{4 \log 2} \\ &\le 1.0805 \end{aligned}$$

The solution region is $\{x \mid x \le 1.0805\}$.

31. $3^{y-1} \le 4^y$

SOLUTION:

$$\begin{aligned} 3^{y-1} &\le 4^y \\ \log 3^{y-1} &\le \log 4^y \\ (y-1)\log 3 &\le y \log 4 \\ \frac{y-1}{y} &\le \frac{\log 4}{\log 3} \\ 1 - \frac{1}{y} &\le \frac{\log 4}{\log 3} \\ \frac{1}{y} &\ge 1 - \frac{\log 4}{\log 3} \\ y &\ge \frac{1}{1 - \frac{\log 4}{\log 3}} \\ &\ge -3.8188 \end{aligned}$$

The solution region is $\{y \mid y \ge -3.8188\}$.

32. $5^{p-2} \ge 2^p$

SOLUTION:

$$\begin{aligned} 5^{p-2} &\ge 2^p \\ \log 5^{p-2} &\ge \log 2^p \\ (p-2)\log 5 &\ge p \log 2 \\ \frac{p-2}{p} &\ge \frac{\log 2}{\log 5} \\ 1 - \frac{2}{p} &\ge \frac{\log 2}{\log 5} \\ \frac{2}{p} &\le 1 - \frac{\log 2}{\log 5} \\ p &\ge \frac{2}{1 - \frac{\log 2}{\log 5}} \\ &\ge 3.5129 \end{aligned}$$

The solution region is $\{p \mid p \ge 3.5129\}$.

Express each logarithm in terms of common logarithms. Then approximate its value to the nearest ten-thousandth.

33. $\log_7 18$

SOLUTION:

$$\log_7 8 = \frac{\log 18}{\log 7} \approx 1.4854$$

34. $\log_5 31$

SOLUTION:

$$\log_5 31 = \frac{\log 31}{\log 5} \approx 2.1337$$

35. $\log_2 16$

SOLUTION:

$$\log_2 16 = \frac{\log 16}{\log 2} = 4$$

36. $\log_4 9$

SOLUTION:

$$\log_4 9 = \frac{\log 9}{\log 4} \approx 1.5850$$

37. $\log_3 11$

SOLUTION:

$$\log_3 11 = \frac{\log 11}{\log 3} \approx 2.1827$$

38. $\log_6 33$

SOLUTION:

$$\log_6 33 = \frac{\log 33}{\log 6} \approx 1.9514$$

39. **PETS** The number n of pet owners in thousands after t years can be modeled by $n = 35[\log_4 (t + 2)]$. Let $t = 0$ represent 2000. Use the Change of Base Formula to solve the following questions.
a. How many pet owners were there in 2010?
b. How long until there are 80,000 pet owners? When will this occur?

SOLUTION:

a. The value of t at 2010 is 10.
Substitute 10 for t in the equation and evaluate.

$$\begin{aligned} n &= 35\log_4(t+2) \\ &= 35\log_4(10+2) \\ &= 35\log_4 12 \\ &= 35\frac{\log 12}{\log 4} \\ &\approx 62.737 \end{aligned}$$

There will be 62,737 pet owners in 2010.

b. Substitute 80 for n and solve for t.

$$\begin{aligned} n &= 35\log_4(t+2) \\ 80 &= 35\log_4(t+2) \\ \frac{80}{35} &= \log_4(t+2) \\ \frac{80}{35} &= \frac{\log(t+2)}{\log 4} \\ \frac{80}{35}\log 4 &= \log(t+2) \\ t+2 &= 10^{\frac{80}{35}\log 4} \\ t &= 10^{\frac{80}{35}\log 4} - 2 \\ &\approx 22 \end{aligned}$$

In 2022, there will be 80,000 pet owners.

40. **CCSS PRECISION** Five years ago the grizzly bear population in a certain national park was 325. Today it is 450. Studies show that the park can support a population of 750.

a. What is the average annual rate of growth in the population if the grizzly bears reproduce once a year?

b. How many years will it take to reach the maximum population if the population growth continues at the same average rate?

SOLUTION:

a. Substitute 325, 450 and 5 for a, $A(t)$ and t in the equation $A(t) = a(1 + r)^t$.

$$450 = 325(1+r)^5$$

Solve for r.

$$\begin{aligned} 450 &= 325(1+r)^5 \\ \frac{450}{325} &= (1+r)^5 \\ \sqrt[5]{\frac{450}{325}} &= (1+r) \\ \sqrt[5]{\frac{450}{325}} - 1 &= r \\ r &\approx 0.067 \end{aligned}$$

The average annual rate is about 0.067 or 6.7%.

b. Substitute 750 for $A(t)$ and evaluate.

$$\begin{aligned} 750 &= 325(1+0.067)^t \\ \frac{750}{325} &= 1.067^t \\ \log\frac{750}{325} &= \log 1.067^t \\ \log 750 - \log 325 &= t \log 1.067 \\ t &= \frac{\log 750 - \log 325}{\log 1.067} \\ &\approx 13 \end{aligned}$$

It will take 8 years to reach the maximum population.

Solve each equation or inequality. Round to the nearest ten-thousandth.

41. $3^x = 40$

SOLUTION:

$$\begin{aligned} 3^x &= 40 \\ \log 3^x &= \log 40 \\ x \log 3 &= \log 40 \\ x &= \frac{\log 40}{\log 3} \\ &\approx 3.3578 \end{aligned}$$

The solution is about 3.3578.

42. $5^{3p} = 15$

SOLUTION:

$$\begin{aligned} 5^{3p} &= 15 \\ \log 5^{3p} &= \log 15 \\ 3p \log 5 &= \log 15 \\ p &= \frac{\log 15}{3 \log 5} \\ &\approx 0.5609 \end{aligned}$$

The solution is about 0.5609.

43. $4^{n+2} = 14.5$

SOLUTION:

$$\begin{aligned} 4^{n+2} &= 14.5 \\ \log 4^{n+2} &= \log 14.5 \\ (n+2)\log 4 &= \log 14.5 \\ n \log 4 + 2\log 4 &= \log 14.5 \\ n \log 4 &= \log 14.5 - 2\log 4 \\ n &= \frac{\log 14.5 - 2\log 4}{\log 4} \\ n &\approx -0.0710 \end{aligned}$$

The solution is about –0.0710.

44. $8^{z-4} = 6.3$

SOLUTION:

$$\begin{aligned} 8^{z-4} &= 6.3 \\ \log 8^{z-4} &= \log 6.3 \\ (z-4)\log 8 &= \log 6.3 \\ z\log 8 - 4\log 8 &= \log 6.3 \\ z\log 8 &= \log 6.3 + 4\log 8 \\ z &= \frac{\log 6.3 + 4\log 8}{\log 8} \\ z &\approx 4.8851 \end{aligned}$$

The solution is about 4.8851.

45. $7.4^{n-3} = 32.5$

SOLUTION:

$$\begin{aligned} 7.4^{n-3} &= 32.5 \\ \log 7.4^{n-3} &= \log 32.5 \\ (n-3)\log 7.4 &= \log 32.5 \\ n\log 7.4 - 3\log 7.4 &= \log 32.5 \\ n &= \frac{\log 32.5 + 3\log 7.4}{\log 7.4} \\ n &\approx 4.7393 \end{aligned}$$

The solution is about 4.7393.

46. $3.1^{y-5} = 9.2$

SOLUTION:

$$\begin{aligned} 3.1^{y-5} &= 9.2 \\ \log 3.1^{y-5} &= \log 9.2 \\ (y-5)\log 3.1 &= \log 9.2 \\ y\log 3.1 - 5\log 3.1 &= \log 9.2 \\ y &= \frac{\log 9.2 + 5\log 3.1}{\log 3.1} \\ y &\approx 6.9615 \end{aligned}$$

The solution is about 6.9615.

47. $5^x \geq 42$

SOLUTION:

$$\begin{aligned} 5^x &\geq 42 \\ \log 5^x &\geq \log 42 \\ x\log 5 &\geq \log 42 \\ x &\geq \frac{\log 42}{\log 5} \\ x &\geq 2.3223 \end{aligned}$$

The solution region is $\{x \mid x \geq 2.3223\}$.

48. $9^{2a} < 120$

SOLUTION:

$$\begin{aligned} 9^{2a} &< 120 \\ \log 9^{2a} &< \log 120 \\ 2a\log 9 &< \log 120 \\ 2a &< \frac{\log 120}{\log 9} \\ a &< \frac{\log 120}{2\log 9} \\ a &< 1.0894 \end{aligned}$$

The solution region is $\{a \mid a < 1.0894\}$.

49. $3^{4x} \leq 72$

SOLUTION:

$$\begin{aligned} 3^{4x} &\leq 72 \\ \log 3^{4x} &\leq \log 72 \\ 4x\log 3 &\leq \log 72 \\ 4x &\leq \frac{\log 72}{\log 3} \\ x &\leq \frac{\log 72}{4\log 3} \\ x &\leq 0.9732 \end{aligned}$$

The solution region is $\{x \mid x \leq 0.9732\}$.

50. $7^{2n} > 52^{4n+3}$

SOLUTION:

$$\begin{aligned} 7^{2n} &> 52^{4n+3} \\ \log 7^{2n} &> \log 52^{4n+3} \\ 2n \log 7 &> (4n+3)\log 52 \\ 2n \log 7 &> 4n \log 52 + 3\log 52 \\ 2n\log 7 - 4n\log 52 &> 3\log 52 \\ n(2\log 7 - 4\log 52) &> 3\log 52 \end{aligned}$$

Note that (2 log 7 – 4 log 52) is a negative number, so when we divide both sides by this, we need to reverse the inequality.

$$\begin{aligned} n &< \frac{3\log 52}{2\log 7 - 4\log 52} \\ n &< -0.9950 \end{aligned}$$

The solution region is $\{n \mid n < -0.9950\}$.

51. $6^p \leq 13^{5-p}$

SOLUTION:

$$\begin{aligned} 6^p &\leq 13^{5-p} \\ \log 6^p &\leq \log 13^{5-p} \\ p\log 6 &\leq (5-p)\log 13 \\ p\log 6 &\leq 5\log 13 - p\log 13 \\ p\log 6 + p\log 13 &\leq 5\log 13 \\ p(\log 6 + \log 13) &\leq 5\log 13 \\ p &\leq \frac{5\log 13}{\log 6 + \log 13} \\ p &\leq 2.9437 \end{aligned}$$

The solution region is $\{p \mid p \leq 2.9437\}$.

52. $2^{y+3} \geq 8^{3y}$

SOLUTION:

$$\begin{aligned} 2^{y+3} &\geq 8^{3y} \\ \log 2^{y+3} &\geq \log 8^{3y} \\ (y+3)\log 2 &\geq 3y\log 8 \\ y\log 2 + 3\log 2 &\geq 3y\log 8 \\ y\log 2 - 3y\log 8 &\geq -3\log 2 \\ y(\log 2 - 3\log 8) &\geq -3\log 2 \\ y &\leq \frac{-3\log 2}{\log 2 - 3\log 8} \\ y &\leq 0.3750 \end{aligned}$$

The solution region is $\{y \mid y \leq 0.3750\}$.

Express each logarithm in terms of common logarithms. Then approximate its value to the nearest ten-thousandth.

53. $\log_4 12$

SOLUTION:

$$\begin{aligned} \log_4 12 &= \frac{\log 12}{\log 4} \\ &\approx 1.7925 \end{aligned}$$

54. $\log_3 21$

SOLUTION:

$$\begin{aligned} \log_3 21 &= \frac{\log 21}{\log 3} \\ &\approx 2.7712 \end{aligned}$$

55. $\log_8 2$

SOLUTION:

$$\begin{aligned} \log_8 2 &= \frac{\log 2}{\log 8} \\ &\approx 0.3333 \end{aligned}$$

56. $\log_6 7$

SOLUTION:

$$\begin{aligned} \log_6 7 &= \frac{\log 7}{\log 6} \\ &\approx 1.0860 \end{aligned}$$

57. $\log_5 (2.7)^2$

SOLUTION:

$$\log_5 7.29 = \frac{\log 7.29}{\log 5} \approx 1.2343$$

58. $\log_7 \sqrt{5}$

SOLUTION:

$$\log_7 \sqrt{5} = \frac{\log \sqrt{5}}{\log 7} \approx \frac{\log 2.236}{\log 7} \approx 0.4135$$

59. **MUSIC** A musical cent is a unit in a logarithmic scale of relative pitch or intervals. One octave is equal to 1200 cents. The formula $n = 1200\left(\log_2 \frac{a}{b}\right)$ can be used to determine the difference in cents between two notes with frequencies a and b.

a. Find the interval in cents when the frequency changes from 443 Hertz (Hz) to 415 Hz.

b. If the interval is 55 cents and the beginning frequency is 225 Hz, find the final frequency.

SOLUTION:

a. Substitute 443 and 415for a and b then evaluate.

$$n = 1200\left(\log_2 \frac{443}{415}\right) \approx 113.03$$

The interval is 113.03 cents.

b. Substitute 55 and 225 for n and a then solve for b.

$$\begin{aligned} 55 &= 1200\left(\log_2 \frac{225}{b}\right) \\ \frac{55}{1200} &= \log_2 \frac{225}{b} \\ 2^{\frac{55}{1200}} &= \frac{225}{b} \\ b &= \frac{225}{2^{\frac{55}{1200}}} \\ &\approx 218 \end{aligned}$$

The final frequency is 218.

Solve each equation. Round to the nearest ten-thousandth.

60. $10^{x^2} = 60$

SOLUTION:

$$\begin{aligned} 10x^2 &= 60 \\ \log 10^{x^2} &= \log 60 \\ x^2 \log 10 &= \log 60 \\ x^2 &= \frac{\log 60}{\log 10} \\ x &= \pm\sqrt{\frac{\log 60}{\log 10}} \\ &= \pm 1.3335 \end{aligned}$$

The solution is about ± 1.3335.

61. $4^{x^2-3} = 16$

SOLUTION:

$$\begin{aligned} 4^{x^2-3} &= 16 \\ \log 4^{x^2-3} &= \log 16 \\ (x^2-3)\log 4 &= \log 16 \\ x^2 - 3 &= \frac{\log 16}{\log 4} \\ x^2 - 3 &= 2 \\ x^2 &= 5 \\ x &= \pm\sqrt{5} \approx \pm 2.2361 \end{aligned}$$

The solutions are $\pm\sqrt{5} \approx \pm 2.2361$.

62. $9^{6y-2} = 3^{3y+1}$

SOLUTION:

$$\begin{aligned} 9^{6y-2} &= 3^{3y+1} \\ \log 9^{6y-2} &= \log 3^{3y+1} \\ (6y-2)\log 9 &= (3y+1)\log 3 \\ 3y+1 &= (6y-2)\frac{\log 9}{\log 3} \\ &= (6y-2)2 \\ 3y+1 &= 12y-4 \\ 9y &= 5 \\ y &= \frac{5}{9} \end{aligned}$$

The solution is about 0.5556.

63. $8^{2x-4} = 4^{x+1}$

SOLUTION:

$$\begin{aligned} 8^{2x-4} &= 4^{x+1} \\ \log 8^{2x-4} &= \log 4^{x+1} \\ (2x-4)\log 8 &= (x+1)\log 4 \\ (2x-4)\frac{\log 8}{\log 4} &= (x+1) \\ (2x-4)\frac{3}{2} &= (x+1) \\ 3(2x-4) &= 2(x+1) \\ 6x-12 &= 2x+2 \\ 4x &= 14 \\ x &= 3.5 \end{aligned}$$

The solution is 3.5.

64. $16^x = \sqrt{4^{x+3}}$

SOLUTION:

$$\begin{aligned} 16^x &= \sqrt{4^{x+3}} \\ \log 16^x &= \log\sqrt{4^{x+3}} \\ x\log 16 &= \log 4^{\frac{x+3}{2}} \\ x\log 16 &= \frac{x+3}{2}\log 4 \\ x\frac{\log 16}{\log 4} &= \frac{x+3}{2} \\ 2x &= \frac{x+3}{2} \\ 4x &= x+3 \\ 3x &= 3 \\ x &= 1 \end{aligned}$$

The solution is 1.

65. $2^y = \sqrt{3^{y-1}}$

SOLUTION:

$$\begin{aligned} 2^y &= \sqrt{3^{y-1}} \\ \log 2^y &= \log\left(3^{y-1}\right)^{0.5} \\ y\log 2 &= (0.5y - 0.5)\log 3 \\ y\log 2 &= 0.5y\log 3 - 0.5\log 3 \\ y\log 2 - 0.5y\log 3 &= -0.5\log 3 \\ y(\log 2 - 0.5\log 3) &= -0.5\log 3 \\ y &= \frac{-0.5\log 3}{\log 2 - 0.5\log 3} \\ y &\approx -3.8188 \end{aligned}$$

The solution is about –3.8188.

66. **ENVIRONMENTAL SCIENCE** An environmental engineer is testing drinking water wells in coastal communities for pollution, specifically unsafe levels of arsenic. The safe standard for arsenic is 0.025 parts per million (ppm). Also, the pH of the arsenic level should be less than 9.5. The formula for hydrogen ion concentration is $pH = -\log H$. (*Hint*: 1 kilogram of water occupies approximately 1 liter. 1 ppm = 1 mg/kg.)

a. Suppose the hydrogen ion concentration of a well is 1.25×10^{-11}. Should the environmental engineer be worried about too high an arsenic content?

b. The environmental engineer finds 1 milligram of arsenic in a 3 liter sample, is the well safe?

c. What is the hydrogen ion concentration that meets the troublesome pH level of 9.5?

SOLUTION:

a. Substitute 1.25×10^{-11} for *H* and evaluate.

$$\begin{aligned} pH &= -\log H \\ &= -\log\left(1.25\times 10^{-11}\right) \\ &\approx 10.9 \end{aligned}$$

Yes. The environmental engineer be worried about too high n arsenic content, since 10.9 > 9.5.

b. 1 milligram of arsenic in a 3 liter sample is $\frac{1}{3}$ ppm.

Substitute $\frac{1}{3}$ for *H* and evaluate.

$$\begin{aligned} pH &= -\log\frac{1}{3} \\ &\approx 0.4771 \end{aligned}$$

Since 0.4771 > 0.025, the well is not safe.

c. Substitute 9.5 for pH and solve for *H*.

$$\begin{aligned} 9.5 &= -\log H \\ -9.5 &= \log H \\ H &= 10^{-9.5} \\ &\approx 3.16\times 10^{-10} \end{aligned}$$

The hydrogen ion concentration is 3.16×10^{-10}.

67. **MULTIPLE REPRESENTATIONS** In this

problem, you will solve the exponential equation $4^x = 13$.

a. TABULAR Enter the function $y = 4^x$ into a graphing calculator, create a table of values for the function, and scroll through the table to find x when $y = 13$.

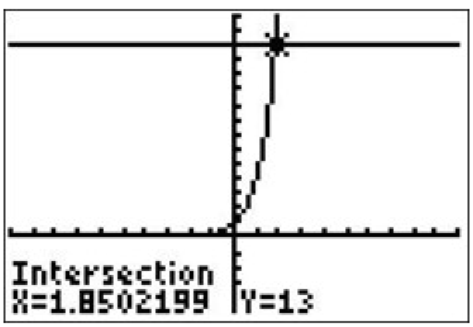
b. GRAPHICAL Graph $y = 4^x$ and $y = 13$ on the same screen. Use the intersect feature to find the point of intersection.

c. NUMERICAL Solve the equation algebraically. Do all of the methods produce the same result? Explain why or why not.

SOLUTION:

a. KEYSTROKES: Y= 4 ^ $\boxed{X,T,\theta,n}$ 2nd [TABLE]
Press ▼ button to scroll down.
The solution is between 1.8 and 1.9.

b. KEYSTROKES: Y= 4 ^ $\boxed{X,T,\theta,n}$ ENTER 1 3 GRAPH
Find the points of intersection.
KEYSTROKES: 2nd [CALC] 5
Press ENTER ENTER ENTER .



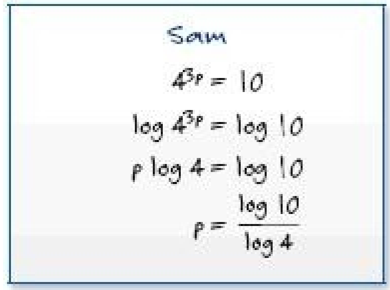
The solution is (1.85, 13).

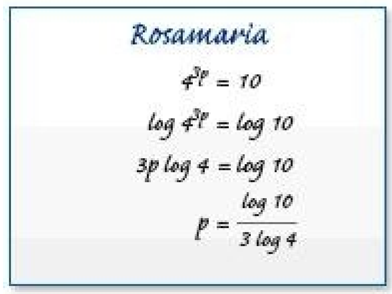
c.

$$\begin{aligned} 4^x &= 13 \\ \log 4^x &= \log 13 \\ x \log 4 &= \log 13 \\ x &= \frac{\log 13}{\log 4} \\ &\approx 1.85 \end{aligned}$$

Yes; all methods produce the solution of 1.85. They all should produce the same result because you are starting with the same equation. If they do not, then an error was made.

68. **CCSS CRITIQUE** Sam and Rosamaria are solving $4^{3p} = 10$. Is either of them correct? Explain your reasoning.





SOLUTION:
Rosamaria; Sam forgot to bring the 3 down from the exponent when he took the log of each side.

69. **CHALLENGE** Solve $\log_{\sqrt{a}} 3 = \log_a x$ for x and explain each step.

SOLUTION:

$\log_{\sqrt{a}} 3 = \log_a x$	Original equation
$\frac{\log_a 3}{\log_a \sqrt{a}} = \log_a x$	Change of Base Formula
$\frac{\log_a 3}{\frac{1}{2}} = \log_a x$	$\sqrt{a} = a^{\frac{1}{2}}$
$2 \log_a 3 = \log_a x$	Multiply numerator and denominator by 2.
$\log_a 3^2 = \log_a x$	Power Property of Logarithms
$3^2 = x$	Property of Equality for Logarithmic Functions
$9 = x$	Simplify

70. **REASONING** Write $\frac{\log_5 9}{\log_5 3}$ as a single logarithm.

SOLUTION:

$$\begin{aligned}\frac{\log_5 9}{\log_5 3} &= \frac{\frac{\log 9}{\log 5}}{\frac{\log 3}{\log 5}} && \text{Change of base formula}\\ &= \frac{\log 9}{\log 5}\cdot\frac{\log 5}{\log 3} && \text{Multiply by reciprocal of denominator.}\\ &= \frac{\log 9}{\log 3} && \text{Cancel common factor of } \log 5.\\ &= \log_3 9 && \text{Change of base formula}\end{aligned}$$

71. **PROOF** Find the values of $\log_3 27$ and $\log_{27} 3$. Make and prove a conjecture about the relationship between $\log_a b$ and $\log_b a$.

SOLUTION:

$\log_3 27 = 3$ and $\log_{27} 3 = \frac{1}{3}$

Conjecture: $\log_a b = \frac{1}{\log_b a}$

Proof: $\log_a b \stackrel{?}{=} \frac{1}{\log_b a}$

Original Statement

$$\frac{\log_b b}{\log_b a} \stackrel{?}{=} \frac{1}{\log_b a}$$

Change of Base Formula

$$\frac{1}{\log_b a} = \frac{1}{\log_b a}$$

Inverse Property of Exponents and Logarithms

72. **WRITING IN MATH** Explain how exponents and logarithms are related. Include examples like how to solve a logarithmic equation using exponents and how to solve an exponential equation using logarithms.

SOLUTION:

Logarithms are exponents. To solve logarithmic equations, write each side of the equation using exponents and solve by using the Inverse Property of Exponents and Logarithms. To solve exponential equations, use the Property of Equality for Logarithmic Functions and the Power Property of Logarithms.

73. Which expression represents $f[g(x)]$ if $f(x) = x^2 + 4x + 3$ and $g(x) = x - 5$?

A $x^2 + 4x - 2$

B $x^2 - 6x + 8$

C $x^2 - 9x + 23$

D $x^2 - 14x + 6$

SOLUTION:

$$\begin{aligned}f[g(x)] &= f(x-5)\\ &= (x-5)^2 + 4(x-5) + 3\\ &= x^2 - 10x + 25 + 4x - 20 + 3\\ &= x^2 - 6x + 8\end{aligned}$$

Option B is the correct answer.

74. **EXTENDED RESPONSE** Colleen rented 3 documentaries, 2 video games, and 2 movies. The charge was \$16.29. The next week, she rented 1 documentary, 3 video games, and 4 movies for a total charge of \$19.84. The third week she rented 2 documentaries, 1 video game, and 1 movie for a total charge of \$9.14.

a. Write a system of equations to determine the cost to rent each item.

b. What is the cost to rent each item?

SOLUTION:

a. Let d, v and m be the number of documentaries, video games and movies.
The system of equation represents this situation is:

$$\begin{aligned}3d + 2v + 2m &= 16.29\\ d + 3v + 4m &= 19.84\\ 2d + v + m &= 9.14\end{aligned}$$

b. The solution of the above system of equation are 1.99, 2.79 and 2.37.

75. **GEOMETRY** If the surface area of a cube is increased by a factor of 9, what is the change in the length of the sides if the cube?
F The length is 2 times the original length.
G The length is 3 times the original length.
H The length is 6 times the original length.
J The length is 9 times the original length.

SOLUTION:

If the surface area of a cube is increased by a factor of 9, the length is 3 times the original length. Therefore, option G is the correct answer.

76. **SAT/ACT** Which of the following *most* accurately describes the translation of the graph $y = (x + 4)^2 - 3$ to the graph of $y = (x - 1)^2 + 3$?
A down 1 and to the right 3
B down 6 and to the left 5
C up 1 and to the left 3
D up 1 and to the right 3
E up 6 and to the right 5

SOLUTION:

The graph move up 6 units and to the right 5. Option E is the correct answer.

Solve each equation. Check your solutions.

77. $\log_5 7 + \frac{1}{2}\log_5 4 = \log_5 x$

SOLUTION:

$$\begin{aligned}\log_5 7 + \frac{1}{2}\log_5 4 &= \log_5 x\\ \log_5 7 + \log_5 4^{\frac{1}{2}} &= \log_5 x\\ \log_5\left(7 \times 4^{\frac{1}{2}}\right) &= \log_5 x\\ x &= \left(7 \times 4^{\frac{1}{2}}\right)\\ &= 7 \times 2\\ x &= 14\end{aligned}$$

The solution is 14.

78. $2\log_2 x - \log_2(x+3) = 2$

SOLUTION:

$$\begin{aligned}2\log_2 x - \log_2(x+3) &= 2\\ \log_2 x^2 - \log_2(x+3) &= 2\\ \log_2\left(\frac{x^2}{x+3}\right) &= 2\\ 2^2 &= \frac{x^2}{x+3}\\ 4 &= \frac{x^2}{x+3}\\ 4x + 12 &= x^2\\ x^2 - 4x - 12 &= 0\\ (x-6)(x+2) &= 0\end{aligned}$$

By Zero Product Property:

$$\begin{aligned}x - 6 = 0 \text{ or } \quad & x + 3 = 0\\ x = 6 \quad \text{or} \quad & x = -3\end{aligned}$$

Logarithms are not defined for negative values. Therefore, the solution is 6.

79. $\log_6 48 - \log_6 \frac{16}{5} + \log_6 5 = \log_6 5x$

SOLUTION:

$$\begin{aligned}\log_6 48 - \log_6 \frac{16}{5} + \log_6 5 &= \log_6 5x\\ \log_6\left(\frac{48}{\frac{16}{5}} \cdot 5\right) &= \log_6 5x\\ \frac{48}{\frac{16}{5}} \cdot 5 &= 5x\\ x &= 15\end{aligned}$$

The solution is 15.

80. $\log_{10} a + \log_{10}(a+21) = 2$

SOLUTION:

$$\begin{aligned}\log_{10} a + \log_{10}(a+21) &= 2\\ \log_{10}(a(a+21)) &= 2\\ a(a+21) &= 10^2\\ a^2 + 21a - 100 &= 0\\ (a+25)(a-4) &= 0\end{aligned}$$

By the Zero Product Property:

$$\begin{aligned}&a - 4 = 0 \text{ or} \quad && a + 25 = 0\\ &a = 4 \quad \text{or} && a = -25\end{aligned}$$

Logarithms are not defined for negative values. Therefore, the solution is 4.

Solve each equation or inequality.

81. $\log_4 x = \frac{1}{2}$

SOLUTION:

$$\begin{aligned}\log_4 x &= \frac{1}{2}\\ x &= 4^{\frac{1}{2}}\\ x &= 2\end{aligned}$$

The solution is 2.

82. $\log_{81} 729 = x$

SOLUTION:

$$\begin{aligned}\log_{81} 729 &= x\\ \frac{\log_3 729}{\log_3 81} &= x\\ \frac{\log_3 3^6}{\log_3 3^4} &= x\\ \frac{6\log_3 3}{4\log_3 3} &= x\\ x &= \frac{3}{2}\end{aligned}$$

The solution is $\frac{3}{2}$.

83. $\log_8 (x^2 + x) = \log_8 12$

SOLUTION:

$$\begin{aligned}\log_8(x^2 + x) &= \log_8 12\\ x^2 + x &= 12\\ x^2 + x - 12 &= 0\\ (x+4)(x-3) &= 0\end{aligned}$$

By the Zero Product Property:

$$\begin{aligned}&x + 4 = 0 \text{ or} \quad && x - 3 = 0\\ &x = -4 \quad \text{or} && x = 3\end{aligned}$$

The solution is –4, 3.

84. $\log_8 (3y - 1) < \log_8 (y + 5)$

SOLUTION:

Logarithms are defined only for positive values. So, the argument should be greater than zero.

$$\begin{aligned}&3y - 1 > 0 \quad && \text{or} \quad && y + 5 > 0\\ &3y > 1 && \text{or} && y > -5\\ &y > \frac{1}{3} && \text{or} && y > -5\end{aligned}$$

Solve the original equation.

$$\begin{aligned}\log_8(3y-1) &< \log_8(y+5)\\ 3y - 1 &< y + 5\\ 2y &< 6\\ y &< 3\end{aligned}$$

The common region is the solution of the given inequality. Therefore, the solution region is $\frac{1}{3} < y < 3$.

85. **SAILING** The area of a triangular sail is $16x^4 - 60x^3 - 28x^2 + 56x - 32$ square meters. The base of the triangle is $x - 4$ meters. What is the height of the sail?

SOLUTION:

The area of a triangle is $A = \frac{1}{2}bh$.

Substitute $16x^4 - 60x^3 - 28x^2 + 56x - 32$ and $x - 4$ for A and b respectively.

$$16x^4 - 60x^3 - 28x^2 + 56x - 32 = \frac{1}{2}(x-4)h$$

Solve for h.

$$\begin{aligned}\frac{2\left(16x^4 - 60x^3 - 28x^2 + 56x - 32\right)}{x-4} &= h\\ \frac{32x^4 - 120x^3 - 56x^2 + 112x - 32}{x-4} &= h\\ \frac{\left(32x^3 + 8x^2 - 24x + 16\right)(x-4)}{x-4} &= h\\ 32x^3 + 8x^2 - 24x + 16 &= h\end{aligned}$$

The height of the sail is $32x^3 + 8x^2 - 24x + 16$.

86. **HOME REPAIR** Mr. Turner is getting new locks installed. The locksmith charges \$85 for the service call, \$25 for each door, and each lock costs \$30.
a. Write an equation that represents the cost for x number of doors.
b. Mr. Turner wants the front, side, back, and garage door locks changed. How much will this cost?

SOLUTION:

a.

$$\begin{aligned}y &= 85 + 25x + 30x\\ &= 85 + 55x\end{aligned}$$

b. Substitute 4 for x and evaluate.

$$\begin{aligned}y &= 85 + 55(4)\\ &= 85 + 220\\ &= 305\end{aligned}$$

This will cost \$305.

Write an equivalent exponential equation.

87. $\log_2 5 = x$

SOLUTION:

$$\begin{aligned}\log_2 5 &= x\\ 2^x &= 5\end{aligned}$$

88. $\log_4 x = 3$

SOLUTION:

$$\begin{aligned}\log_4 x &= 3\\ 4^3 &= x\end{aligned}$$

89. $\log_5 25 = 2$

SOLUTION:

$$\begin{aligned}\log_5 25 &= 2\\ 5^2 &= 25\end{aligned}$$

90. $\log_7 10 = x$

SOLUTION:

$$\begin{aligned}\log_7 10 &= x\\ 7^x &= 10\end{aligned}$$

7-6 Common Logarithms

91. $\log_6 x = 4$

SOLUTION:

$$\begin{aligned} \log_6 4 &= x \\ 6^x &= 4 \end{aligned}$$

92. $\log_4 64 = 3$

SOLUTION:

$$\begin{aligned} \log_4 64 &= 3 \\ 4^3 &= 64 \end{aligned}$$